Course content for MT1820, Matrix Algebra

Prerequisites:

Aims:
This course aims to give students a working knowledge of basic linear algebra, with an emphasis on an approach via matrices and vectors. The course introduces some of the basic theoretical and computational techniques of matrix theory, and illustrates them by examples.

Learning outcomes:
On completion of the course, students should be able to:
- appreciate the power of vector methods, use vector methods to describe three-dimensional space, and apply scalar and vector products of two vectors and triple products appropriately;
- understand the notions of field, vector space and subspace;
- calculate the determinant of an nxn matrix; calculate the inverse of a non-singular matrix;
- appreciate the significance of the characteristic polynomial of a matrix, compute the eigenvalues and eigenspaces of a matrix, and diagonalise it when possible;
- understand the notions of linear independence and dimension;
- reduce a matrix to row-reduced echelon form.

Course content:
Vectors in R^3: vectors as directed line segments; addition, scalar multiplication, parallelogram law. Dot product, length, distance, perpendicular vectors, angle between vectors, $u.v=|u|\,|v| \cos t$. Lines and planes in R^3. Cross product, area $|u \times v|$ and volume $|(u \times v).w|$.
2 × 2 matrices over a field: determinants, inverses, rotation and reflection matrices. Eigenvalues and eigenvectors, the characteristic polynomial and trace. Diagonalisation $P^{-1}AP$ with applications.
3 × 3 matrices: permutations, compositions, parity. Definition of the determinant of an nxn matrix, cofactors, row and column expansion, the adjugate matrix, inverse of a non-singular matrix. Examples of characteristic polynomial and diagonalisation of 3 × 3 matrices A. Statement of the properties of determinants, including $|AB| = |A|\,|B|$.
Vector spaces: axioms, linear independence, span, dimension. Subspaces.
Row-reduction: elementary row operations, echelon and row-reduced echelon form, rank of a matrix.
Applications: solution of systems of linear equations, deriving a basis from a spanning set, computing the inverse of a matrix.