Endogenous Entry in Markets with Adverse Selection*

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Abstract
Since Akerlof’s (1970) seminal paper the existence of adverse selection due to asymmetric information about quality is well-understood. Yet two questions remain. First, given the negative implications for trading and welfare, how do such markets come into existence? And second, why have many studies failed to find direct or indirect evidence of adverse selection? In addressing the first question directly we shed some light on the second.

We consider a market in which firms make an observable investment that generates products of a quality that becomes known only to the firm. Entry has the tendency to lower prices, which may lead to adverse selection. The implied price collapse limits the amount of entry so that high prices are supported in the market equilibrium, which results in above normal profits.

While contributing to our understanding of markets with asymmetric information and adverse selection, the model also provides insight into the question of why markets with adverse selection are empirically hard to identify. The analysis suggests that rather than observing the canonical market collapse, such markets are instead characterized by less entry than would be empirically predicted and above normal profits even in markets with low measures of concentration.

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1 Introduction

The inefficiencies associated with adverse selection are well known. The basic idea, introduced in Akerlof’s (1970) seminal paper, is familiar: There is a market in which products are differentiated only by their quality. If consumers cannot observe the quality of individual goods, then by the law of one price, all qualities are sold for the same price. If firms’ costs are increasing in quality, then at that single price the highest quality products may not be offered, whereas lower quality ones are. Poor quality drives out good quality and the amount exchanged is inefficiently low, perhaps even zero.

Notwithstanding the many applications and extensions of the basic model, two questions concerning such markets remain. The first is theoretical in nature: Given that especially high quality producers suffer the consequences of adverse selection, the question arises of how high quality firms find themselves in such an unenviable situation. Equilibrium reasoning suggests that forward looking firms should be able to anticipate and avoid such unfavorable outcomes.

The second puzzle concerning adverse selection is an empirical one. As Riley (2002) notes, research seeking empirical support for the potential role of introductory prices, advertising or warranties in overcoming the adverse selection problem draw at best only mixed conclusions. Remarkably, these studies do, however, provide evidence for some of the underlying assumptions of the basic model, namely that even when products are differentiated by quality, they may be subject to the law of one price so that higher quality does not command a price-premium.\(^1\) Moreover, these findings also suggest that high quality is not actually driven off the market. Indeed, many other studies seeking direct or indirect empirical verification for adverse selection in various settings have similarly found only relatively weak evidence.\(^2\)

In this paper we address the first puzzle directly by considering endogenous entry into the market. In doing so, we are able to shed some light on the second puzzle, showing that the two questions may actually be related. A two-stage game is used to model markets in which adverse selection can arise. In the first stage firms make an observable fixed investment in

\(^1\)See Gerstner (1985), Hjorth-Andersen (1991), Jones and Hudson (1996), or Ackerberg (2003); for contrasting findings see Wiener (1985).

quality. Similar to Daughety and Reinganum (1995), the quality of the product resulting from the investment is random and unobservable to the consumers. Then, firms who enter the market find themselves in the second stage competing in a market characterized by the salient features of adverse selection.

We show that if adverse selection takes hold, then even incremental entry may have substantial price effects, as the market collapses with bad quality driving out good quality, which has dramatic implications for profitability. Indeed, this mechanism can be viewed as a possible manifestation of the notion of ruinous or destructive competition. While recent work in this context focuses on uncertain demand (see, e.g., Deneckere et al., 1997) some, including legal scholars and policy makers (for instance, OECD, 2008, or Hovenkamp, 1989), see ruinous competition tied specifically to a deterioration in quality. In anticipation of this, firms rationally refrain from entering so that adverse selection and the associated market collapse coupled with a deterioration of quality does not arise in the market.

When latent adverse selection manifests itself in this way it results in ex ante positive profits in equilibrium. That is, the potential for adverse selection works as a barrier to entry. An implication of this is that it would be difficult to find empirical support for adverse selection, even though it is a salient feature of the market studied. Indeed, empirical support for the presence of latent adverse selection might be found in indirect evidence such as otherwise unexplained supra-normal profits, which may help to explain why above average rates-of-return are often observed in industries with uncertainty (e.g., Caballero and Pindyck, 1996, Guiso and Parigi, 1999, or Ghosal and Loungani, 2000).

