Course content for MT3280, Non-linear Dynamical Systems: Routes to Chaos

Prerequisites:

MT1720 and MT1820

Aims:

To introduce the fundamentals of the analysis of nonlinear dynamical systems and, in particular, to investigate whether the behaviour of a nonlinear system can be predicted from the corresponding linear system.

Learning outcomes:

On completion of the course, students should be able to:

- identify and classify the critical points for both discrete and continuous dynamical systems;
- understand when and why the direct and indirect Liapunov methods are appropriate and use them both;
- understand when a limit cycle can, and cannot, occur and prove the non-existence as appropriate;
- recognise the role of the linear system in predicting the long-term behaviour of the nonlinear system.

Course content:

Systems of first order linear differential equations: Similarity types for 2x2 matrices and their connection with linear systems. Classification of two-dimensional linear phase portraits. Extension to three dimensions.

Nonlinear differential equations: Liapunov's stability analysis, periodic solutions and limit cycles, Poincaré-Bendixson theorem. Applications to problems from physics, biology and economics.

Non-linear difference equations: Poincaré surface of section, stability of critical points, routes to chaos.