Course content for MT3860, Groups and Group Actions

Prerequisites:

MT1810 and MT1820

Aims:

Learning outcomes:

- 1. understand and apply the fundamental concepts of group theory;
- 2. recognise and construct group homomorphisms and quotients;
- 3. know basic examples of groups and group actions;
- 4. count the number of orbits and determine their sizes in specific group actions;
- 5. apply the concept of a group action to count discrete patterns.

Course content:

Groups: cycle structure and sign of a permutation; symmetric and alternating groups; group axioms; subgroups; order of a group and of group elements; cosets and Lagrange's theorem; homomorphisms, normal subgroups, quotient groups; isomorphism theorems; key examples, such as: cyclic groups, dihedral groups (symmetries of regular polygons)

and symmetries of the Platonic solids, matrix groups.

Group actions: definition of a group action and connection with permutations; Cayley's theorem; orbits and stabilisers; the size of an orbit and the number of orbits; Burnside's lemma.

Applications such as: conjugacy classes and centralisers; counting problems concerning discrete patterns.

Further topics as time permits, such as: p-groups, Sylow theorems; simplicity of the alternating groups.