Dynamics of Holographic Superfluids

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Physics in the City

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Outline of Talk

Bhaseen, Gauntlett, Simons, Sonner and Wiseman, *"Holographic Superfluids and the Dynamics of Symmetry Breaking"*

Phys. Rev. Lett. **110**, 015301 (2013)

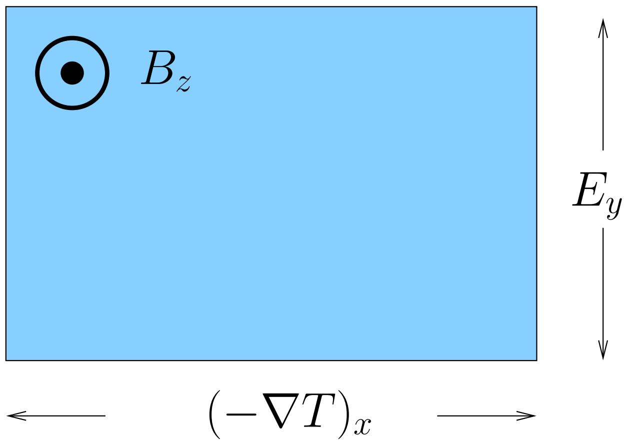
- Background, motivation and directions
- Exploiting holography in and out of equilibrium
- Far from equilibrium processes in correlated systems
- Quantum quenches
- Dynamical phase diagram for a BCS superconductor
- Different regimes of behavior depending on quench parameters
- Collective oscillations
- Non-equilibrium holographic superfluids
- Current status and future developments

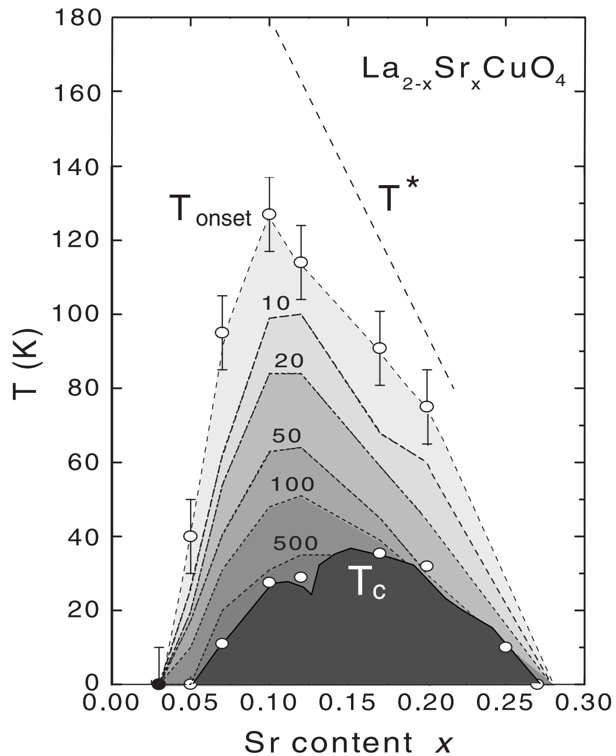
Nernst Effect in the Cuprates

Xu, Ong, Wang, Kakeshita and Uchida, Nature **406**, 486 (2000)

$$\nu \equiv \frac{E_y}{(-\nabla T)_x B}$$

$$\nu = \frac{1}{B} \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

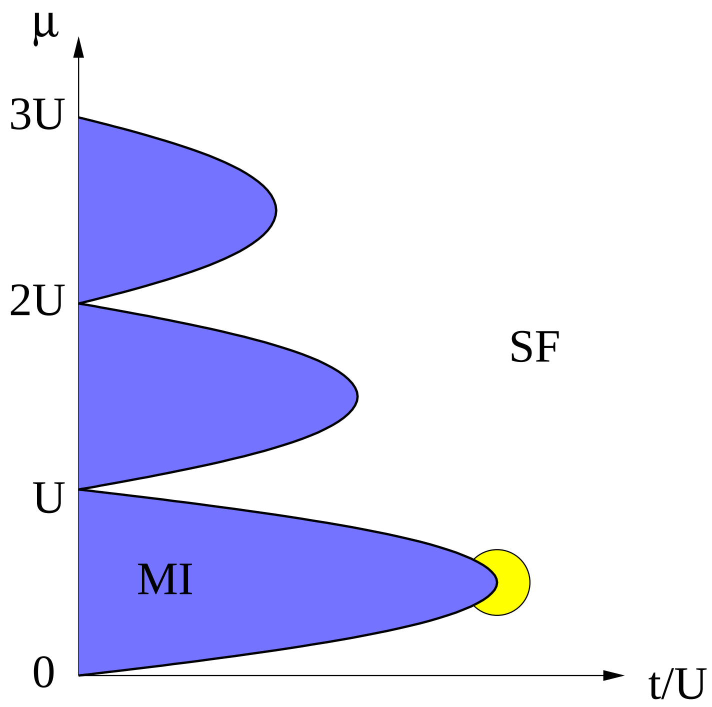




PRB **73**, 024510 (2010) Numbers indicate ν in nV/KT

The Bose–Hubbard Model

$$H = -t\sum_{\langle ij\rangle}(a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2}\sum_i n_i(n_i-1) - \mu\sum_i n_i$$



