Course content for MT3880, Further Linear Algebra and Modules

Prerequisites:
MT2800 and MT2830

Aims:
The language and concepts of linear algebra are of widespread use within mathematics and also in the social and natural sciences. The purpose of the course is to cover some further topics in linear algebra and to provide an introduction to the theory of modules. If a matrix cannot be diagonalised, one uses alternative matrix representations, such as the Jordan canonical form or the rational canonical form. This course introduces these canonical forms, and aims to explain why they are important. To understand their importance, some of the basic theory of modules over a Euclidean domain is introduced. Roughly speaking, a module bears the same relationship to a vector space as a ring does to a field. Key aims of the course include to treat canonical forms of linear operators and to develop the basic theory of modules over a Euclidean ring, and to link concrete matrix manipulations to conceptual interpretations.

Learning outcomes:
1. determine the Jordan canonical form and rational canonical form of a suitable matrix;
2. compute the characteristic and the minimum polynomial of a square matrix and to draw conclusions regarding its canonical forms;
3. understand and apply the basic theory of modules over a Euclidean domain in its relation to matrix canonical forms.

Course content:
Linear algebra: revision of the matrix representation of a linear transformation and change of basis, Euclidean domains; elementary operations; generalised eigenspaces, Jordan canonical form for matrices with split characteristic polynomial. If time permits, Smith normal form.
Modules: modules, submodules and homomorphisms; annihilators, cyclic modules and direct sums; free modules; structure theorem for finitely generated modules over a Euclidean domain; primary decomposition.
Applications and further topics: connection between normal forms of matrices and module theory; companion matrix and extended companion matrix; rational canonical form and primary rational canonical form; minimal polynomials; further topics as time permits, for example the Cayley-Hamilton theorem.