Coauthorship and the Measurement of Individual Productivity

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Abstract. We propose a new index of individual scientific productivity that formally accounts for coauthorship, separating individual from collective productivity. In contrast to existing measures, our index, CoScore, reflects the complete coauthorship network, not only the publication record of the author being ranked. CoScore uses the varying levels of success of all academic partnerships to infer, simultaneously, an author’s productivity and her credit on each of her papers. Crucially, the productivities of all authors are determined endogenously via the solution of a fixed point problem. We show that CoScore is well defined and provide axiomatic foundations for the associated rule used to allocate credit on coauthored papers. We illustrate CoScore for a large database of papers in economics.

Keywords: Consistency, Citation indices, Ranking methods.

1. Introduction

Over the last decades, teams have come to dominate the production of scientific research, both in terms of quantity and impact (Wuchty et al., 2007). The pattern is particularly striking in science and engineering, where more than eighty percent of the papers are now coauthored, but is also significant in the social sciences. In economics, the average number of authors has increased from 1.3 to 2.2 authors per paper in the last fifty years (Card and DellaVigna, 2013), while articles with more than three authors, a rare occurrence before the nineties, now represent over forty percent of all published articles (Hamermesh, 2013). Research has become a truly collective endeavour. Yet, scientists still have to be evaluated on an individual basis for important decisions such as
hiring, tenure or funding. It is therefore essential to properly account for coauthorship in quantifying individual scientific productivity, disentangling individual from collaborative productivity. Unfortunately, existing indices either ignore coauthorship, assigning full credit for a joint paper to each of its coauthors (Garfield, 1972; Hirsch, 2005; Lehmann et al., 2006; Perry and Reny, 2016), or only correct for it ad hoc, dividing credit equally among coauthors (Marchant, 2009).

The biases of these approaches are well known. Assigning full credit to all coauthors, artificially inflates their publication record by overrating the value of coauthored papers (Price, 1981). This favors researchers with multiple collaborators or belonging to large teams and gives perverse incentives for artificial coauthorship. Dividing credit uniformly corrects this bias but dilutes the credit due to the intellectual leaders of a paper, ignoring that authors’ contributions may differ substantially (Hirsch, 2007; Hagen, 2008; Shen and Barabási, 2014). The same concern applies to the various proposals to allocate credit when authors are listed by order of importance or by other field-specific conventions (Hagen, 2008; Stallings et al., 2013).

The deeply collaborative nature of modern research means that individual productivity cannot be accurately quantified without assigning credit for joint research. At the same time, credit cannot be allocated without evaluating the productive input of the coauthors involved on a paper. That is, the twin problems of measuring individual productivity and allocating credit for joint research can only be solved jointly.

We propose a new index of individual productivity, CoScore, capturing the relationship between credit allocation and individual productivity. A scientist’s score is defined as her average yearly productivity, the sum of her credit on all of her papers divided by her academic age. In turn, the credit for each paper is distributed proportionally to a fixed power of its coauthors’ scores. Crucially, the scores of all authors are determined endogenously and simultaneously as the solution of a fixed point problem. The premise is that stronger authors are likely to have contributed more to the success of the paper and should therefore be given, as an approximation, relatively more credit for their joint papers.¹ The strength of a researcher in turn depends on the credit she receives on all of the papers she contributes to.

¹See Hirsch (2007) and Shen and Barabási (2014) for more work relying on this assumption and its justification.
Formally, an academic database consists of a collection of papers $C$ and a group of authors $N$. Each paper $p$ is described by a measure of scientific worth $w(p)$ and its group of coauthors $S(p)$. The worth can be thought of as the number of citations or another measure of impact comparable across papers. Each author $i$ is described by her list of papers $C_i$ and her academic age $a_i$. For any such database, the CoScore $s \in \mathbb{R}_+^N$ is such that, for each $i,$

$$s_i = \frac{1}{a_i} \sum_{p \in C_i} w(p) \frac{s_i^\alpha}{\sum_{j \in S(p)} s_j^\alpha},$$

where $\alpha \in [0, 1]$ is a fixed parameter which determines how much credit should be allocated to more productive authors. CoScore is well defined because the system of equations (1) has a unique solution. The existence of a solution follows from Brouwer’s fixed point theorem; uniqueness comes from the fact that a solution of (1) maximizes a strictly concave function over an open convex set which can have at most one maximizer (see Theorem 3 in the appendix for a proof).\(^2\) When $\alpha$ equals 1, credit is allocated proportionally to each coauthor’s endogenous productivity. We call the corresponding index the proportional CoScore. As $\alpha$ gets smaller, the allocation of credit between coauthors becomes more egalitarian. At the extreme, when $\alpha$ equals 0, credit on joint papers is always allocated equally among coauthors, independently of their productivities. We call the corresponding index the egalitarian score.

When $\alpha$ is positive, an author’s CoScore reflects the structure of the entire coauthorship network and the research records of possibly all scientists in the database.\(^3\) The central insight is that allocating credit on a given paper requires assessing the relative productivity of all of its authors. Therefore, the credit assigned to a scientist on a paper depends on the credit assigned to all of her coauthors on all of their papers, which is itself determined by their coauthors’ credit on all their respective papers, and so forth. Hence, the scientist’s score and her assigned credit depends on the identity of her coauthors.

\(^2\)In order for CoScore to be well defined, we assume throughout that the aggregate solo contribution of each author, however small, is positive. In practice, this amounts to assigning a single individual citation to each author in the database.

\(^3\)Here, we take the coauthorship network to be the weighted hypergraph where nodes represent different scientists and hyperedges represent partnerships of varying weights (the number of citations). This is in contrast with Newman (2001, 2004), where the coauthorship network is defined as the undirected graph where nodes represent different scientists and two nodes are connected whenever the corresponding scientists have coauthored.
This is in sharp contrast to existing indices that can be computed for each author in isolation and can only control for coauthorship anonymously, based solely on the number of coauthors in a publication (Marchant, 2009) or, if available, their position in the author order (Sekercioglu, 2008).

The endogenous formulation of CoScore as a fixed point and its intensive use of the coauthorship network are reminiscent of various measures of network centrality used to rigorously quantify socioeconomic phenomena such as reputation, importance, influence, popularity, and the quality of scholarly papers. Most closely, eigenvector centrality (Bonacich, 1972) defines the centrality of an individual to be proportional to the sum of the centralities of the individuals connected to her. Such measures have proved extremely useful in many different contexts, as shown in particular with the PageRank algorithm used by Google to rank the relevance of web pages (Page et al., 1998) or the recursive methods to rank journals (Pinski and Narin, 1976; Palacios-Huerta and Volij, 2004). Although CoScore relies on similar insights, it differs substantially from network centrality because it is not defined on a network but on a weighted hypergraph, the database of papers. From a technical point of view, a major difference is also that CoScore cannot be defined in terms of a system of linear equations.

We illustrate CoScore for a large database of papers in economics, using citations as a measure of scientific worth. As expected, we observe a sharp divide between the proportional CoScore and the egalitarian score: accounting for the identity of coauthors in a given paper, not only their number, yields significant differences both in rankings and scores. For authors with at least 5 papers in the database, scores differ on average by 27, 4%, rankings by 7, 6%. In contrast with the egalitarian score, the proportional CoScore tends to favor authors who have proved their individual quality by publishing either (i) with multiple groups of coauthors, (ii) with relatively weaker coauthors and/or (iii) single authored papers. Conversely, authors who have not been able to publish outside of a given team or only with stronger coauthors tend to loose credit for their joint work. Although CoScore cannot be computed independently for each author, since it requires looking at the whole database of papers, its implementation is very straightforward. Like PageRank, it can be applied to extremely large databases (possibly all scientific papers) in a systematic way, and provides a concrete alternative to measure individual productivity.

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4For a survey on network centrality measures see Jackson (2008) and Newman (2010).
We provide axiomatic foundations for CoScore in the benchmark case where credit is allocated proportionally to each coauthor’s endogenous productivity. More precisely, we show that the associated rule to allocate credit on each paper, which we call the CoScore rule, is uniquely characterized by three logically independent axioms: consistency, invariance to merging papers, and invariance to merging coauthors (Theorem 1).

Consistency, requires the distribution of credit to remain invariant when an author is taken away from the database, and the value of every paper she has contributed to is reduced by the amount she was previously allocated. Conversely, if an author were to be added to existing papers, her credit on every paper should be equal to the corresponding increase in the worth of the paper, while the credit of the other coauthors should remain identical. Similar consistency properties, which allow to relate databases of different sizes, have been used in the literature on the measurement of intellectual influence (Palacios-Huerta and Volij, 2004), network centrality (Dequiedt and Zenou, 2014) and, more extensively, in resource allocation and game theory (Thomson, 2011).

The second property, invariance to merging papers, requires that authors not benefit from either merging or splitting papers with the same coauthors into papers of equal total value. The allocation of credit on joint papers should only depend on the aggregate value produced by each possible sets of coauthors, not on the particular breakdown of that value in papers of different worth. The axiom is related to a property proposed in Perry and Reny (2016), depth relevance, which prevents authors from increasing their score by splitting a paper into two papers of equal total value.