Fisher, Weichman, Grinstein and Fisher, PRB **40**, 546 (1989)

$$L = \int d^D x\, |\partial_\mu \Phi|^2 - m^2|\Phi|^2 - \frac{u_0}{3}|\Phi|^4$$

Quantum Boltzmann Equation

Real time & finite T Damle & Sachdev, PRB **56**, 8714 ('97)

Opposite charge particles + applied field + interactions $|\Phi|^4$

$$\left(\frac{\partial}{\partial t} \pm Q\mathbf{E}(t).\frac{\partial}{\partial \mathbf{k}}\right) f_\pm(\mathbf{k},t) = -\frac{2u_*^2}{9}\int d\mu\, \mathcal{F}_\pm$$

$$d\mu \equiv \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\frac{d^dk_3}{(2\pi)^d}\frac{1}{16\,\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k}_1}\varepsilon_{\mathbf{k}_2}\varepsilon_{\mathbf{k}_3}} \times$$
$$(2\pi)^d\delta^d(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)(2\pi)\delta(\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{k}_1}-\varepsilon_{\mathbf{k}_2}-\varepsilon_{\mathbf{k}_3}).$$

$$\begin{aligned}\mathcal{F}_\pm(\mathbf{k},\mathbf{k_1},\mathbf{k}_2,\mathbf{k}_3) \equiv\ & 2f_\pm(\mathbf{k})f_\mp(\mathbf{k}_1)[1+f_\pm(\mathbf{k}_2)][1+f_\mp(\mathbf{k}_3)] \\ & +f_\pm(\mathbf{k})f_\pm(\mathbf{k}_1)[1+f_\pm(\mathbf{k}_2)][1+f_\pm(\mathbf{k}_3)] \\ & -2[1+f_\pm(\mathbf{k})][1+f_\mp(\mathbf{k}_1)]f_\pm(\mathbf{k}_2)f_\mp(\mathbf{k}_3) \\ & -[1+f_\pm(\mathbf{k})][1+f_\pm(\mathbf{k}_1)]f_\pm(\mathbf{k}_2)f_\pm(\mathbf{k}_3)\end{aligned}$$

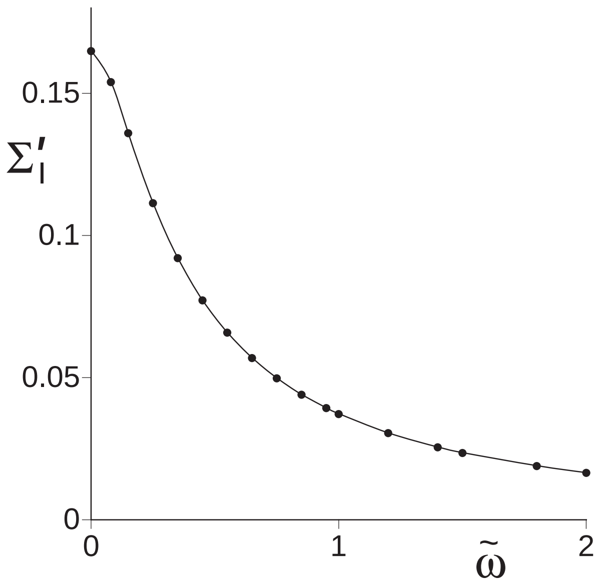
We have suppressed the t dependence

$\varepsilon_{\mathbf{k}} = \sqrt{\mathbf{k}^2+m^2}$ $\quad$ $\epsilon = 3-d$ $\quad$ $u_* = \frac{24\pi^2\epsilon}{5}$ $\quad$ $m^2 = \frac{4\epsilon T^2}{15}$

Universal Transport

Damle and Sachdev, PRB **56**, 8714 (1997)

$$\sigma(\omega) = \frac{(2e)^2 T^{d-2}}{\epsilon^2} \Sigma\left(\frac{\omega}{\epsilon^2 T}\right) \qquad d = 3 - \epsilon$$