The basic intuition behind why latent adverse selection affects entry is straightforward as is illustrated in the following example. There are ten price-taking firms each of which sells one indivisible unit. Firms possess a technology that either produces a high quality product at cost $2.20$, or a low quality product at cost of $1.50$. Demand for high quality goods is given by

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3In contrast, in Daughety and Reinganum (1995) it is a monopolist that makes an ex ante investment, which generates a distribution from which a random quality is drawn.

4As Hovenkamp (1989) notes, this point has long been acknowledged, but has, to our knowledge, not been formally modeled; see, for instance, Jenks (1888, 1889) or Jones (1914, 1920).

5Of course, an immediate corollary to this is that our model predicts that under unforeseen negative demand shocks markets do experience dramatic crashes that can be interpreted as ruinous competition coupled with a deterioration of quality.
\[ P = 7(1 - 0.05Q); \] that of low quality goods is \[ P = 2(1 - 0.05Q). \] The quality of an individual firm’s product is unobservable, but it is known that half of the firms offer high quality, so demand for goods of unknown quality is given by \[ \tilde{P} = (0.5 \times 7 + 0.5 \times 2)(1 - 0.05Q) = 4.5(1 - 0.05Q). \] The equilibrium price for the goods of unobservable quality when all ten firms produce one unit each is \[ P^{\ast} = 4.5(1 - 0.05 \times 10) = 2.25, \] which is sufficient to cover the cost of both low quality and high quality producers, yielding an expected profit of 0.40.

A potential entrant is contemplating this market. The quality of the entrant’s product is equally likely to be high or low \textit{ex ante} (e.g., this may be the result of R\&D that is required to enter the industry) so its expected costs are \[ 0.5 \times 1.50 + 0.5 \times 2.20 = 1.85. \] Should the firm enter this market? Remarkably, the answer is no.

Since the entrant’s level of quality is unobservable, market demand is—as before—given by \( \tilde{P} \). If the firm enters the market, the price with eleven units on offer in the market is thus \[ P^{\ast} = 4.5(1 - 0.05 \times 11) \approx 2.03. \] While this price is above the firm’s \textit{ex ante} expected costs of 1.85, it is insufficient to cover a high quality firm’s cost of 2.20. \textit{Ceteris paribus}, this need not be of concern to the eleventh firm, as it might contemplate entry in anticipation of becoming a low-quality producer. However, upon entry \textit{some} high quality firm is driven out of the market, so the average quality of the remaining output is decreased, which implies a decrease of demand, reinforcing the reduction in price. In other words, adverse selection takes holds of the market and—as can readily be verified for our illustrative example—all high quality firms leave the market.\textsuperscript{6} With only low quality firms in the market demand is given by \( P \), and so the price is no greater than \[ P = 2(1 - 0.05 \times 5) = 1.50. \textsuperscript{7} \] Consequently, low quality firms can at best only cover their costs and therefore make zero profit.

In sum, despite prices being well above cost in the ten-firm market, if an additional firm enters the market adverse selection sets in and the price plunges from 2.25 to 1.50. Hence, no investment made to enter the market—no matter how small—can be recovered. As a consequence no entry takes place: the latent adverse selection in the market serves as an entry barrier protecting above normal profits and, in equilibrium, there is no adverse selection

\textsuperscript{6}Consecutive market prices upon exit of 1, 2, 3, 4, 5 high-quality firms are approximately 2.13, 2.17, 2.14, 2.00, 1.69; none of which are sufficient to cover the costs of producing high quality.