The third property, invariance to merging coauthors, requires an author’s credit on a paper to depend only on the strength of her own research record and the aggregate research records of her coauthors on the paper. This can be interpreted as saying that an author’s credit in a paper depends only on her own productivity and the aggregate productivity of all of her coauthors in that paper. Thus what matters for how much credit an author receives is not how many coauthors the paper has but how good these coauthors are as a whole.

We also provide a second characterization replacing invariance to merging coauthors by the following proportionality condition: the credit assigned to each author should be proportional to her single authored contribution when the database only involves two
authors, and both authors have exactly one paper on their own and one paper together. We show that the three axioms are uniquely satisfied by the CoScore rule (Theorem 2).

Although we formulate CoScore for the problem of measuring individual scientific productivity, our approach applies more generally to any situation where teams of individuals produce an observable output but where individual contributions cannot be perfectly and objectively quantified. What is the contribution of a football player to the success of her team? What is the contribution of a manager to her firm’s profits? What is the contribution of an actor to the box-office revenue of a movie? CoScore can be used in any of these examples where it provides a new and meaningful approach to measuring individual productivity, disentangling individual from collective productivity.

The remainder of this paper is organized as follows: Section 2 illustrates CoScore for a large database of papers in economics. Section 3 introduces the formal problem and provides two axiomatic characterizations for the rule used by the proportional CoScore to allocate credit on coauthored papers. Section 4 discusses various extensions to the CoScore indices. Section 5 concludes. All of the proofs are included in the appendix.

2. THE PRODUCTIVITY OF ECONOMISTS

We illustrate CoScore for the field of Economics, where authors are usually listed alphabetically, and the specific role of each author in a paper is not specified. We use Thomson Reuters’s Web of Science database to extract all published articles with at least one citation from 33 major journals in Economics over the period 1970 – 2015 (see appendix for details). The database includes 73,732 articles, for a total of 29,226 different authors. As previously observed in other scientific databases (Seglen, 1992), the distributions are extremely skewed, with a small number of authors producing a large fraction of both papers and citations. Authors have on average 4.32 papers, for a total of 168 citations, while the median author only has 2 papers and 22 citations. The share of coauthored papers increases steadily from only 20% in 1970 to almost 80% in 2015, with the average number of coauthors going from 1.25 to 2.31. Papers with more than two authors, a rare occurrence before the nineties, represent almost 40% of all papers published in 2015. The increasing prominence of coauthorship also translates

5Here, the database is biased towards more productive researchers because of the restriction to top journals. Since more productive researchers are less likely to publish in lower ranked journals, more of their papers are represented in the database.
in the fraction of coauthored citations, which exhibits the same trend as the fraction of coauthored papers. Figure 1 shows the evolution of the fraction of coauthored papers and citations since 1970.

We compute the proportional CoScore by repeatedly iterating the associated fixed point operator, starting from a vector of equal scores. The process converges quickly to the desired fixed point (see Appendix). As illustrated in Figure 2, we observe a sharp divide between the proportional CoScore and the egalitarian score: accounting for the identity of coauthors in a given paper, not only their number, yields significant differences both in rankings and scores. For authors with at least 5 papers in the database, scores differ on average by 27.4%, rankings by 7.6%. Compared to the egalitarian score, the proportional CoScore tends to favor authors who have proved their individual quality by having published either (i) with multiple groups of coauthors, (ii) with relatively weaker coauthors and/or (iii) single authored papers. Conversely, the proportional CoScore tends to be unfavorable to authors who have a limited number of stronger coauthors.

The changes are the most dramatic for authors who have a very high fraction of single-authored work, since they have by definition the most to lose and the most to win from credit allocation. R. Blundell, who has 74 papers for a total of 8,113 citations, goes from being ranked 90th with the egalitarian score to being ranked 31st with the proportional CoScore, while his score is almost doubled. Despite a very small fraction of single authored citations (0.1%), Blundell benefits from having worked with a large number of coauthors (66 coauthors), thus showing his ability to produce high quality work independently of the strength of his coauthors (some of them being very productive as well). Andrei Shleifer, who has 77 papers for a total of 19,711 citations in the database, goes from
being ranked 10th with the egalitarian score to being ranked 1st with the proportional CoScore. Shleifer, who has a small fraction of single authored work (5%), benefits from his main coauthors having published their most significant work only in collaboration with him. In contrast, Shleifer has been able to produce highly cited papers with many different groups of coauthors. As a result, Shleifer obtains a bigger share of the credit for these joint works. Typically, researchers who have not been able to demonstrate their productivity outside of a given team tend to lose credit for their joint papers.

![Figure 2](image)

**Figure 2.** Distribution of CoScore (blue) and the Egalitarian Score (red) for a random sample of authors with at least 10 papers in the database. Both scores are bounded above by the average yearly number of citations (green) and below by the average yearly number of single-authored citations (yellow). Authors are displayed according to their egalitarian rank. Scales are logarithmic.

Although having a large number of coauthors or a strong individual record is usually beneficial, it does not necessarily lead to an improved score. What matters ultimately for CoScore is the identity of a researcher’s coauthors. For instance, Avinash Dixit, who has written 73 papers for a total of 7035 citations together with 27 different coauthors and has more than 50% of single authored citations, satisfies all the conditions mentioned before. Yet, his proportional CoScore is inferior to his egalitarian score (his ranking drops from 60th to 111th) because some of his most cited work was written in collaboration with Joseph Stiglitz, Nobel prize winner and ranked 5th according to the proportional CoScore. Figure 4 illustrates the coauthorship network of two prominent economists.

In contrast to existing indices, CoScore cannot be computed independently for each author, since it requires looking at the whole database of papers. However, as illustrated
Figure 3. Coauthorship networks of J. Stiglitz (left, Nobel 2001) and J. Tirole (right, Nobel 2014). Lines represent collaborations between pairs of authors, darker lines reflecting a higher number of joint citations. Nodes represent authors whose degree of separation from Stiglitz and Tirole (at the center) respectively is at most 2. For each author (i) the size of the inner gray circle is proportional to the number of single-authored citations, (ii) the size of the outer gray circle is proportional to the total number of citations and (iii) the size of the orange ring is proportional to the number of jointly authored citations credited to the author by CoScore. The scales are specific to each network.

Here for the field of economics, its implementation is very straightforward. Like PageRank, it can be applied to extremely large databases (possibly all scientific papers) in a systematic way, and provides a concrete alternative to measure individual productivity.

3. Axiomatic foundations

We focus in this section on the simplest case where authors do not differ in their academic age. We provide two axiomatic characterizations of the rule used by the proportional CoScore to allocate credit on joint papers.

3.1. Model. As before, a database is specified by a collection of papers and each paper is described by its group of coauthors and its scientific worth. We now introduce a general model to study variations in papers, their authors, and worth: The papers constituting a database are drawn from a countable index set $\mathbb{C}$. The collection of finite subsets of $\mathbb{C}$ is denoted by $\mathbb{C}$. Similarly, authors are drawn from a countable set $\mathbb{N}$ and the collection of finite subsets of $\mathbb{N}$ is denoted by $\mathbb{N}$. For each $N \in \mathbb{N}$, a database involving $N$ is a triple $D \equiv (C, w, S)$ where

$$C \in \mathbb{C}, \quad w : C \to \mathbb{R}_+, \quad S : C \to 2^N.$$
Database $D$ is thus described by the collection of papers $C$ and, for each paper $p$ in $C$, a worth $w(p)$ and a set of authors $S(p)$. For each $i \in N$, let $C_i$ denote the papers in $C$ involving $i$ as an author or coauthor; for each $S \subseteq N$, let $C_S = \{p \in C|S(p) = S\}$ and note that $C_{\{i\}}$ is contained in $C_i$ and that the inclusion is strict whenever $i$ has coauthored. For each $N \in \mathcal{N}$, let $\mathcal{D}^N$ denote the domain of databases involving the authors in $N$.

A score is a systematic procedure which associates to every database $D \in \mathcal{D}^N$ a profile of individual productivity scores $s(D) \in \mathbb{R}^N_+$. We refer to the $i$th coordinate of $s(D)$, denoted by $s_i(D)$, as the score of score of author $i$ when there is no room for confusion.

A problem closely related to assigning scores is that of allocating the credit of a given paper among its coauthors. A rule is a systematic procedure which distributes the worth of each paper among its coauthors. Formally, a rule $m$ is a function such that, for each $D = (C, w, S) \in \mathcal{D}^N$,

$$m(D) \in Z(D) \equiv \bigtimes_{p \in C} \left\{ x \in \mathbb{R}^N_+ \mid \sum_{i \in S(p)} x_i = w(p), \ x_{N \setminus S(p)} = 0_{N \setminus S(p)} \right\}.$$  

For each $p \in C$ and each $i \in S(p)$, $m^p_i(D)$ denotes the credit attributed to individual $i$ on paper $p$. A rule allocates credit for a paper among its coauthors so that the credit received by all adds up to the paper’s worth, $\sum_{i \in S(p)} m^p_i(D) = w(p)$. Following Liebowitz (2013), we thus rule out the possibility that, for example, in a two-author paper, each author receives more than one half of the credit for the paper. Authors only get credit for papers they have contributed to.