Two spatial dimensions $\sigma(0) \approx 1.037 \left(\frac{4e^2}{h}\right)$

Crossover between hydrodynamic and collisionless regimes

Transport near Quantum Critical Points

Linear response for SF-MI transition in Bose–Hubbard

Damle and Sachdev, PRB **56**, 8714 (1997)

$$\sigma(\omega) = \frac{(2e)^2 T^{d-2}}{\epsilon^2} \Sigma\left(\frac{\omega}{\epsilon^2 T}\right) \qquad d = 3 - \epsilon \qquad \sigma(0) \approx 1.037 \left(\frac{4e^2}{h}\right)$$

Bhaseen, Green and Sondhi, PRL **98**, 166801 (2007)

$$\alpha_{xy} = \frac{\mathcal{S}}{B} \qquad \bar{\kappa}_{xx} \simeq g\epsilon^2 \frac{T^{d+3}}{(2e)^2 B^2} \qquad g \approx 5.5$$

Hartnoll, Kovtun, Müller and Sachdev, PRB **76**, 144502 (2007)

$$\bar{\kappa}_{xx}(B) = \frac{TS^2}{B^2 \sigma_{xx}(0)}$$

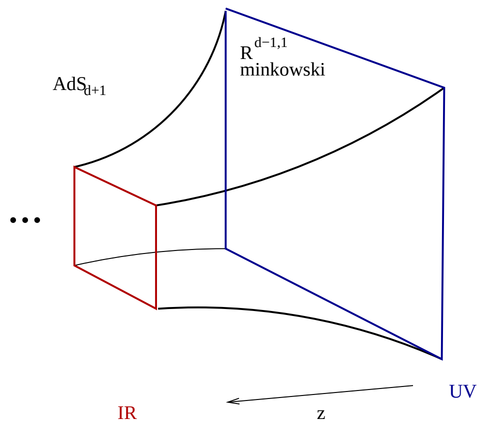
Relativistic hydrodynamics & AdS/CFT link all coeffs

QBE   Müller *et al*, PRB (2008)   Bhaseen *et al*, PRB (2009)

Viscosity/Entropy | **Quark Gluon Plasma** | **Graphene**

AdS/CFT Correspondence

For an overview see for example John McGreevy, *Holographic duality with a view toward many body physics*, arXiv:0909.0518



Generating function for correlation functions

$$Z[\phi_0]_{\rm CFT} \equiv \langle e^{-\int d\mathbf{x}dt\, \phi_0(\mathbf{x},t)\mathcal{O}(\mathbf{x},t)}\rangle_{\rm CFT}$$

Gubser–Klebanov–Polyakov–Witten

$$Z[\phi_0]_{\rm CFT} \simeq e^{-S_{\rm AdS}[\phi]}|_{\phi\sim\phi_0 \; at \; z=0}$$

$$\phi(z) \sim z^{d-\Delta}\phi_0(1+\ldots) + z^{\Delta}\phi_1(1+\ldots)$$

Fields in AdS $\leftrightarrow$ operators in dual CFT $\quad \phi \leftrightarrow \mathcal{O}$

Utility of Gauge-Gravity Duality

Quantum dynamics **Classical Einstein equations**

Finite temperature **Black holes**

Real time approach to finite temperature quantum dynamics in interacting systems, with the possibility of anchoring to $1+1$ and generalizing to higher dimensions

Non-Equilibrium **Beyond linear response**

Temporal dynamics in strongly correlated systems

Combine analytics with numerics

Dynamical phase diagrams

Organizing principles out of equilibrium

BCS Quench Dynamics

Barankov, Levitov, Spivak   Yuzbashyan, Tsyplyatyev, Altshuler

Time dependent BCS Hamiltonian

$$H = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}} a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda(t)}{2} \sum_{\mathbf{p},\mathbf{q}} a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow} a_{-\mathbf{q},\downarrow} a_{\mathbf{q},\uparrow}$$

Pairing interactions turned on abruptly

$$\lambda(t) = \lambda\theta(t)$$

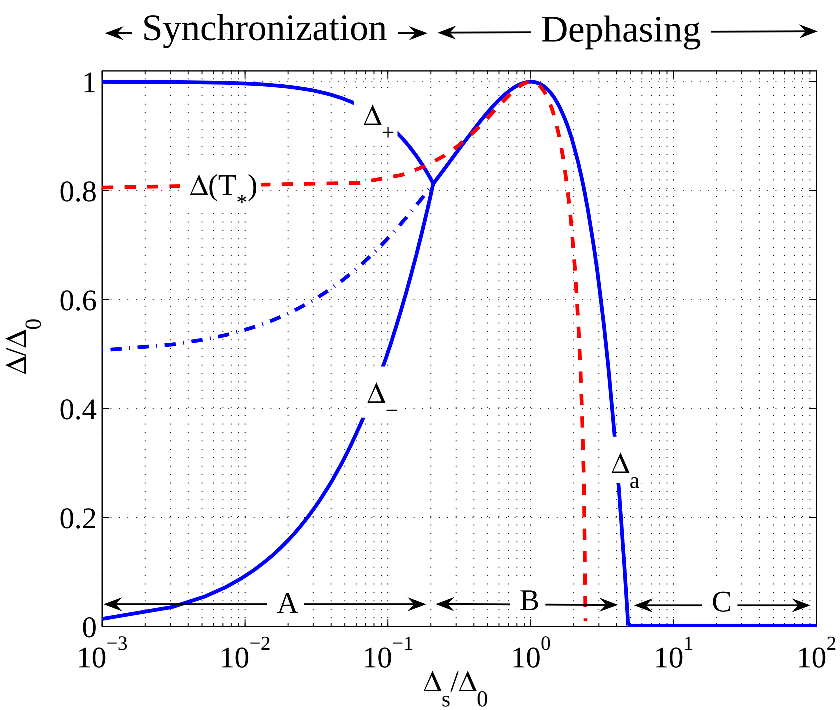
Generalized time-dependent many-body BCS state