\textsuperscript{7}There are at least the original five, but possibly six low quality firms in the market depending on the investment outcome of the entrant.
in the market: all firms—high and low quality alike—produce and sell their output.\(^8\)

The underlying mechanism that generates the result is that prices are a function of both the quantity and the average quality sold in the market. Entry reduces prices due to increased quantity, but the price reduction triggers adverse selection, further eroding profit, rendering initial entry costs unrecoverable. As a consequence, the long run entry equilibrium may result in positive profit while trade in the market does not exhibit adverse selection. That these insights are not merely a peculiarity of the illustrative example is demonstrated in the more general framework that is introduced after relating the main idea to recent work on adverse selection.

Since Akerlof’s seminal paper much of the theoretical literature has actually departed from his analysis by focusing on monopoly settings,\(^9\) which, of course, preclude entry. Less work has considered non-monopolistic environments.\(^10\) Most recently, however, Hendel and Lizzeri, (1999), Johnson and Waldman, (2003), Hendel, et al., (2005) reconsider Akerlof’s example of used cars in the context of a durable goods market. A major insight of these papers is that the used car market is tied to the new car market. However, supply in the new car market in these models is perfectly elastic at the given constant marginal cost of production so that the effects that we find cannot arise.

The general framework is introduced in Section 2, followed by the analysis of the equilibrium, in which we differentiate between the classic case of adverse selection resulting in a complete elimination of high quality from the market and a milder form of partial adverse selection in which some high quality remains on offer. Welfare and potential policy implications of the equilibrium are presented in Section 2.3, where it is shown that while the absence of adverse selection raises welfare compared to increased entry coupled with adverse selec-

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\(^8\)It should be noted that in keeping with our general set-up in the remainder of the paper, we have chosen the potential entrant’s R&D efforts in the illustrative example to yield a stochastic quality outcome. However, the limited entry/positive profit equilibrium derived in the example holds even if the firm is known to produce high quality, provided that consumers cannot distinguish which firm produced which units in the market. Specifically, for the case that consumers anticipate an additional unit of high quality in the market, demand is given by \( P = \left( \frac{6}{17} \frac{7}{17} \right) (1 - 0.05Q) = \frac{52}{17}(1 - 0.05Q) \), which with an output of 11 units in the market yields a price of approximately 2.13, i.e., below the cost of high quality so that the potential entrant refrains from entry, even if it is known that his product will raise the average quality in the market.


\(^10\)Notable exceptions are Wilson (1980) and Cooper and Ross (1984).
tion, welfare still falls short of second-best levels. However, there is a revenue-neutral policy that allows the achievement of the second-best welfare optimum. Section 2.4 concludes the basic analysis with some technical conditions that differentiate markets with the potential for milder forms of adverse selection. In Section 3 we examine the robustness of the findings by considering alternative frameworks. In particular, we examine the case of generalized quality distributions (3.1), Cournot competition (3.2) and monopolistic firms (3.3). This is followed in Section 4 by some concluding remarks.

2 Entry and Welfare in the Base Model

In this section we present the basic model with a binary distribution of quality. We consider the entry equilibrium, while distinguishing markets in which under adverse selection no high quality is traded (“classic” adverse selection) from those in which some high quality producers continue to sell (“mild” adverse selection). Thereafter we analyze the welfare properties of the equilibrium configurations and propose a revenue-neutral welfare enhancing tax cum subsidy scheme that results in the attainment of the second-best welfare optimum. We conclude with a discussion of technical conditions that differentiate markets in which mild adverse selection may occur from markets where this phenomenon does not arise.

2.1 The Base Model

Consider a two-period model of a market for a good of which the quality characteristics are inherently unobservable. In the first period firms install a fixed level of capacity, normalized to 1, by making an investment outlay of \( \iota \geq 0 \). The investment cost may reflect discovery costs associated with securing requisite inputs or basic research and development outlays. The quality of the resulting output of the firm is unknown \( \text{ex ante} \), but has a binomial distribution: there is a probability \( \gamma \) that a firm’s product is of high quality and \( (1 - \gamma) \) is the probability that it is of low quality. That is, after making the investment, nature selects each firm to be of high quality with probability \( \gamma \). At the end of the first period, the firm observes the quality that it can produce after its investment outlay of \( \iota \) is sunk.

