Course content for MT2940, Real Analysis

Prerequisites:
MT1940

Aims:

- To explain the rigorous definition of limit of a function of a positive integer variable;
- To discuss convergence of series, including power series;
- To discuss the concepts of continuity and differentiability of functions of a real variable *x*;
- To show how the Riemann integral is constructed.

Learning outcomes:
On completion of the course, students should be able to:

- quote the Weierstrass definition of a limit and verify it in simple cases;
- use standard tests to investigate the convergence of commonly occurring series;
- specify the power series of standard functions;
- understand the Intermediate Value Theorem and the Mean Value Theorems;
- understand the constructive approach of the Riemann integral.

Course content:
Sequences and series: Sequences that tend to a limit; absolute convergence of series; use of comparison and ratio tests for absolute convergence; absolute convergence implies convergence. Conditional convergence, alternating series test, rearrangement of an alternating series.
Differentiation: Formal definition of a limit of a function, with connection to continuity. The intermediate value theorem. Differentiability at a point – definition and geometric interpretation, with examples. Differentiability implies continuity. Derivative of a sum, product, quotient, and the chain rule (with application to inverse functions). Differentiability on an open interval; Rolle's theorem, Mean Value Theorem, Cauchy's Mean Value Theorem, with applications including l'Hôpital's rule. Taylor's theorem with (one) remainder.
Power series: Existence of radius of convergence, and use of ratio test to find it. Power series can be differentiated term-by-term within the circle of convergence. Formal definition and properties of exp, sin, cos, etc., and (using the inverse function) ln, arcsin etc. Periodicity of sin and cos.
Riemann integral: Upper and lower sums, leading to definition and properties of Riemann integral. Fundamental theorem of calculus. Integral test for convergence of series.