Course content for MT5491, Topology

Prerequisites:
UG real analysis

Aims:
To introduce students to the basic concepts of metric and topological spaces, and to some aspects of low-dimensional topology.

Learning outcomes:
1. Understand what it means for knots and links to be equivalent, understand the concept of a knot invariant, and be able to use some invariants to distinguish knots and links.
2. Understand the defining properties of a metric space, and determine whether a given function defines a metric; understand some basic concepts of metric spaces.
3. Understand the definition of a topological space, and be able to verify the axioms in examples.
4. Understand the concepts of subspace, product spaces, quotient spaces, Hausdorff space, homeomorphism, connectedness and compactness.
5. Understand the notions of Euler characteristic, orientability and apply these to classify closed surfaces.
6. Demonstrate independent learning skills

Course content:
Knot theory: knot and link diagrams, the Reidemeister moves, 3-colourings of knot diagrams, n-colourings of knot diagrams.
Metric spaces: definition of metric spaces; examples of metric spaces, open and closed sets, further topics may include: compactness, Cantor set, continuous maps.
Topological spaces: (motivated by properties of open sets in a metric space), examples of topological spaces, subspaces, connectivity, Hausdorff property, continuous functions, homeomorphisms, paths, path-connectedness, product topology, compactness, quotient spaces.
Surfaces: identification spaces, connected sums, orientability, triangulations, Euler characteristic, standard examples including the sphere, the cylinder, the torus, the Möbius band, the projective plane, and the Klein bottle; the classification of surfaces.
If time permits one or more of the following topics: homotopy, fixed point theorems, further knot theory