$$|\Psi(t)\rangle = \prod_{\mathbf{p}} \left[u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow} \right] |0\rangle$$

Integrable

Regimes of BCS Quench Dynamics

Barankov and Levitov, *"Synchronization in the BCS Pairing Dynamics as a Critical Phenomenon"*, PRL **96**, 230403 (2006)



Initial pairing gap $\boxed{\Delta_s}$ Final pairing gap $\boxed{\Delta_0}$

(A) Oscillations between $\Delta_\pm$ (B) Underdamped approach to Δ_a

(C) Overdamped approach to $\Delta = 0$

Emergent temperature T^* and gap $\Delta(T^*)$

Holographic Superconductor

Gubser, *"Breaking an Abelian Gauge Symmetry Near a Black Hole Horizon"*, Phys. Rev. D. **78**, 065034 (2008)

Hartnoll, Herzog, Horowitz, *"Building a Holographic Superconductor"*, PRL **101**, 031601 (2008)

Abelian Higgs Coupled to Einstein–Hilbert Gravity

$$S = \int d^D x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F_{ab} F^{ab} - |D_a \psi|^2 - m^2 |\psi|^2 \right]$$

$$F_{ab} = \partial_a A_b - \partial_b A_a \quad D_a = \partial_a - iqA_a \quad \Lambda = -\tfrac{D(D-1)}{4L^2}$$

Finite temperature **Black hole**

Below critical temperature black hole with $\psi = 0$ unstable

Black hole with charged scalar hair $\psi \neq 0$ becomes stable

Spontaneous U(1) symmetry breaking

Non-Equilibrium AdS Superconductors

Murata, Kinoshita & Tanahashi, *"Non-Equilibrium Condensation Process in a Holographic Superconductor"*, JHEP 1007:050 (2010)

Start in unstable normal state described by a Reissner–Nordström black hole and perturb

Temporal dynamics from supercooled to superconducting

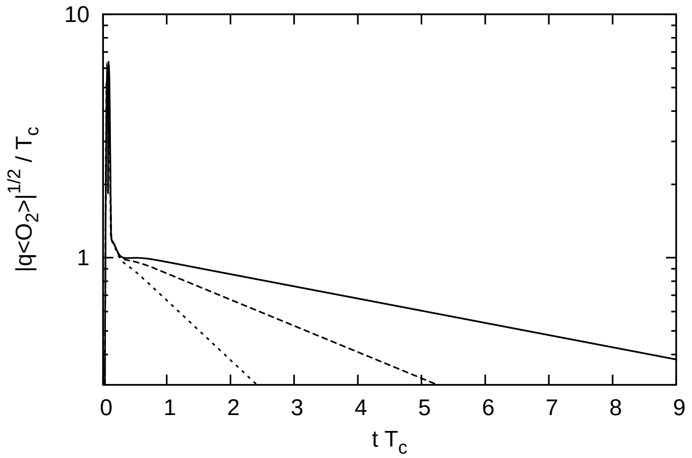
Time Evolution of the Order Parameter

Murata, Kinoshita & Tanahashi, JHEP 1007:050 (2010)

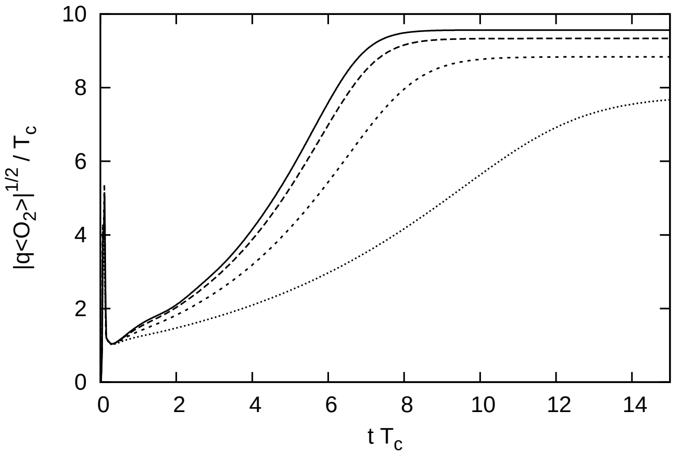
Start in normal state (Reissner–Nordström) & perturb

