Course content for MT2800, Linear Algebra

Prerequisites:
MT1820

Aims:
To develop the matrix theory covered in MT1820

Learning outcomes:
On completion of the course, the student should be able to:

- understand rank and nullity of a linear map and the connection between them;
- calculate orthonormal bases;
- diagonalise real symmetric matrices;
- check positivity of a real symmetric matrix in various ways;
- apply the least squares method.

Course content:
Laplace expansion of determinants, minors and cofactors. Bases and change of basis. Linear maps and their matrix representations. Range and kernel. Row rank, column rank, rank and nullity. Sylvester's law of nullity. Diagonalisation of a diagonalisable matrix. Algebraic and geometric multiplicities. Introduction to Jordan normal form.
Abstract definition of an inner product. Orthonormal bases. Gram-Schmidt orthogonalisation procedure. Symmetric and positive definite matrices. Quadratic and positive definite forms. Bringing a quadratic form into canonical form. Testing positivity of a real symmetric matrix.
Change between orthonormal bases. Orthogonal matrices. Unitary matrices, hermitian matrices. Spectral theorem over the complex numbers. Singular value decomposition of real matrices.
Further topics may include: Cramer's rule. Block matrices. Sylvester's law of inertia. Least squares approximation. Moore-Penrose inverse.