Our main proposal, the $\text{CoScore}$ rule, allocates credit on each paper proportionally to the individual productivity of each coauthor, as (endogenously) measured by the sum of the author’s credits on all of her papers: for each $D = (C, w, S) \in \mathcal{D}^N$, each $p \in C$, and each $i \in S(p)$,

$$m^p_i(D) = \frac{\sum_{q \in C_i} m^q_i(D)}{\sum_{j \in S(p)} \sum_{q \in C_j} m^q_j(D)} w(p).$$ (2)

The $\text{CoScore rule}$ is closely related to the $\text{CoScore}$ index since it can equivalently be defined as the rule which allocates the value of each paper proportionally to the proportional
CoScore of each coauthor. The CoScore rule is well defined because the proportional CoScore is itself well defined.

Other alternative rules may also be considered. The egalitarian rule allocates the value of each paper equally among all its coauthors: \( m_i^p(D) = \frac{w(p)}{|S(p)|} \). When assigning credit on a paper, the egalitarian rule disregards any information about the publication records of its coauthors. Any two coauthors are deemed equally deserving of the paper’s authorship.

The solo rule allocates the value of each paper proportionally to the value of the (aggregate) solo contribution: for each \( D = (C, w, S) \in D^N \), each \( p \in C \), and each \( i \in S(p) \),

\[
m_i^p(D) = \frac{\sum_{q \in C_{\{i\}}} w(q)}{\sum_{j \in S(p)} \sum_{q \in C_{\{j\}}} w(q)} w(p).
\]

When assigning credit on a paper, the solo rule makes inferences on authorship based only on the worth of the individual contributions of its coauthors. For example, if researchers Alice and Bob have similar individual contributions, they will receive similar shares of their joint work, regardless of the fact that Alice also coauthored several other important papers while Bob didn’t.

**Notation.** For each \( N \in \mathcal{N} \) and each \( D = (C, w, S) \in D^N \), let \( C(D) \subseteq 2^C \) denote the subsets of papers which share exactly the same group of coauthors and let \( S(D) \subseteq 2^N \) denote the subsets of authors which share exactly the same list of coauthored papers:

\[
C(D) = \{ C' \subseteq C \mid S(p) = S(q) \ \forall \ p, q \in C' \} \quad \text{and} \quad S(D) = \{ N' \subseteq N \mid C_i \setminus C_{\{i\}} = C_j \setminus C_{\{j\}} \ \forall \ i, j \in N' \}.
\]

3.2. Characterization. We now introduce the three axioms characterizing the CoScore rule. The first axiom, consistency, concerns the response of the rule to the withdrawal of authors from the database. For example, starting from the database of all papers in economics and upon removing all economic historians, we would like to assess the distribution of credit among the remaining authors. Removing economic historians requires removing papers written exclusively by them from the database. Papers written by economic historians and other economists must remain in the database. However, the

\[\text{As defined in system of equations (1) for } \alpha = 1, \text{ assuming that } a_i = 1 \text{ for each } i \in N.\]
credit assigned to the economic historians involved in these papers needs to be deducted from these papers before we can reassess the allocation of credit among the remaining economists. Consistency, requires the distribution of credit among other economists to remain identical after economic historians have been removed and the value of each paper reduced accordingly.

Thus, if the assignment of credit is considered desirable for a group of authors, then it should still be considered desirable if an author is taken away from the database and the worth of every paper is reduced by the amount she was previously assigned. Conversely, consistency means that if an author were to be added to existing papers, her credit would be equal to the corresponding increase in the value of the paper, while the credit of the other authors would remain unchanged.

Formally, the database obtained from $D = (C, w, S) \in \mathcal{D}^{N}$ upon the departure of $k \in N$ starting from credit assignment $x \in Z(D)$, $c^x_k(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{N \setminus \{k\}}$, is such that

i. $\tilde{C} = C \setminus C\{k\}$,

ii. for each $p \in \tilde{C}$, $\tilde{w}(p) = w(p) - x^p_k$ and $\tilde{S}(p) = S(p) \setminus \{k\}$.

**Consistency:** For each $D \in \mathcal{D}^{N}$, $k \in N$ and $p \in \tilde{C}$, $m^p_{N \setminus \{k\}}(D) = m^p_{N \setminus \{k\}}(c^m(D))$.

Thus, a rule $m$ is consistent if the assignment of credit among the authors in $N \setminus \{k\}$ is the same in databases $D$ and $c^m_k(D)$. A related consistency condition is used in the axiomatic characterization of the “invariant” journal ranking of Palacios-Huerta and Volij (2004).7 Other conceptually related conditions have also been central in the analysis of resource allocation and game theory.8 Consistency is satisfied by the egalitarian and the CoScore rules but not by the solo rule.

The second axiom, invariance to merging papers, prevents authors from increasing their credit by either fragmenting or consolidating papers with the same group of coauthors. For example, no author should be given incentives to split one of her solo works, say an Econometrica article, into two Journal of Economic Theory articles of equal total worth. More generally, no author should either gain or lose credit if a paper is split into lesser

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7This ranking relies on the same ideas at the core of PageRank (Page et al., 1998), the procedure used by Google to rank web pages.

8See Thomson (2011) for a survey and Thomson (2012) for a discussion of the ethical content of consistency-type requirements.
papers with the same group of coauthors and a total value equal to that of the original paper. In other words, the allocation of credit on joint papers should only depend on the *aggregate* value produced by each possible sets of coauthors, not on the particular breakdown of that value in papers of different worth. Perry and Reny (2016) propose a related property, “depth relevance”, which requires that an author’s score does not increase upon splitting an article in her publication list into two articles with the same total number of citations. In contrast, invariance to merging papers prevents authors from both splitting and merging papers.

Formally, consider a database $D = (C, w, S) \in \mathcal{D}^N$ and a set of papers $C' \subseteq C$ with the same coauthors, $C' \in \mathcal{C}(D)$. The database obtained by merging $C'$ into a single paper $r$, $c^{C'\rightarrow r}(P) = (\tilde{C}, \tilde{w}, \tilde{S})$, is such that

i. $\tilde{C} = [C \setminus C'] \cup \{r\}$,

ii. for each $p \in C \setminus C'$, $\tilde{S}(p) = S(p)$ and $\tilde{w}(p) = w(p)$,

iii. for each $p \in C'$, $\tilde{S}(r) = S(p)$ and $\tilde{w}(r) = \sum_{p \in C'} w(p)$.

**Invariance to merging papers:** For each $D = (C, w, S) \in \mathcal{D}^N$, $C' \in \mathcal{C}(D)$, and $r \notin C \setminus C'$, $m^r(c^{C'\rightarrow r}(D)) = \sum_{p \in C'} m^p(D)$ and $m^p(c^{C'\rightarrow r}(D)) = m^p(D)$ for each $p \in C \setminus C'$.

Thus, a rule $m$ satisfies invariance to merging papers if the credit allocated to any author for the merged paper $r$ in the new database $c^{C'\rightarrow r}(D)$ is equal to the total credit that was previously allocated to her for all papers in $C'$ while the allocation of credit on all other papers remains the same. Invariance to merging papers is satisfied by the egalitarian, the solo, and the CoScore rules.

The third axiom captures the idea that an author’s credit on a paper should depend only on the strength of her own research record and the *aggregate* research records of her coauthors on the paper. More precisely, we consider the implication of merging the research records of a proper sub-group of coauthors in a paper. This means that each author in this group is replaced by a new single author in the database whose research portfolio is comprised of all the articles previously authored by the members of the subgroup. Our axiom, invariance to merging coauthors, requires the allocation of credit among all remaining authors to be the same. To avoid introducing a new coauthor in
papers written only by some members of the sub-group, we only impose the property on coauthors who share the exact same list of coauthored articles.

Formally, to define invariance to merging coauthors, consider a database $D = (C, w, S) \in \mathcal{D}^N$ and a group of authors $I \subseteq N$. Suppose that each author in $I$ has the same list of coauthored papers, $I \in \mathcal{S}(D)$. The database obtained by merging authors in $I$ into author $k$, $c_{I \rightarrow k}(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{N \setminus I \cup \{k\}}$, is such that each paper involving a coauthor in $I$ is authored by $k$ instead, everything else equal:

i. $\tilde{C} = C$,
ii. $\tilde{w} = w$, and
iii. for each $p \in \tilde{C}$, $\tilde{S}(p) = \left[ S(p) \setminus I \right] \cup \{k\}$ if $I \cap S(p) \neq \emptyset$.
\[ = S(p) \text{ otherwise} \]

**Invariance to merging coauthors:** For each $D \in \mathcal{D}^N$, $I \in \mathcal{S}(D)$, $k \notin N \setminus I$, and $p \in C$, $m^p_{N \setminus I}(c_{I \rightarrow k}(D)) = m^p_{N \setminus I}(D)$.