In the second period market exchange takes place. Since quality is unobservable, the
market clears at one price, \( P \). Firms act as price takers vis-à-vis that price and make a production decision that maximizes their market profit (gross of entry costs, which are sunk at this stage) \( \pi := P - c \), given their costs, \( c \). The cost of producing a unit of low quality is given by \( c \); whereas a firm selling a unit of high quality incurs a cost of \( \bar{c} \), with \( 0 < c < \bar{c} \). The cost of a unit of high quality either represents a production cost or can be thought of as an opportunity cost, as is done, for example in Daughety and Reinganum (2005). This latter case occurs, for instance, if there is an alternative use for the product, as is the case in Akerlof’s (1970) archetypal paper in which used car owners may choose to keep their cars, or horse-breeders who choose to hold on to some yearlings (Chezum and Wimmer, 1997); or if high-quality products can be sold at a given price in an alternative market in which quality is independently verified—as is the case with vintners who can sell their grapes to a négociant, rather than selling under their own label (Lonsford, 2002a,b), or electronics manufacturers who sell their products to name brands for retail (see, e.g., Financial Times Information, 2000). The opportunity cost interpretation may also be particularly relevant in international trade settings in which high-quality producers in less developed economies in which certification is not fully credible have the ability to access foreign markets with established certification measures.\(^{11}\)

Inverse demand for products of (known) high quality is given by \( P(Q) \), whereas demand for products of low quality is \( P(Q) \left( < P(Q), \forall Q \right) \)—both twice continuously differentiable and strictly decreasing. While firms know the quality of their product, the quality characteristics of any given good on offer is unobservable to consumers. At the beginning of the second period, consumers know the number of firms that invested in order to sell in the market, but not each firm’s output. Consumers also know the \textit{ex ante} distribution of quality that can be delivered, given by \( \gamma \). On the basis of this consumers form beliefs about the quality composition of overall market supply. Letting \( \alpha \) denote the consumers’ perception of the fraction of high quality products on offer (which can differ from \( \gamma \), depending on firms’

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\(^{11}\)The two interpretations of the the unit cost for high quality (i.e., production costs or opportunity costs) are isomorphic whenever the investment outlay is not so low that firms invest solely in the hopes of obtaining high quality for the alternate use. Consequently, all the derived results continue to hold under the opportunity cost interpretation provided that the critical thresholds on \( \iota \) derived in the paper are shifted by the amount of this added profit opportunity.
production choices), market demand is

\[ P(Q, \alpha) := \alpha \overline{P}(Q) + (1 - \alpha) \underline{P}(Q). \quad (1) \]

Since both demand for high quality goods \( \overline{P} \) and demand for low quality goods \( \underline{P} \) are strictly decreasing, \( P(Q, \alpha) \) is strictly decreasing in its first argument. And, since \( \overline{P} > \underline{P} \), market demand is increasing in its second argument, reflecting the greater willingness to pay for higher quality. In equilibrium, consumers have rational expectations about the expected (i.e., average) quality on offer so that \( \alpha \) correctly reflects the average quality of the goods in the market.

We assume that selling some high-quality goods is efficient, (i.e., \( \overline{c} < P(0, 1) \)) and that producing some low-quality goods is also efficient (i.e., \( \underline{c} < P(0, 0) \)). As a result, \( Ec := \gamma \overline{c} + (1 - \gamma) \underline{c} < P(0, \gamma) \) so that there is positive demand for at least some output, given the prior distribution on quality. Finally, since we are interested in market constellations in which adverse selection may occur we assume for convenience that when beliefs rule out high quality, the market price is insufficient to cover the costs of producing high quality, i.e., \( P(0, 0) < \overline{c} \).

Our assumptions characterizing the market equilibrium are standard in the literature on adverse selection, yet these do not always identify a unique equilibrium. In particular, there is the possibility of a coordination failure in which high-quality firms under-produce for no reason other than consumers do not expect them to produce. In order to assure that firms’ entry decisions are not driven by equilibrium selection, we use the Pareto selection criterion to eliminate all but one market equilibrium, whenever multiple equilibrium configurations exist. This means that we restrict attention to the equilibrium with the greatest average quality of output in the market.\(^{12}\) As a result of this assumption, for any number of firms in the market there is a unique equilibrium price \( P^*(n) \), which implies a well-defined expectation of market profit for the \( n^{th} \) firm prior to entry, i.e., \( E\pi(n) = P^*(n) - Ec = P^*(n) - [\gamma \overline{c} + (1 - \gamma) \underline{c}] \).

