COURSE SPECIFICATION FORM
for new course proposals and course amendments

| Department/School: | Mathematics | Academic Session: | 2017-18 |
| :---: | :---: | :---: | :---: |
| Course Title: | Number Theory | Course Value: <br> (UG courses = unit value, <br> PG courses = notional learning hours <br> hours) | 0.5 unit |
| Course Code: | MT3110 | Course JACS Code: <br> (Please contact Data <br> Management for advice) | G100 |
| Availability: <br> (Please state which teaching <br> terms) | Term 2 | Status: | Optional Condonable |
| Pre-requisites: | MT1810 | Co-requisites: |  |
| Co-ordinator: |  |  |  |
| Course Staff: |  |  |  |
| Aims: | To acquaint students with some of the elementary tools used to analyse the additive and multiplicative structures of the set of integers. |  |  |
| Learning Outcomes: | On completion of this course students should: <br> - Be confident in handling congruences, including the use of the Chinese Remainder theorem, and the Fermat-Euler theorem; <br> - Be able to manipulate arithmetic functions such as $\mathrm{T}(\mathrm{n}), \mu(\mathrm{n}), \varphi(\mathrm{n})$ and $\mathrm{o}(\mathrm{n})$; and derive some of their basic properties; <br> - Be able to prove the existence of primitive roots modulo a prime and use them in solving certain congruences; <br> - Be able to test for quadratic residues, and use them to answer questions on primes in arithmetic progressions, and representing numbers as sums of two squares; <br> - Be able to find the continued fraction expansion of real numbers, in particular quadratic irrationals, and apply continued fractions to the solution of Pell's equation. |  |  |
| Course Content: | Introduction. Revision of material seen in previous years. Integers, primes, factorization, congruences including Chinese remainder theorem and the Fermat-Euler theorem. Arithmetic functions. Introduction to the functions $\mathrm{T}(\mathrm{n}), \mu(\mathrm{n}), \varphi(\mathrm{n})$ and $\mathrm{o}(\mathrm{n})$; product and summation form, Mobius inversion. <br> Primitive roots. The order of an integer modulo $p$, primitive roots modulo $p$, the proof of their existence, the index of an integer modulo $p$. <br> Quadratic residues. Legendre's symbol. Euler's criterion for a quadratic residue. Gauss's Lemma. The law of quadratic reciprocity. Applications to quadratic congruences and primes in arithmetic progressions. <br> Continued fractions. Definition. Diophantine approximation. Quadratic irrationals and Pell's equation. <br> Arithmetic functions II. The function $\mathrm{r}(\mathrm{n})$ - representing numbers as sums of two squares. |  |  |
| Teaching \& Learning Methods: | 33 hours of lectures and examples classes. <br> 117 hours of private study, including work on problem sheets and examination preparation. <br> This may include discussions with the course leader if the student wishes. |  |  |
| Key Bibliography: | Introduction to the Theory of Numbers - I Niven, H S Zuckerman and H L Montgomery. $5^{\text {th }}$ edition (Wiley 1991). Library Ref. 512.91 NIV <br> Elementary Number Theory in Nine Chapters - J JTattersall. (Cambridge 2005) Library Ref. 512.91 TAT |  |  |
| Formative Assessment \& Feedback: | Formative assignments in the form of 8 problem sheets. The students will receive feedback as written comments on their attempts. |  |  |
| Summative Assessment: | Exam: (100\%) A two hour paper. Coursework: None |  |  |

Updated September 2017
The information contained in this course outline is correct at the time of publication, but may be subject to change as part of the Department's policy of continuous improvement and development. Every effort will be made to notify you of any such changes.