Numerical solution of Einstein equations

$$\langle \mathcal{O}_2(t) \rangle \equiv \sqrt{2}\psi_2(t)$$



$T/T_c = 1.1, 1.2, 1.4$



$T/T_c = 0.2, 0.4, 0.6, 0.8$

$T > T_c$ **Relaxation to N** $|\langle \mathcal{O}_2(t) \rangle| = C \exp(-t/t_{\text{relax}})$

$T < T_c$ **Evolution to SC** $|\langle \mathcal{O}_2(t) \rangle| = C_1 \exp(-t/t_{\text{relax}}) + C_2$

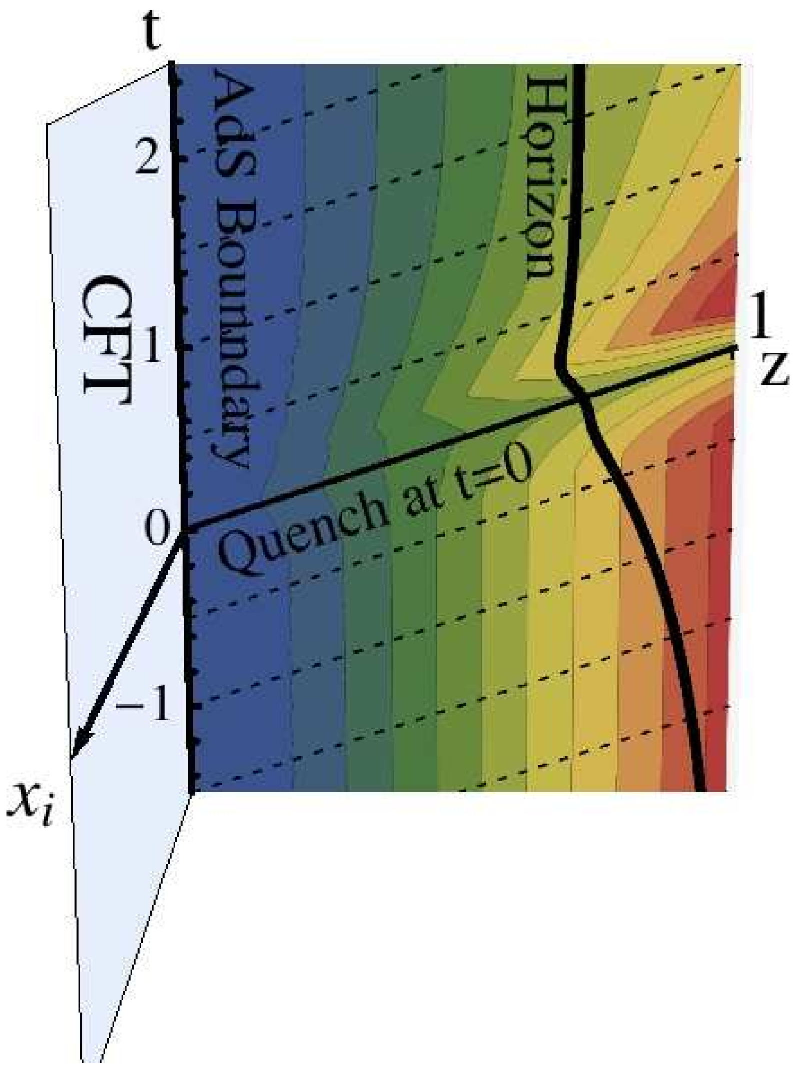
Questions

Where are the oscillations?

- Is there always damping or are there different regimes?
- Are the dynamics of holographic superconductors related to condensed matter systems or intrinsically different?
- What is the role of coupling to a large number of critical degrees of freedom?
- What is the role of large N and strong coupling?
- What is their influence on the emergent timescales?
- Do thermal fluctuations reduce the amplitude of oscillations?
- Beyond BCS and mean field dynamics using AdS/CFT?
- What is the dynamics at short, intermediate and long times?
- What happens if one quenches from a charged black hole?

AdS/CMT far from equilibrium?