Thus, a rule $m$ satisfies invariance to merging coauthors if the allocation of credit for authors in $N \setminus I$ remains identical after authors in $I$ have been merged into author $k$. As an implication, the credit allocated to author $k$ must be equal to the the total credit that used to be allocated to all individuals in $I$. **Invariance to merging coauthors** is satisfied by the solo and the CoScore rules, but is violated by the egalitarian rule.

**Theorem 1.** The only rule satisfying consistency, invariance to merging papers, and invariance to merging coauthors is the CoScore rule.

The properties in Theorem 1 are logically independent. See appendix C.

Our last property, proportionality, requires credit to be allocated proportionally to each coauthor’s single authored contribution when the database only involves two authors, and both authors have exactly one paper on their own and one paper together.

**Proportionality:** For each $\{i, j\} \in \mathcal{N}$ and each $D = (C, w, S) \in \mathcal{D}^{\{i,j\}}$ where $C = \{p_i, p_j, p\}$, $S(p_i) = \{i\}$, $S(p_j) = \{j\}$, and $S(p) = \{i, j\}$, $m^p_i(D)w(p_j) = m^p_j(D)w(p_i)$.

In contrast to the CoScore and solo rules, the egalitarian rule does not satisfy proportionality. The CoScore rule is uniquely characterized by consistency, invariance to merging papers and proportionality.
Theorem 2. The only rule satisfying consistency, invariance to merging papers, and proportionality is the CoScore rule.

The properties in Theorem 2 are logically independent. See appendix C.

4. Discussion

4.1. Non-alphabetical authorship. We have formulated CoScore for academic fields where authors are listed alphabetically, such as economics or mathematics. However, in many other fields, the order of authors is meaningful, reflecting either the importance or the nature of their contribution to the paper. In the biological sciences, for example, most recognition typically goes to the the first and last authors, while the remaining authors’ contributions can be harder to determine. Increasingly, journals also require the capacity of each coauthor (research design, research, data analysis, writing, etc) to be specified (Allen et al., 2014). Although conventions vary widely, even across sub-fields, this information is indicative of the contribution of each author and should play a role in the allocation of credit. CoScore can naturally be extended to account for ordered authorship and the capacity of each coauthor. A first alternative is allocating relatively more credit to authors who have a higher rank or capacity, by introducing paper-specific weights in the right hand side of the CoScore formula. A second alternative is separating the allocation of credit across groups of authors who have contributed to the paper in similar capacities.\(^\text{10}\) Each paper is subdivided into several artificial papers, one for each group, where the number of citations assigned to each paper (or group) depends on the relative importance of the associated capacity.\(^\text{11}\) CoScore can then be applied to the resulting enlarged database.

4.2. Topic-specific experience. The issue of quantitatively allocating credit on a paper was recently considered by Shen and Barabási (2014) who suggest a simple algorithm based on citation patterns. Their assumption is that authors with more experience on the topic of the paper should receive more credit; their algorithm determines this topic-specific experience and allocates credit accordingly. A researcher, however, may often contribute

\(^\text{10}\)It is common for a paper to state that several of its authors contributed in the same capacities and for its authors not to be ranked strictly, for example, having two or more lead authors. A paper’s authors can thus be ordered into contribution classes, whereby authors in the highest class are considered the most significant contributors, authors in the second highest class are the second most significant, and so forth.

\(^\text{11}\)The credit allocation across groups of authors could follow the proposals of Stallings et al. (2013) or Hagen (2008).
more despite having relatively less topic-specific experience. This is particularly true for intellectual leaders, who usually have broad interests, and would therefore lose most of the credit for their coauthored work.\textsuperscript{12} Such a procedure would thus favor specialized researchers, discouraging interdisciplinary work and atypical collaborations, which have been shown to play an important role in the production of scientific knowledge (Uzzi et al., 2013). In contrast, \textit{CoScore} exploits the information contained in the whole database of papers to infer the quality of each scientist, independently of the considered topic. As in the case of ordered authorship, it can be extended to incorporate topic-specific experience, for example, by allocating relatively more credit to authors who have more experience on the topic of the paper, as measured by Shen and Barabási’s algorithm.

4.3. Beyond citations. Following a long tradition (Garfield, 1972), we measure the quality of a scientific paper by its total number of citations. Citations provide an objective metric that is based on observable data and can be used systematically on a large scale. However, citations have been shown to suffer from various shortcomings: (i) citation patterns and intensities differ substantially across fields and even sub-fields, making inter field comparisons problematic (Radicchi et al., 2008; Ellison, 2013), (ii) citations take time to accumulate, underestimating the value of recent papers (Wang et al., 2013), and (iii) citation counts treat each citation equally, ignoring that citations originating from more influential papers reflect a higher value. In response, various alternative metrics have been proposed, respectively: (i) normalizing the citation count by the average per-paper citations in the field (sub-field) to insure comparability (Radicchi et al., 2008), (ii) measuring the quality of the journal where the paper was published (Palacios-Huerta and Volij, 2014) or discounting the citation count by the age of the paper, and (iii) giving more weight to citations originating from more influential papers, as measured by recursive network centrality measures (Pinski and Narin, 1976; Palacios-Huerta and Volij, 2004). Depending on the academic field and practices, other appropriate metrics, relying on possibly different forms of information, may also be considered (Evans and Foster, 2011). \textit{CoScore} can be computed with any of these alternative measures of paper quality, for which it provides complementary assessments of a scientist’s productivity.

\textsuperscript{12}Jean Tirole (Nobel 2014), for instance, has coauthored in topics ranging from game theory, macroeconomics, industrial organization, finance to psychology.
5. Conclusion

CoScore contributes to an array of indices that have thus far not been able to properly account for the growth of coauthorship. As all existing indices, CoScore summarizes a scientist’s record into a one dimensional measure of scientific productivity, which inevitably entails some loss of information. Since authors contributions are not observable, it must necessarily involve some degree of approximation. However, scientists still need to be evaluated and compared by universities and government agencies for hiring, tenure and funding; this should be done according to objective aggregation indices, reflecting the increasing level of coauthorship. As discussed in the previous section, CoScore is flexible and can be adjusted to capture additional information such as ordered authorship, topic specific experience, and alternative measures of paper quality. Although we focus here on the benchmark case where credit is allocated proportionally to each coauthor’s productivity, CoScore may also be computed for other, possibly more appropriate, values of parameter $\alpha$. An important objective for future research will thus be to determine which of the CoScore indices performs best, either in terms of predicting future productivity (Hirsch, 2007) or emulating job market outcomes (Ellison, 2013). Another important avenue of research will be to investigate the application of CoScore to other forms of joint production (boards of directors, team sports, artistic collaborations, etc.), or to the valuation of different attributes in consumption products (conjoint analysis).

References


Note that such a calibration will require a very comprehensive dataset (possibly all papers in economics) as the score of each author depends on the entire database. Focusing on the complete cv of a subset of authors, as is done in Hirsch (2007) or Ellison (2013), would not be completely meaningful for investigating the performance of CoScore.


Pinski, G. and F. Narin (1976). Citation influence for journal aggregates of scientific publications: theory, with application to the literature of physics. *Information Processing and Management* 12.


Appendix A. CoScore is well defined

**Theorem 3.** Let $\alpha \in [0, 1]$, $a \in \mathbb{R}^N_{++}$, and suppose that each individual has at least one single authored contribution. Then, the system of equations (1) has a unique solution.

**Proof.** We first consider the case where $\alpha > 0$. Note that by assumption, since for each $i \in N$ there exists $p \in C$ such that $S(p) = \{i\}$ and $w(p) > 0$, any solution of system (1) is necessarily in $\mathbb{R}^N_{++}$. Let $\phi : \mathbb{R}^N_{++} \rightarrow \mathbb{R}^N_{++}$ denote the function defined by setting, for each $x \in \mathbb{R}^N_{++}$ and each $i \in N$,

\[
\phi_i(x) = \frac{1}{a_i} \sum_{p \in C_i} w(p) \frac{x_i^\alpha}{\sum_{j \in S(p)} x_j^\alpha}.
\]

We now show that $\phi$ has a unique fixed point. Let $W = \sum_{p \in C} w(p)$ and, for each $i \in N$, let $W_i = \sum_{p \in C \setminus S(p) = \{i\}} w(p)$. Thus, $W$ denotes the total scientific worth in the database and $W_i$ denotes the total independent contribution of author $i$. Note that, for each $x \in \mathbb{R}^N_{++}$ and each $i \in N$, $a_i \phi_i(x) \geq W_i > 0$ and $\sum_{i \in N} a_i \phi_i(x) = W$. Thus, letting

\[
K = \left\{ x \in \mathbb{R}^N \mid \forall i \in N, \ W_i \leq a_i x_i \leq W \right\},
\]

we can redefine $\phi$ as a function mapping $K$ into $K$. Note that $K$ is compact and convex. Furthermore, $\phi$ is continuous on $K$. Thus, by Brouwer’s fixed point theorem, $\phi$ has at least one fixed point. We now prove that this fixed point is unique. Let $x^*$ denote any one of such fixed points. By (3) and since the coordinates of $x^*$ are positive, for each $i \in N$,

\[
\sum_{p \in C_i} w(p) \frac{x_i^{\alpha-1}}{\sum_{j \in S(p)} x_j^\alpha} = a_i.
\]

