

Course content for MT2830, Rings and Factorisation

Prerequisites:

MT1810 and MT1820

Aims:

One of the most significant developments in 20th century mathematics was the influx of algebra into all parts of mathematics, including classical areas such as number theory. The course introduces and develops the elementary theory of rings. It shows how this theory can be applied to study the problem of factorising integers into primes and shows how this situation can be generalised in a natural way. The course places an emphasis on concrete examples, such as rings of integers and polynomial rings; it aims to develop the skill of connecting these concrete examples with general mathematical concepts, and to practice algebraic reasoning.

Learning outcomes:

On completion of the course, students should be able to:

- understand and apply the fundamental concepts of ring theory;
- know basic examples of rings;
- investigate the structure and detect properties of explicit rings;
- recognise and construct ring homomorphisms and quotients;
- know and apply key theorems such as Kronecker's theorem on field extensions, and the Chinese Remainder Theorem.

Course content:

Commutative ring theory: Rings and subrings; homomorphisms and ideals; factor rings; the first isomorphism theorem for rings and the correspondence theorem; prime ideals, maximal ideals and their use in constructing fields; Kronecker's theorem that every field can be extended to include a root of a given irreducible polynomial.

Factorisation, applications and further topics: Integers and polynomial rings; zero-divisors and group of units; irreducibles and factorisation; unique factorisation domains; principal ideal domains; the Gaussian integers as an example of a Euclidean ring; quotients of integers and polynomial rings, including their groups of units, via the Chinese Remainder Theorem; if time permits, suitable further topics, for instance, simple primality tests, applications to algebraic geometry, or the Hilbert basis theorem.