Setup



Homogeneous isotropic dynamics of CFT from Einstein equations

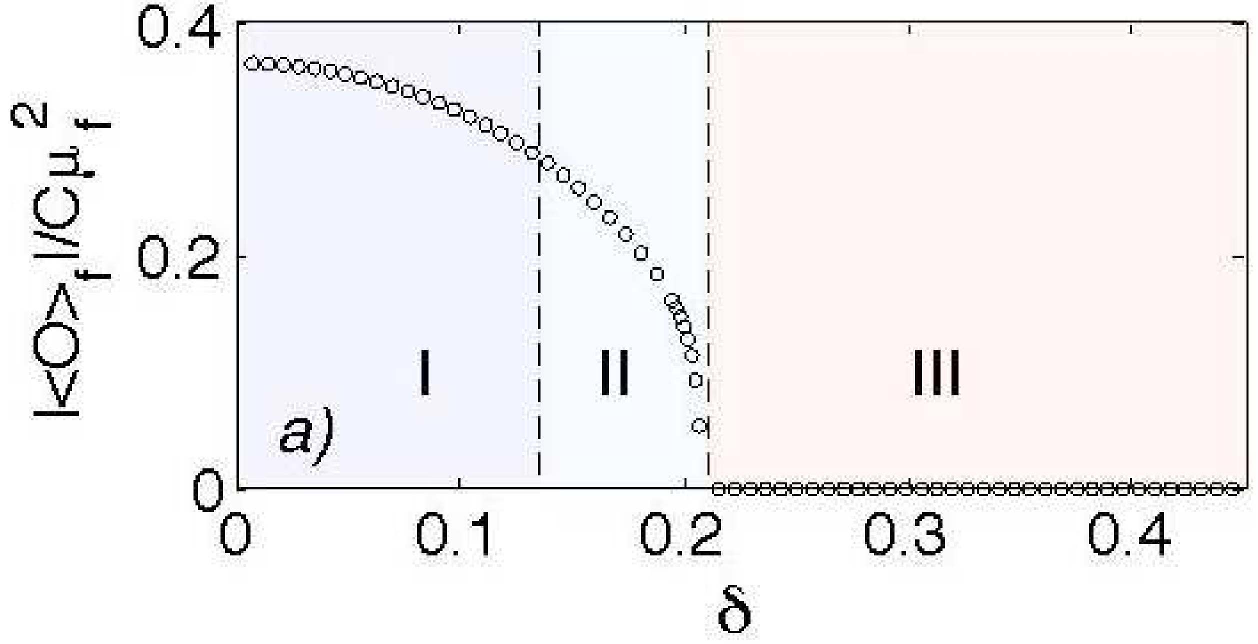
Dynamical Phase Diagram

Conjugate field pulse

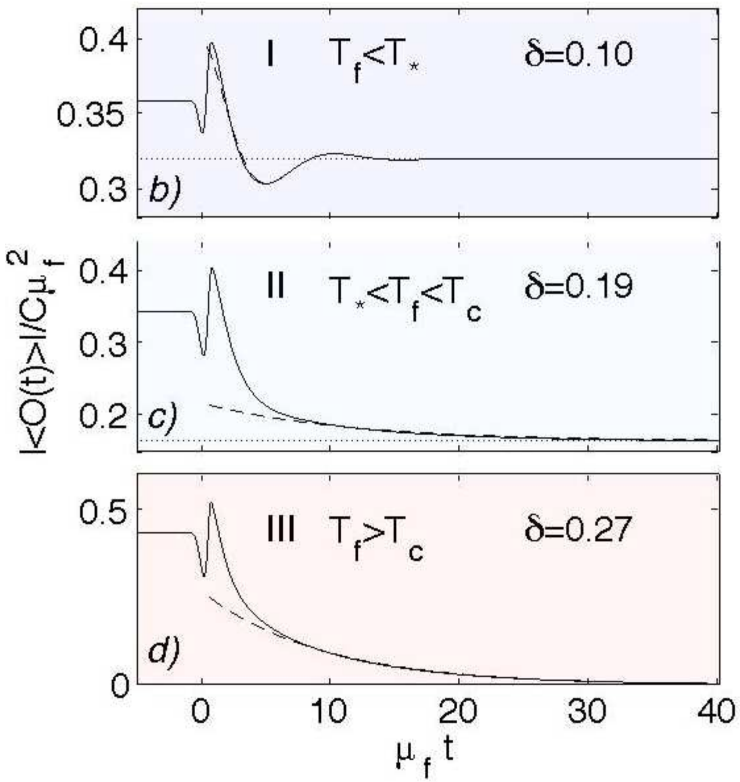
AdS_4/CFT_3 $\qquad \psi(t,z) = \psi_1(t)z + \psi_2(t)z^2 + \ldots$

$$\psi_1(t) = \bar{\delta} e^{-(t/\bar{\tau})^2}$$

$$\delta = \bar{\delta}/\mu_i \qquad \tau = \bar{\tau}/\mu_i \qquad \tau = 0.5 \qquad T_i = 0.5 T_c$$