Let $f : \mathbb{R}^N_{++} \rightarrow \mathbb{R}$ denote the function defined by setting, for each $x \in \mathbb{R}^N_{++}$,

\[
f(x) = -\sum_{i \in N} a_i x_i + \frac{1}{\alpha} \sum_{p \in C} w(p) \ln \left( \sum_{j \in S(p)} x_j^\alpha \right).
\]

We first show that $f$ is strictly concave. Let $g : \mathbb{R}^N_{++} \rightarrow \mathbb{R}$ be defined, for each $x \in \mathbb{R}^N_{++}$, by $g(x) = \frac{1}{\alpha} \sum_{i \in N} W_i \ln (x_i^\alpha)$. The function $g$ is strictly concave since its Hessian is
negative definite and, for each $i \in N$, $W_i > 0$. Let $D \subseteq C$ consist of all of the papers in $C$ with at least two coauthors: $p \in D$ if and only if $|S(p)| \geq 2$. Let $h : \mathbb{R}_{++}^N \to \mathbb{R}$ be defined, for each $x \in \mathbb{R}_{++}^N$, by:

$$h(x) = -\sum_{i \in N} a_i x_i + \frac{1}{\alpha} \sum_{p \in D} w(p) \ln \left( \sum_{i \in S(p)} x_i^\alpha \right).$$

As the sum and composition of concave functions, $h$ is concave. Note that $f = g + h$. Since the sum of a strictly concave function and a concave function is strictly concave, $f$ is strictly concave.

By the strict concavity of $f$ and the fact that $\mathbb{R}_{++}^N$ is open and convex, $x^{**}$ is the unique maximizer of $f$ over $\mathbb{R}_{++}^N$ if and only if $\nabla f(x^{**}) = 0$. That is, for each $i \in N$,

$$0 = \frac{\partial f}{\partial x_i}(x^{**}) = -a_i + \sum_{p \in C_i} w(p) \frac{x_i^{**\alpha - 1}}{\sum_{j \in S(p)} x_j^{**\alpha}}.$$  

Thus, by (4), $x^* = x^{**}$. Recall that $x^*$ is an arbitrary fixed point of $\phi$. By uniqueness of $x^{**}$, we conclude that the fixed point of $\phi$ is indeed unique.

It remains to consider the case where $\alpha = 0$. The system of equations (1) reduces to, for each $i \in N$,

$$s_i = \frac{1}{a_i} \sum_{p \in C_i} \frac{w(p)}{|S(p)|}.$$  

The system thus (trivially) has a unique solution.

**APPENDIX B. PROOF OF THEOREM 2**

**Lemma 1.** The CoScore rule satisfies Consistency, Invariance to merging papers, Invariance to merging coauthors, and proportionality.

**Proof.** Let $\bar{m}$ denote the CoScore rule and $\bar{s}$ denote the score defined in (1) for $\alpha = 1$ and $a_i = 1$ for each $i \in N$. Let $N \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{D}^N$.

**Consistency**

Let $k \in N$, $x \equiv \bar{m}(D)$, and $y \equiv \bar{m}(c_k^\xi(D))$. Recall that $c_k^\xi(D) = (\tilde{C}, \tilde{w}, \tilde{S})$ is such that:

$$- \tilde{C} = C \setminus C_{\{k\}}$$
- For each \( p \in \tilde{C} \) such that \( p \in C_k \), \( \bar{w}(p) = w(p) - x_k^p \).

- For each \( p \in \tilde{C} \) such that \( p \notin C_k \), \( \bar{w}(p) = w(p) \).

- For each \( p \in \tilde{C} \), \( \tilde{S}(p) = S(p) \setminus \{k\} \).

Then, for each \( p \in C_k \setminus C_{\{k\}} \), and each \( i \in S(p) \setminus \{k\} \),

\[
y_i^p = [w(p) - x_k^p] \sum_{q \in C_i} \frac{y_j^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} y_j^q}
\]

\[
= w(p) \left[ 1 - \frac{\sum_{q \in C_k} x_k^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q} \right] \frac{\sum_{q \in C_i} y_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} y_j^q}
\]

\[
= w(p) \left[ \frac{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q} \right] \frac{\sum_{q \in C_i} y_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} y_j^q}
\]

while for each \( p \in C \setminus C_k \), and each \( i \in S(p) \),

\[
y_i^p = w(p) \sum_{q \in C_i} \frac{y_j^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} y_j^q}
\]

On the other hand, for each \( p \in C_k \setminus C_{\{k\}} \) and each \( i \in S(p) \setminus \{k\} \),

\[
x_i^p = w(p) \left[ \frac{\sum_{q \in C_i} x_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q} \right] = w(p) \left[ \frac{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q} \right] \frac{\sum_{q \in C_i} x_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}
\]

while for each \( p \in C \setminus C_k \), and each \( i \in S(p) \),

\[
x_i^p = w(p) \sum_{q \in C_i} \frac{x_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}
\]

Let \( z \in Z(r_k^p(D)) \) be such that, for each \( p \in \tilde{C} \) and each \( i \in N \setminus \{k\} \), \( z_i^p = x_i^p \). Note that, by (7) and (8), \( z \) satisfies the system of equations in (5) and (6). By (2), (5) and (6) uniquely define \( y \). Thus, \( z = y \). Thus, for each \( i \in N \setminus \{k\} \), \( \tilde{m}_i(c_k^p(D)) = \tilde{m}_i(D) \), so \( \tilde{m} \) satisfies consistency.
Invariance to merging papers

Let $C' \in \mathcal{C}(D)$ and recall that $c^{C' \rightarrow r}(P) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^N$ is such that:

- $\tilde{C} = [C \setminus C'] \cup \{r\}$ where $r \notin C \setminus C'$.
- $\tilde{w}(r) = \sum_{q \in D} w(q)$ and, for each $q \in C \setminus C'$, $\tilde{w}(q) = w(q)$.
- For each $q \in \tilde{C}$, $S(q) = \tilde{S}(q)$.

From (1), $\bar{s}(c^{C' \rightarrow r}(D)) = \bar{s}(D)$. By definition of $\bar{m}$, for each $p \in C'$ and each $i \in S(p)$,

$$\sum_{q \in D} \bar{m}_i^q(D) = \sum_{q \in D} w(q) \frac{\bar{s}_i(D)}{\sum_{j \in S(p)} \bar{s}_j(D)} = \tilde{w}(r) \frac{\bar{s}_i(c^{C' \rightarrow r}(P))}{\sum_{j \in S(p)} \bar{s}_j(c^{C' \rightarrow r}(D))} = \bar{m}_i^r(c^{C' \rightarrow r}(D))$$

while for each $\hat{p} \in C \setminus C'$ and each $i \in S(\hat{p})$,

$$\bar{m}_{\hat{p}}^i(D) = w(p) \frac{\bar{s}_i(D)}{\sum_{j \in S(p)} \bar{s}_j(D)} = \tilde{w}(\hat{p}) \frac{\bar{s}_i(c^{C' \rightarrow r}(P))}{\sum_{j \in S(p)} \bar{s}_j(c^{C' \rightarrow r}(D))} = \bar{m}_i^\hat{p}(c^{C' \rightarrow r}(D)).$$

Thus, $\bar{m}$ satisfies invariance to merging papers.

Invariance to merging coauthors

Let $I \in \mathcal{S}(D)$, $k \in \mathbb{N} \setminus N$, and recall that $c_{I \rightarrow k}(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{[N \setminus I] \cup \{k\}}$ is such that:

- $\tilde{C} = C$,
- for each $p \in \tilde{C}$, $\tilde{w}(p) = w(p)$,
- for each $p \in \tilde{C}$, $\tilde{S}(p) = [S(p) \setminus I] \cup \{k\}$ if $I \cap S(p) \neq \emptyset$ and $\tilde{S}(p) = S(p)$ otherwise.

By definition of the CoScore rule, for each $p \in C$ and each $r \in S(p)$:

$$\bar{m}_r^p(D) = w(p) \frac{\sum_{q \in C_r} \bar{m}_i^q(D)}{\sum_{t \in S(p)} \sum_{q \in C_t} \bar{m}_i^q(D)} \quad (9)$$

Let $z \in Z(c_{I \rightarrow k}(D))$ be such that, for each $p \in \tilde{C}$,

$$z_k^p = \sum_{i \in I} \bar{m}_i^p(D) \quad \text{and, for each } r \in \tilde{S}(p) \setminus \{k\}, \quad z_r^p = \bar{m}_r^p(D).$$

Then, by (9), for each $p \in \tilde{C}$ and each $r \in \tilde{S}(p)$,

$$z_r^p = \tilde{w}(p) \frac{\sum_{q \in C_r} z_i^q}{\sum_{t \in S(p)} \sum_{q \in C_t} z_i^q}.$$
which means \( z \) satisfies the system of equations (2) for database \( c_{I \rightarrow k}(D) \). By (2), since \( CoScore \) is well defined, \( \tilde{m}(c_{I \rightarrow k}(D)) = z \). Thus, \( \tilde{m} \) satisfies invariance to merging coauthors.