For purposes of greater clarity, we treat the number of firms \( n \) as coming from a continuum. Consequently, any above-normal profit equilibrium is not due to the well-known

\(^{12}\)Wilson (1980) considers the possibility of multiple equilibrium configurations and notes that these can be Pareto-ranked by increasing prices; Rose (1993), however, finds that generally a unique equilibrium emerges. In our model there is multiplicity, including the possibility of more than one equilibrium with the same price so that average quality, rather than price, yields the relevant Pareto-ranking.
integer constraint problem, but is a general characteristic of the equilibrium that occurs even when \( n \) is an integer. Moreover, for expositional expediency, we characterize symmetric equilibrium configurations in which firms choose mixed strategies over binary production plans, i.e., they choose a probability with which they either produce at full capacity, or shut down. Other equilibrium configurations, involving asymmetric pure strategies, or fractional capacity utilization rates, yield identical insights.

2.2 Endogenous Entry and Market Equilibrium

The entry equilibrium is determined by the last firm that expects to recover its entry costs of \( \iota \) upon entering the market.\(^{13}\) Due to downward sloping demand for a given quality composition, as firms enter the market the increase in supply drives down the market price. Consequently firms’ expected market profits are diminished upon entry. Entry continues up to the point where the marginal firm’s expected market profit upon entering \( E\pi \) no longer exceeds its entry cost of \( \iota \). We consider how this process plays out in the equilibrium of the entry game, and what the implications of the entry equilibrium are on the market equilibrium.

We first consider high entry costs and demonstrate that in the resulting zero-profit entry equilibrium no adverse selection occurs in the market. Second, we consider lower entry costs (including the possibility of zero entry costs) and examine markets in which adverse selection leads to all high quality being taken off the market so only low quality is traded.\(^{14}\) We refer to this market outcome as “classic” adverse selection, and show that the possibility of classic adverse selection may function as a barrier to entry so that the entry equilibrium is associated with positive (long-run) profits and there is no adverse selection in the market.

We conclude this subsection by considering a milder form of adverse selection in which some, but not all high quality is taken off the market. We show that with small (possibly even zero) entry costs adverse selection may still function as an entry barrier, resulting in

\(^{13}\)In the special case of costless entry, the Pareto criterion yields that firms refrain from entering when they are indifferent between entering or not.

\(^{14}\)Using the Pareto selection criterion in conjunction with our assumption that it is efficient to produce some low quality precludes a complete market collapse. However, these cases are easily subsumed in the current analysis.
positive long-run expected profit in concurrence with mild adverse selection; while classic adverse selection is prevented from occurring in the market equilibrium. We leave for later a more technical discussion of the conditions on demand, costs, and quality that allow for mild adverse selection to occur.

2.2.1 High Entry Cost: Zero Profit and No Adverse Selection

The entry equilibrium is determined once the marginal firm is left without positive overall expected profit when contemplating incurring the investment outlay of $\iota$, given its expected market profit upon entering. Thus, if entry costs $\iota$ are large, a firm must expect high market profits upon entry in order to enter the market. Since expected costs of the firm are exogenous, the only way to support high expected profits is through a high market price. However, a price that is high enough to induce entry of the last firm, may be sufficiently high so that all firms in the market—regardless of their quality and cost characteristics—can cover their costs of production. Consequently, there is no adverse selection in the market when prices are sufficiently high. This leads to our first result concerning endogenous entry in markets with the potential for adverse selection; namely that high entry costs imply a zero-profit entry condition and a market in which there is no adverse selection. Formally,

Proposition 1 (High Entry Costs Prevent Adverse Selection) There exists $\tau$ such that for all $\iota \geq \tau$,

1. there is no adverse selection in the market, i.e., all high quality producers are active in the market and the average quality in the market is characterized by $\gamma$;

2. firms make zero expected profit, i.e., $E\pi - \iota = 0$; and

3. if entry takes place, the equilibrium number of firms $n^*$ is implied by $P(n^*, \gamma) = \iota + Ec$.