Three Dynamical Regimes



(I) Damped Oscillatory to SC

$$\langle \mathcal{O}(t) \rangle \sim a + be^{-kt} \cos[l(t - t_0)]$$

(II) Over Damped to SC

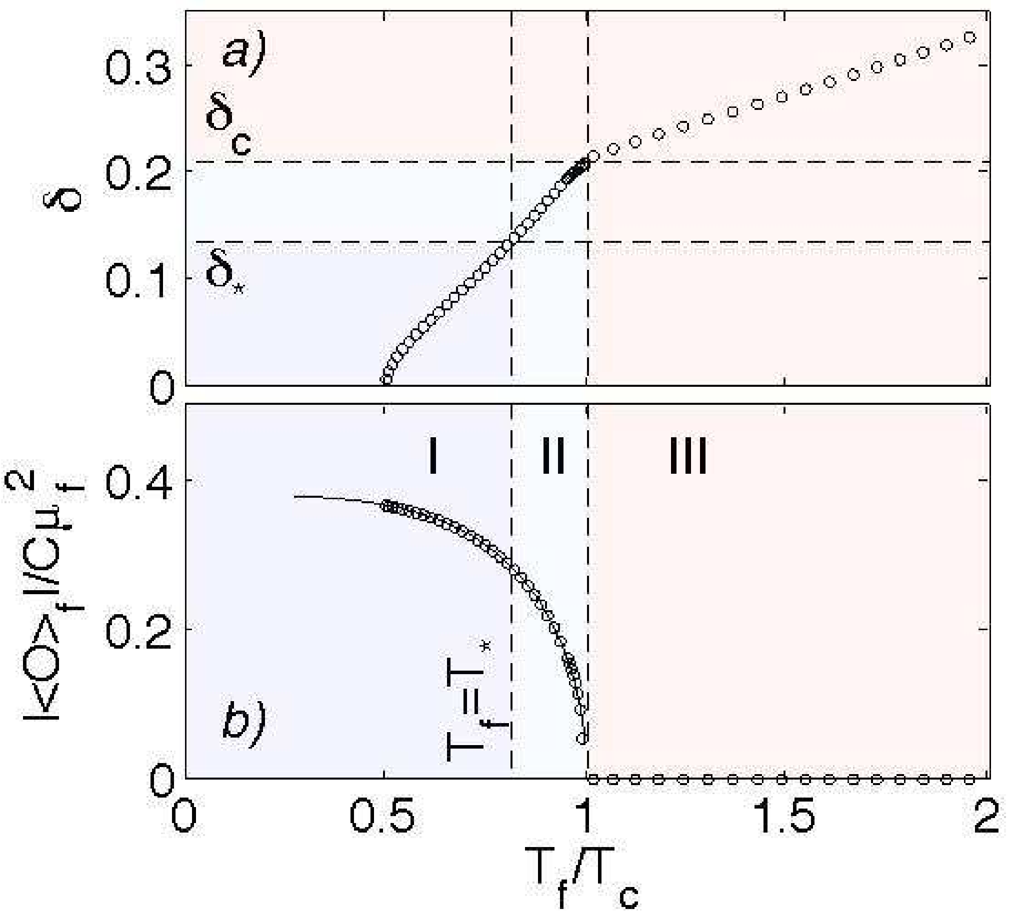
$$\langle \mathcal{O}(t) \rangle \sim a + be^{-kt}$$