**Proportionality**

Suppose that \( N = \{i, j\} \), \( C = \{p_i, p_j, p\} \), \( S(p_i) = \{i\} \), \( S(p_j) = \{j\} \), and \( S(p) = \{i, j\} \). Then, it follows from the system of equations (2) that:

\[
\frac{\tilde{m}_{i}^{p}(D)}{\tilde{m}_{j}^{p}(D)} = \frac{w(p_i) + \tilde{m}_{i}^{p}(D)}{w(p_j) + \tilde{m}_{j}^{p}(D)}
\]

and thus \( m_{i}^{p}(D)w(p_j) = m_{j}^{p}(D)w(p_i) \). Thus, \( \tilde{m} \) satisfies proportionality.

We introduce two additional properties that will be useful in the proof of Theorem 2. The first of these reflects the requirement that the indexing of papers is irrelevant to the credit assignment. Only the worth of papers and the coauthorship relations are taken into consideration:

**Neutrality:** For each \( N \in \mathcal{N} \) and each pair \( D = (C, w, S), D' = (C', w', S') \in \mathcal{D}^{N} \), if there is a bijection \( \sigma : C \rightarrow C' \) such that, for each \( p \in C \), \( w'(\sigma(p)) = w(p) \) and \( S'(\sigma(p)) = S(p) \), then, for each \( p \in C \) and each \( i \in S'(\sigma(p)) \), \( m_{i}^{\sigma(p)}(D') = m_{i}^{p}(D) \).

The second property specifies that the name of authors bears no influence on the credit assignment. Again, only the worth of papers and the coauthorship pattern are taken into consideration:

**Anonymity:** For each pair \( N, N' \in \mathcal{N} \), each \( D = (C, w, S) \in \mathcal{D}^{N} \), and each \( D' = (C', w', S') \in \mathcal{D}^{N'} \), if \( C = C' \), \( w = w' \), and there is a bijection \( \pi : N \rightarrow N' \) such that, for each \( p \in C \), \( S'(p) = \pi(S(p)) \), then, for each \( p \in C \) and each \( i \in S(p) \), \( m_{i}^{p}(D) = m_{\pi(i)}^{p}(D') \).

**Remark 1.** Invariance to merging papers implies neutrality. Invariance to merging coauthors implies anonymity.

**Proof of Theorem 2.** Let \( m \) denote a rule satisfying consistency, invariance to merging papers, and proportionality. Let \( \tilde{m} \) denote the \( CoScore \) rule. By Lemma 1, it suffices to prove that \( m = \tilde{m} \).

**Step 1:** For each \( N \in \mathcal{N} \) such that \( |N| = 2 \) and each \( D \in \mathcal{D}^{N} \), \( m(D) = \tilde{m}(D) \).
Let \( \{i, j\} \in \mathcal{N} \) and \( D = (C, w, S) \in \mathcal{D}^{N} \). Let \( K \) denote the total value of the joint papers in \( D \), \( K = \sum_{p \in C_{\{i,j\}}} w(p) \). Let \( Q \subseteq \mathcal{D}^{N} \) consist of all databases \( Q = (\hat{C}, \hat{w}, \hat{S}) \in \mathcal{D}^{N} \) such that:

- \( \hat{C}_{\{i\}} = C_{\{i\}} \) and \( \hat{C}_{\{j\}} = C_{\{j\}} \).
- for each \( p \in \hat{C}_{\{i\}} \cup \hat{C}_{\{j\}} \), \( \hat{w}(p) = w(p) \).
- \( \sum_{p \in \hat{C}_{\{ij\}}} \hat{w}(p) = K \).

Note that \( D \in \mathcal{Q} \). The argument proceeds in three steps.

**Step 1A:** For each \( D' = (C', w', S') \), \( D'' = (C'', w'', S'') \in \mathcal{Q} \), each \( p' \in C'_{\{i,j\}} \), and each \( p'' \in C''_{\{i,j\}} \), \( w'(p') = w''(p'') \) implies \( m'(D') = m''(D'') \).

Let all of the notation be as in the statement of Step 1A. Without loss of generality, suppose that \( |C'_{\{i,j\}}| \geq 2 \) and \( |C''_{\{i,j\}}| \geq 2 \).\(^{14}\) Let \( \hat{D} = (\hat{C}, \hat{w}, \hat{S}) \in \mathcal{Q} \) be such that \( \hat{C}_{\{i,j\}} = \{p', \hat{p}\} \), \( \hat{w}(p) = w'(p') \), and \( \hat{w}(\hat{p}) = K - w'(p') \). By invariance to merging papers, merging all the papers in \( C'_{\{i,j\}} \setminus \{p'\} \) into \( \hat{p} \), \( m'(D') = m'(\hat{D}) \). Similarly, let \( \hat{D} = (\hat{C}, \hat{w}, \hat{S}) \in \mathcal{Q} \) be such that \( \hat{C}_{\{i,j\}} = \{p'', \hat{p}\} \), \( \hat{w}(p) = w''(p'') \), and \( \hat{w}(\hat{p}) = K - w''(p'') \). By invariance to merging papers, merging all the papers in \( C''_{\{i,j\}} \setminus \{p''\} \) into \( \hat{p} \), \( m''(D'') = m''(\hat{D}) \). By neutrality, which is implied by invariance to merging papers (Remark 1), since the databases \( \hat{D} \) and \( \hat{D} \) differ only in the indexing of papers, \( m'(D') = m'(\hat{D}) = m''(\hat{D}) = m''(D'') \).

**Step 1B:** For each pair \( p, q \in C \), \( \frac{m_{1}^{p}(D)}{m_{1}^{q}(D)} = \frac{w(p)}{w(q)} \).

Let \( f : [0, K] \rightarrow \mathbb{R} \) be such that

\[
\forall x \in [0, K] \quad f(x) = m_{1}^{p}(Q)
\]

where \( Q = (\hat{C}, \hat{w}, \hat{S}) \in \mathcal{Q} \) and \( p \) is any paper in \( \hat{C} \) such that \( \hat{w}(p) = x \) and \( \hat{S}(p) = \{i, j\} \).

By Step 1A, \( f \) is well defined. Furthermore, by invariance to merging papers,

\[
\forall x, y \in [0, K] \quad x + y \leq K
\]

Thus, \( f \) satisfies the Cauchy functional equation and there is a constant \( c_{K} \) such that, for each \( x \in [0, K] \), \( f(x) = c_{K}x \) (see Theorem 3 on page 48 of Aczél, 2006).

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\(^{14}\)Otherwise, we necessary have \( |C'_{\{i,j\}}| = |C''_{\{i,j\}}| = 1 \) and the result simply follows from neutrality which, by Remark 1, is implied by invariance to merging papers.
for any \( p, q \in C \), \( m^p_i(D) = f(w(p)) = c_k w(p) \) and \( m^q_i(D) = f(w(q)) = c_k w(q) \). This establishes Step 1B.

**Step 1C:** Let \( w_{\{i\}} \) and \( w_{\{j\}} \) denote the total worth of \( i \) and \( j \)’s individual contributions, respectively.\(^{15}\) For each \( q \in C_{\{ij\}} \), \( m^q_i(D) = \frac{w_{\{i\}}}{w_{\{i\}} + w_{\{j\}}} w(q) \).

Let \( D^* = (C^*, w^*, S^*) \in D^N \) be such that

\[
C^* = \{ p_i, p_i, p_{ij} \}
\]

\[
S^*(p_i) = \{ i \}, \quad S^*(p_j) = \{ j \}, \quad S^*(p_{ij}) = \{ i, j \},
\]

\[
w^*(p_i) = w_{\{i\}}, \quad w^*(p_j) = w_{\{j\}}, \quad w^*(p_{ij}) = \sum_{p \in C_{\{i,j\}}} w(p).
\]

By invariance to merging papers,

(10) \[ m^{p_{ij}}_i(D^*) = \sum_{p \in C_{\{i,j\}}} m^p_i(D). \]

By proportionality,

(11) \[ m^{p_{ij}}_i(D^*) = \frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} w^*(p_{ij}). \]

By Step 1B, for each pair \( p, q \in C \),

\[ m^p_i(D) = \frac{w(p)}{w(q)} m^q_i(D). \]

Thus, by (10),

\[ m^{p_{ij}}_i(D^*) = \sum_{p \in C_{\{i,j\}}} \frac{w(p)}{w(q)} m^q_i(D) = \sum_{p \in C_{\{i,j\}}} w(p) = \frac{m^q_i(D)}{w(q)} w^*(p_{ij}). \]

Thus, by (11),

\[ \frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} = \frac{m^q_i(D)}{w(q)}. \]

Thus, for each \( q \in C_{\{i,j\}} \),

\[ m^q_i(D) = \frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} w(q) = \frac{w_{\{i\}}}{w_{\{i\}} + w_{\{j\}}} w(q) = \tilde{m}^q_i(D). \]

\(^{15}\) Thus, \( w_{\{i\}} = \sum_{p \in C_{\{i\}}} w(p) \).
We conclude that \( m(D) = \tilde{m}(D) \).