Proof. Suppose first that no adverse selection occurs in the market. Then market demand is given by $P(Q, \alpha) = P(n, \gamma)$. Since $P(n, \gamma)$ is decreasing in $n$, entry ceases once the price has declined to the point where it is just sufficient to cover the expected production cost $Ec$ and the entry cost $\iota$, i.e., equilibrium entry is defined by $P(n^*, \gamma) = \iota + Ec$, proving the third statement. At this price, expected market profits are $E\pi = P(n^*, \gamma) - Ec = \iota$,
proving the second statement. In order to confirm our initial supposition that no adverse selection takes place, thereby completing the proof, it must be the case that the market price is sufficiently high to cover the production costs of the high quality producer. That is, 
\[ P(n^*, \gamma) = \iota + Ec = \iota + \gamma \bar{c} + (1 - \gamma)\zeta \geq \bar{c}. \]
Let \( \bar{\iota} = (1 - \gamma)(\bar{c} - \zeta). \)

In this equilibrium *ex ante* profits are zero and all firms produce at full capacity. It follows that under endogenous entry adverse selection is not observed when there are sufficiently high entry costs despite the salient features of adverse selection being present. To put this more succinctly: when entry costs are high, few firms enter. And when few firms enter, a high price is sustained. And when the price is high, all firms can cover their costs. Finally, when all firms can cover their costs there is no adverse selection.

Note the critical threshold of entry costs identified in the proposition, \( \bar{\iota} = (1 - \gamma)(\bar{c} - \zeta). \) This threshold is exactly equal to the *ex ante* expected profit of a firm when the market price only just covers the cost of producing high quality, i.e., when \( P = \bar{c} \) so that only low quality producers obtain positive profit. We now turn to how endogenous entry affects markets when entry costs are lower.

### 2.2.2 Positive Profit and No Adverse Selection

In Proposition 1 entry costs are identified that are so high that entry into the market is restricted and only few firms enter. As a result of the small number of firms in the market, prices are sufficiently high to support full production by high quality producers so that adverse selection does not occur in the market. We now suppose that entry costs are below the threshold identified in Proposition 1 and show that the resulting increase in entry still need not result in adverse selection. Indeed, despite lower entry costs, latent adverse selection can serve as an effective entry barrier under which no adverse selection occurs in the market and the long-run entry equilibrium entails above normal profit, as is the case in the example in the introduction.

In order to demonstrate this, note first that for adverse selection to *not* occur all firms must be offering their output for sale. This only happens if the resulting market price is no lower than the cost of producing high quality. Let \( \overline{n} \) denote the largest number of firms that
the market can sustain under full production without adverse selection setting in. It follows that $\pi$ is implicitly given by

$$P(\pi, \gamma) = \bar{c}. \tag{2}$$

Firms’ expected market profits (i.e., gross of entry costs $\iota$, but before production costs are known) at $\pi$ are given by

$$E\pi (\pi) = P(\pi, \gamma) - [\gamma\bar{c} + (1 - \gamma)c] = (1 - \gamma)(\bar{c} - c). \tag{3}$$

This is the critical threshold on entry costs, identified in Proposition 1, above which entry falls short of levels that may trigger adverse selection. We now derive the entry equilibrium when entry costs are below this level of ex ante expected market profit. That is, we consider cases in which, in contrast to Proposition 1, $\iota < \bar{\tau}$.