(III) Over Damped to N

$$\langle \mathcal{O}(t) \rangle \sim be^{-kt}$$

Asymptotics described by black hole quasi normal modes

Approach to Thermal Equilibrium

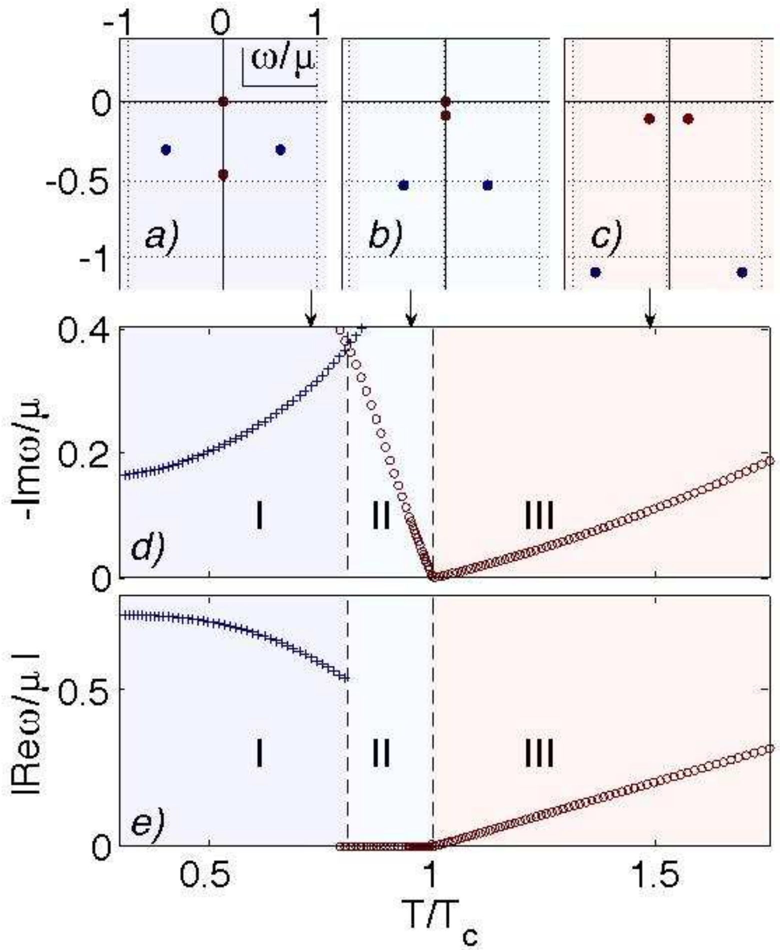


Collapse on to Equilibrium Phase Diagram

Emergent temperature scale T_* within superfluid phase

Quasi Normal Modes

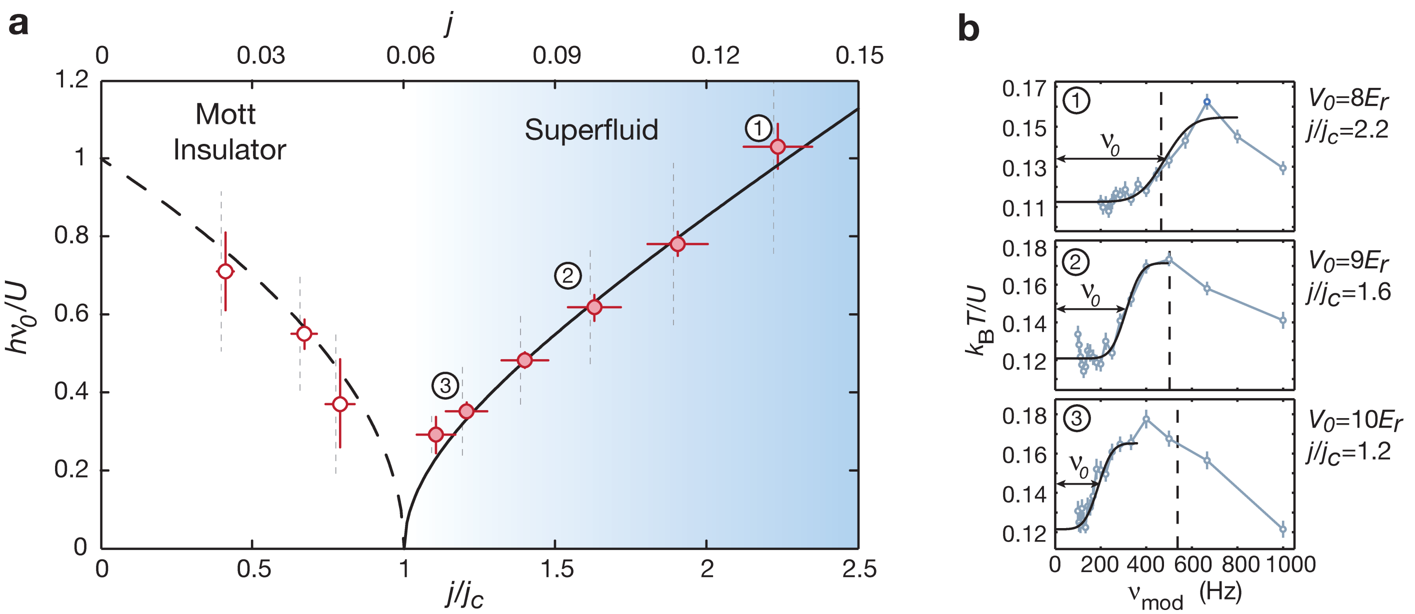
$$|\langle \mathcal{O}(t) \rangle| \simeq |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|$$



Three regimes and an emergent T_*

Recent Experiments

Endres *et al*, *"The 'Higgs' Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition"*, Nature **487**, 454 (2012)



Conclusions

- AdS/CFT is a potential tool for non-equilibrium dynamics
- Allows access to real time dynamics over entire time interval
- Amenable to both numerical and analytical treatments
- Emergent temperatures, black hole formation and equilibration
- AdS_3 allows contact with integrable systems in 1+1 dimensions
- Extensions to higher dimensional systems

Developing new tools and organizing principles

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