**Step 2:** For each \( N \in \mathcal{N} \) and each \( D \in \mathcal{D}^N \), \( m(D) = \tilde{m}(D) \).

**Induction hypothesis:** Let \( n \) denote a positive integer and suppose that, for each \( N \in \mathcal{N} \) such that \( |N| \leq n \) and each \( D \in \mathcal{D}^N \), \( m(D) = \tilde{m}(D) \).

Let \( N \in \mathcal{N} \) be such that \( |N| = n + 1 \), \( D = (C, w, S) \in \mathcal{D}^N \), and \( x \equiv m(D) \). For each \( k \in N \), let \( N_k = N \setminus \{k\} \). Let \( k \in N \) and consider database \( c_k^x(D) \). By consistency,

\[
(12) \quad \text{for each } p \in C \setminus C_{\{k\}} \text{ and each } i \in S(p) \setminus \{k\}, \quad m^p_i(c_k^x(D)) = x^p_i.
\]

By the induction hypothesis, since \( |N_k| = n \), for each \( p \in C \setminus C_{\{k\}} \) and each \( i \in S(p) \setminus \{k\} \),

\[
m^p_i(c_k^x(D)) = \tilde{m}^p_i(c_k^x(D)).
\]

Thus, by the definition of \( \tilde{m} \), for each \( p \in C \setminus C_{\{k\}} \) and each \( i \in S(p) \setminus \{k\} \),

\[
m^p_i(c_k^x(D)) = (w(p) - x^p_k) \frac{\sum_{q \in C_i} m^q_i(c_k^x(D))}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} m^q_j(c_k^x(D))}
\]

where, abusing notation, if \( p \in C_k \), we let \( x^p_k = 0 \). Thus, by (12), for each \( p \in C \setminus C_k \) and each \( i \in S(p) \setminus \{k\} \),

\[
x^p_i = (w(p) - x^p_k) \frac{\sum_{q \in C_i} x^q_i}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x^q_j}
\]

where, again, abusing notation, if \( p \in C_k \), we let \( x^p_k = 0 \). Thus, for each \( p \in C \setminus C_{\{k\}} \) and each pair \( i, j \in S(p) \setminus \{k\} \),

\[
\frac{x^p_j}{x^p_i} = \frac{\sum_{q \in C_j} x^q_j}{\sum_{q \in C_i} x^q_i}.
\]

Repeating the same argument for each \( k \in N \), for each \( p \in C \) and each pair \( i, j \in S(p) \),

\[
x^p_j = x^p_i \frac{\sum_{q \in C_j} x^q_j}{\sum_{q \in C_i} x^q_i}.
\]
Thus, for each \( p \in C \) and each \( i \in S(p) \), summing over \( j \in S(p) \),
\[
    w(p) = \sum_{j \in S(p)} x_j^p = x_i^p \frac{\sum_{q \in C_i} x_q^p}{\sum_{q \in C_i} x_q^i}.
\]
Thus, for each \( p \in C \) and each \( i \in S(p) \),
\[
    x_i^p = w(p) \frac{\sum_{q \in C_i} x_q^p}{\sum_{j \in S(p)} \sum_{q \in C_j} x_q^j}.
\]
Hence, \( x \) satisfies the system of equations (2) defining \( \tilde{m}(D) \). However, \( \tilde{m}(D) \) is the only solution to the system of equations (2). Thus, in fact, \( \tilde{m}(D) = x = m(D) \), as desired. \( \blacksquare \)

**Lemma 2.** If \( m \) satisfies neutrality and invariance to merging coauthors, then it satisfies proportionality.

**Proof.** Suppose that \( m \) satisfies neutrality and invariance to merging coauthors. By Remark 1, \( m \) also satisfies anonymity. For each \( N \in \mathcal{N} \), let \( \mathcal{Q}^N \subseteq \mathcal{D}^N \) consist of all databases such that each author \( i \in N \) has exactly one single-authored paper, denoted by \( p_i \), and one joint paper coauthored by every author in \( N \), denoted by \( p \). Let \( \{i, j\} \in \mathcal{N} \) and \( D = (C, w, S) \in \mathcal{Q}^{\{i, j\}} \). For each \( a \in [0, w(p_i) + w(p_j)] \), let \( D_a = (C^a, w^a, S^a) \in \mathcal{Q}^{\{i, j\}} \) be such that
\[
    w^a(p_i) = a, \quad w^a(p_j) = w(p_i) + w(p_j) - a \quad \text{and} \quad w^a(p) = w(p).
\]
Note that \( D = D_{w(p_i)} \). Let \( a', a'' \in [0, w(p_i) + w(p_j)] \) be such that \( a' + a'' = a \). Let \( i', i'' \in \mathbb{N} \setminus \{i, j\} \) and \( D_a' = (C', S', w') \in \mathcal{Q}^{\{i', i'', j\}} \) be such that
\[
    w'(p_{i'}) = a', \quad w'(p_{i''}) = a'', \quad w'(p_j) = w(p_j), \quad w'(p) = w(p).
\]
Since \( w'(p_{i'}) + w'(p_{i''}) = w^a(p_i) \), by invariance to merging coauthors,
\begin{equation}
    m_i^a(D_a) = m_i^a(D_a') + m_{i'}^a(D_a').
\end{equation}
Similarly, let \( j', j'' \in \mathbb{N} \setminus \{i, j\} \) and \( D_a'' = (C'', S'', w'') \in \mathcal{Q}^{\{i, j', j''\}} \) be such that
\[
    w''(p_i) = a', \quad w''(p_j) = a'', \quad w''(p_{j''}) = w(p_j), \quad w''(p) = w(p).
\]
Since \( w''(p_{j'}) + w''(p_{j''}) = w^a(p_j) \), by invariance to merging coauthors,
\[
    m_j^a(D_a) = m_j^a(D_a'').
\]
By anonymity and neutrality,
\[ m^p_i(D_a') = m^p_i(D_a). \]

Combining the two previous equations, \( m^p_i(D_a') = m^p_i(D_a) \). Similarly, \( m^p_i(D_a'') = m^p_i(D_a'') \). Thus, by (13) and since \( a = a' + a'' \),
\[ m^p_i(D_a + a'') = m^p_i(D_a') + m^p_i(D_a''). \]

Define \( f : [0, w(p_i) + w(p_j)] \rightarrow [0, w(p)] \) by setting, for each \( a \in [0, w(p_i) + w(p_j)] \),
\[ f(a) = m^p_i(D_a). \] Then, \( f(a' + a'') = f(a') + f(a'') \). Thus, \( f \) satisfies the Cauchy functional equation and there is a constant \( c_D \) such that, for each \( a \in [0, w(p_i) + w(p_j)] \), \( f(a) = c_D a \) (Theorem 3, page 48, Aczél, 2006). Since \( D = D_{w(p_i)} \), \( m^p_i(D) = c_D w(p_i) \). Furthermore, by anonymity and neutrality, \( m^p_j(D) = m^p_j(D_{w(p_j)}) = c_D w(p_j) \). We conclude that
\[ m^p_i(D) w(p_j) = m^p_j(D) w(p_i) \]

That is, \( m \) satisfies proportionality.

**Proof of Theorem 1.** By Lemma 1, the CoScore rule satisfies the properties in Theorem 1. Conversely, let \( m \) denote a rule satisfying these properties. By Remark 1, \( m \) satisfies neutrality. Thus, by Lemma 2, \( m \) satisfies proportionality. Thus, by Theorem 2, \( m \) is the CoScore rule.

APPENDIX C. LOGICAL INDEPENDENCE

**Proposition 1.** Consistency, invariance to merging papers, and proportionality are logically independent.

**Proof.** First note that the egalitarian rule satisfies consistency and invariance to merging papers but not proportionality while the solo rule satisfies invariance to merging papers and proportionality but not consistency. We conclude the proof by exhibiting a rule that satisfies consistency and proportionality but not invariance to merging papers.

Let \( \tilde{m} \) denote the CoScore rule. Let \( i \in \mathbb{N} \) and \( x : C \rightarrow \{0,1\} \). Let \( N \in \mathcal{N} \) and \( D = (C, w, S) \in \mathcal{D}^N \). If \( i \in N \), let \( z(D) \in Z(D) \) be such that, for each \( p \in C \), \( z^p_i(D) = x(p) w(p) \). Define the rule \( \tilde{m} \) as follows:
1\textsuperscript{st} Case: \(i\) appears in at most 2 papers in \(C\). Then, for each paper \(p \in C\),
\[
\hat{m}^p(D) = \hat{m}^p(D).
\]

2\textsuperscript{nd} Case: \(i\) appears in at least 3 papers in \(C\). Then, for each paper \(p \in C\),
\[
\hat{m}_i^p(D) = x(p)w(p) \\
\hat{m}_j^p(D) = \hat{m}_j^p(c_i^z(D)) \quad \text{for each} \; j \neq i.
\]

First note that proportionality only applies in Case 1. Therefore, by Lemma 1, \(\hat{m}\) satisfies proportionality. Let \(D = (C, w, S) \in \mathcal{D}^N\) be such that \(i \in N\) and that there are two papers \(p\) and \(q\) in \(C_i\) with \(i \in S(p) = S(q)\) and \(w(p) = w(q)\). Then, since \(x(\cdot)\) may be chosen in an arbitrary way, we can write:
\[
\hat{m}_i^p(D) = x(p)w(p) \neq x(q)w(q) = \hat{m}_i^q(D)
\]
thus violating invariance to merging papers (which implies that two identical papers are shared in the same way).