As noted, if firms in excess of $\pi$ enter the market, then adverse selection takes place. In the current analysis we restrict attention to the classic case of adverse selection known from the literature in which high quality is driven out and only poor quality remains to be traded in the market.\footnote{Necessary and sufficient conditions for this case are given in Subsection 2.4.}

**Proposition 2 (Classic Adverse Selection as an Entry Barrier)** Suppose that for any $n > \pi$ the market suffers from classic adverse selection so that only low quality is traded in the market. Then there exists $\iota \in [0, \tau)$ such that for all $\iota \in [\iota, \bar{\tau})$,

1. there is no adverse selection in the market, i.e., all high quality producers are active in the market and the average quality in the market is characterized by $\gamma$;

2. firms make positive expected profit, i.e., $E\pi - \iota > 0$; and

3. the equilibrium number of firms is $n^* = \pi$.

**Proof.** An implication of Proposition 1 is that since $\iota < \bar{\tau}$ entry assuredly takes place at least till $\pi$. Beyond that, take the proposed entry equilibrium as given and consider the marginal firm at $n^*$. Since $n^* = \pi$, incremental entry triggers adverse selection which, by assumption, leads to all high quality producers shutting down. Thus, the only possibility of
obtaining positive market profit for the marginal firm is if it is a low quality (and hence low cost) producer. Upon entry, if all low quality firms produce, the resulting market price is $P((1 - \gamma)\pi, 0)$. If $P((1 - \gamma)\pi, 0) \leq c$, then firms make no profit so that incremental entry beyond $n^* = \pi$ does not pay off, even under the assumption of costless entry. Hence, let $\xi = 0$ and all three statements of the proposition follow readily.

Suppose instead that $P((1 - \gamma)\pi, 0) > c$. Then, if the marginal firm enters and is a low quality producer, its profit is $P((1 - \gamma)\pi, 0) - c$. Hence, the marginal firm’s expected market profit prior to but conditioned on incremental entry is given by $(1 - \gamma) (P((1 - \gamma)\pi, 0) - c)$. Let $\xi = (1 - \gamma) (P((1 - \gamma)\pi, 0) - c)$ and it is clear that entry beyond $\pi$ does not take place.

Note finally that since $P((1 - \gamma)\pi, 0) < P(0, 0) < \tau$, it follows that $\xi < \tau$. This establishes that long-run profits are positive, since $E\pi(n^*) = E\pi(\pi) = P(\pi, \gamma) - Ec = \tau - (\gamma\tau + (1 - \gamma)c) = (1 - \gamma)(\tau - c) = \tau > \xi$. ■

Proposition 2 demonstrates that even with entry costs that do not limit entry to a zero-profit equilibrium, adverse selection need not occur in the market, as the potential for adverse selection itself can work as an effective entry barrier. Having assumed that it is efficient to produce some low quality, a complete collapse (i.e., a no-trade equilibrium) as in Akerlof’s (1970) paper does not occur (although we can easily also allow for this outcome, and the insights follow even more readily). Nevertheless, when $\xi = 0$ entry is limited to $n^* = \pi$, resulting in potentially substantial above-normal expected profit of $(1 - \gamma)(\tau - c)$ even in the long run equilibrium with costless entry.

Propositions 1 and 2 suggest that when one considers entry in markets with the characteristic features of adverse selection, then adverse selection does not in fact take hold of the market whenever entry costs are above $\xi$, where—depending on characteristics of demand, ex ante quality and costs—$\xi$ can be arbitrarily small, or even zero. If entry costs are high, then the entry equilibrium is characterized by the common zero-profit condition. However, if entry costs are low, the latent adverse selection leads to a long-run equilibrium in which firms’ average market profits are above the cost of entry. These results may provide the explanation for why empirical research frequently fails to uncover direct or indirect evidence of adverse selection. However, the propositions suggest alternative tests for these markets,
namely either high entry costs serving as a barrier to entry which prevents adverse selection from taking hold of the market (Proposition 1); or above normal profit without additional entry (Proposition 2).

In the analysis thus far (in particular in Proposition 2) we have restricted attention to instances in which what we termed classic adverse selection may affect the market, namely that adverse selection implies that high quality producers shut down and only low quality is on offer in the market. We have characterized how such latent adverse selection serves as an entry barrier that prevents adverse selection and preserves above normal profit in the long run equilibrium. If, instead, one considers milder forms of adverse selection, entry may take place beyond $n$, while above-normal long-run profit remain a feature of the equilibrium. We now discuss this case.