Finally, we show that \(\hat{m}\) satisfies consistency. Let \(N \in \mathcal{N}\) and \(D = (C, w, S) \in \mathcal{D}^N\).

Let \(k \in N\) and consider the database \(c_k^{\hat{m}(D)}(D) = (\tilde{C}, \tilde{w}, \tilde{S})\).

1\textsuperscript{st} Case: \(i\) appears in at most 2 papers in \(C\). Then, whether \(k = i\) (\(i\) is taken out) or \(k \neq i\), \(i\) still appears in at most 2 papers in \(\tilde{C}\). Therefore, for each \(p \in C\) and each \(j \in N \setminus \{k\}\):
\[
\hat{m}_j^p(D) = \hat{m}_j^p(D) = \hat{m}_j^p(c_k^{\hat{m}(D)}(D)) = \hat{m}_j^p(c_k^{\hat{m}(D)}(D))
\]
since the CoScore rule \(\hat{m}\) satisfies consistency.

2\textsuperscript{nd} Case: \(i\) appears in at least 3 papers in \(C\).

2.1: \(k = i\). Then, since \(i\) doesn’t appear in any paper in \(\tilde{C}\) (\(i\) has been taken out), we have \(\hat{m}_j^p(c_k^{\hat{m}(D)}(D)) = \hat{m}_j^p(c_k^{\hat{m}(D)}(D))\). Furthermore, since \(c_k^{\hat{m}(D)}(D) = c_k^{z(D)}(D)\), we also have: \(\hat{m}_j^p(D) = \hat{m}_j^p(c(D)) = \hat{m}_j^p(c_k^{\hat{m}(D)}(D))\) so that \(\hat{m}_j^p(D) = \hat{m}_j^p(c_k^{\hat{m}(D)}(D))\). We conclude that \(\hat{m}\) satisfies consistency.

2.2: \(k \neq i\). Then \(i\) still appears in at least 3 papers in \(\tilde{C}\). Therefore, for each \(p \in C\),
\[
\hat{m}_i^p(c_k^{\hat{m}(D)}(D)) = x(p) \left(w(p) - \hat{m}_k^p(D)\right) = x(p)w(p) = \hat{m}_i^p(D)
\]
For each \( j \in N \setminus \{i, k\} \),

\[
\tilde{m}_j^p(c_k^n(D)(D)) = \tilde{m}_j^p(c_i^{z(c_k^n(D))}(D))(c_k^n(D)(D))
\]

Furthermore, since \( i \) is allocated the same share of paper \( p \) in databases \( c_k^n(D) \) and \( D \):

\[
c_i^{z(c_k^n(D))}(D)(D)) = c_i^{z(D)}(c_k^n(D)(D))
\]

and since (a) reducing \( D \) first by taking out \( k \) (according to \( m(D) \)) then by taking out \( i \) (according to \( z(D) \)) is equivalent to (b) reducing \( D \) first by taking out \( i \) (according to \( z(D) \)), then by taking out \( k \) (according to \( m(c_i^z(D)(D)) \)), which is also equivalent to (c) reducing \( D \) first by taking out \( i \) (according to \( z(D) \)), then by taking out \( k \) (according to \( m(c_i^z(D)(D)) \)):

\[
c_i^{z(D)}(c_k^n(D)(D)) = c_k^n(c_i^{z(D)}(D)) = c_k^n(c_i^{z(D)}(D)) = c_k^n(c_i^{z(D)}(D))
\]

We conclude that

\[
\tilde{m}_j^p(c_k^n(D)(D)) = \tilde{m}_j^p(c_k^n(c_i^{z(D)}(D))(c_i^{z(D)}(D))) = \tilde{m}_j^p(c_i^{z(D)}(D)) = \tilde{m}_j^p(D),
\]

using the fact that \( m \) satisfies consistency. Thus, \( m \) satisfies consistency.

**Proposition 2.** Consistency, invariance to merging papers, and invariance to merging coauthors are logically independent.

**Proof.** First note that the egalitarian rule satisfies consistency and invariance to merging papers but not invariance to merging coauthors while the solo rule satisfies invariance to merging papers and invariance to merging coauthors but not consistency. We conclude the proof by exhibiting a rule that satisfies consistency and invariance to merging coauthors but not invariance to merging papers.

Let \( q \in C \). For each \( D = (C, w, S) \in D^N \), define the following associated databases:

- \( D_{-q} = (C', w', S') \in D^N \) such that \( C' = C \setminus \{q\} \) and, for each \( p \in C' \), \( w'(p) = w(p) \)
  and \( S'(p) = S(p) \). Thus, \( D_{-q} \) denotes the database obtained from \( D \) by removing paper \( q \).
- \( D_{+q} = (C', w', S') \in D^N \) such that \( C' = \{q\} \cup \bigcup_{i \in N} C_{\{i\}} \) and, for each \( p \in C' \), \( w'(p) = w(p) \)
  and \( S'(p) = S(p) \). Thus, \( D_{+q} \) denotes the database obtained from \( D \) by removing all joint papers except perhaps for \( q \).
Define the rule \( \dot{m} \) as follows: for each \( D = (C, w, S) \in \mathcal{D}^N \),

\[
\dot{m}^q(D) = \dot{m}^q(D_{+q}) \quad \text{and} \quad \dot{m}^p(D) = \dot{m}^p(D_{-q}) \quad \text{for each} \quad p \in C \setminus \{q\}.
\]

Rule \( \dot{m} \) violates invariance to merging papers because two papers with the same worth and the same group of authors (one of them being \( q \)) may be shared in different ways. It satisfies invariance to merging coauthors and consistency because the CoScore rule does.

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**Appendix D. Data**

The data was retrieved from Thomson Reuters’s Web of Science database on February 10th 2016 (http://ipscience.thomsonreuters.com/product/web-of-science/). We extracted all published articles with at least one citation from the following 33 major journals in Economics over the period 1970 – 2015:

- *Econometrica*,
- *Quarterly Journal of Economics*,
- *Journal of Economic Literature*,
- *American Economic Review*,
- *Journal of Political Economy*,
- *Review of Economic Studies*,
- *Journal of Monetary Economics*,
- *Journal of Economic Theory*,
- *Games and Economic Behavior*,
- *Journal of Economics Perspectives*,
- *Journal of Econometrics*,
- *Rand Journal of Economics*,
- *Economic Theory*,
- *Journal of Labor Economics*,
- *Journal of Human Resources*,
- *Journal of Public Economics*,
- *Journal of Risk and Uncertainty*,
- *International Economic Review*,
- *Journal of Applied Econometrics*,
- *European Economic Review*,
- *International Journal of Game Theory*,
- *Social Choice and Welfare*,
- *Journal of Environmental Economics and Management*,
- *Economic Journal*,
- *Journal of International Economics*,
- *Journal of Economic Dynamics and Control*,
- *Journal of Mathematical Economics*,
- *Economic Inquiry*,
- *Scandinavian Journal of Economics*,
- *Economics Letters*,

The Web of Science database is not entirely consistent regarding the name of the authors. Authors are sometimes identified with their first initial only, all their initials and in some cases their full name. We choose here to retain the first initial only, which prevents the same author from being identified under two or more labels but means that authors with the exact same name and first initial are collapsed together. Note that the
number of citations provided by Web of Science is usually smaller than the one of Google Scholar.

We compute CoScore by repeatedly iterating the fixed point operator $\phi$ (see Appendix A), starting from a vector of equal scores. We assign an extra 10 single authored citations to all authors in the database to compensate for the restricted set of journals. In doing so, we ensure that every author has at least one single-authored citation, so that CoScore is indeed well defined. The process converges quickly to the desired fixed point. After a hundred iterations, $\|\phi(x) - x\|_{\infty} = 0.0332 \leq 0.001\|x\|_{\infty}$, after a thousand iterations, $\|\phi(x) - x\|_{\infty} = 4.10^{-8} \leq 3.10^{-9}\|x\|_{\infty}$. Table 1 gives the top 15 economists according to CoScore.

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<th>Rank</th>
<th>Scholars</th>
<th># Papers</th>
<th>Solo cit. (%)</th>
<th># Coauthors</th>
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<th>Eg. Score</th>
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Table 1. Top 15 economists according to CoScore. Solo cit. gives the fraction of single authored citations.