### 2.2.3 Positive Profit with Mild Adverse Selection

Proposition 2 is concerned with the case of classic adverse selection in which all high quality producers exit and only low quality producers remain, should adverse selection set in. However, recall that given our assumption of downward sloping demand prices are a function of not only the average quality in the market, but also of the quantity on offer in the market. And thus, firms shutting down and exiting has—all else equal—the tendency to increase prices in the market. Hence, if entry beyond $n$ takes place so that the price is insufficient to cover the expense of producing high quality when all firms produce, some high quality firms (albeit not necessarily all) opt out of production. As this reduces the quantity on offer in the market, the price tends to rise, possibly allowing those high quality firms that did not opt out of production to cover their production costs.

An illustration of this can be found with a minor modification of the example given in the introduction. Thus, if in that example the cost of producing high quality is lower, say, 2.10 rather than 2.20, then the eleventh firm will enter the market, knowing full well that if it obtains high quality either itself or a rival high quality firm will no longer produce, resulting in a price of about 2.13. However, entry of a twelfth firm would not take place, since this would surely trigger classic adverse selection and render any investment outlay unrecoverable, regardless of the firm’s type. Hence the long run equilibrium would, once
again, be characterized by classic adverse selection serving as an entry barrier that preserves above normal profit. And while mild adverse selection is a feature of the market equilibrium; classic adverse selection is not.

Whether such an adjustment can take place in any given market depends critically on how firms’ choices affect the composition of quality and quantity in the market. To formalize this, recall market demand as a function of quantity and average quality, given in Equation 1 and reproduced here:

\[ P(Q, \alpha) = \alpha \overline{P}(Q) + (1 - \alpha)\underline{P}(Q). \]  (4)

The feature of demand that underlies Proposition 1 is that \( P(Q, \alpha) \) is decreasing in its first argument. In contrast, the proof of Proposition 2 relies on demand being increasing in its second argument. Specifically, notice that in markets that exhibit classic adverse selection with entry beyond \( \pi \), as considered in Proposition 2, \( \alpha \) takes on the value of either \( \gamma \) (no adverse selection) or 0 (classic adverse selection). This discontinuity (\( \alpha \) switching from \( \gamma \) to 0) when adverse selection sets in is central to the positive profit result in Proposition 2. In departure from the previous analysis, we now consider situations in which both \( Q \) and \( \alpha \) may vary continuously as firms in the market alter their production plans continuously—potentially leading to mild adverse selection.

If entry beyond \( \pi \) takes place and all firms in the market produce, then—by definition of \( \pi \)—the price is below \( \overline{c} \), so high quality producers make negative profit in the market. Consequently, at least some high quality producers will refrain from producing, which reduces market output. Since \( P(Q, \alpha) \) is decreasing in its first argument, the reduction of output—all else equal—yields a higher market price. Note, however, that all else is not equal: as only high quality producers cease production the positive quantity effect is countered by a negative quality effect since the average quality of the goods on offer deteriorates. With this we formalize the notion of “mild” adverse selection.

Lemma 1 (Mild Adverse Selection) A market has the potential for an equilibrium with mild adverse selection whenever for some \( n > \pi \) there exists \( \kappa \in (0, 1) \) such that

\[ P\left((1 - \gamma + \kappa \gamma)n, \frac{\kappa \gamma}{1 - \gamma + \kappa \gamma}\right) = \overline{c}; \]  (5)

in which case \( \kappa \) is the proportion of high quality firms in the market that produce.
Proof. First, by definition of \( \bar{n} \) there exists a full-production equilibrium with no adverse selection for \( n \leq \bar{n} \). Thus, given the Pareto equilibrium selection criterion, mild adverse selection cannot occur for any \( n \leq \bar{n} \), so we require that \( n > \bar{n} \) (which precludes \( \kappa = 1 \)).

Second, note that \( P < \bar{c} \) cannot be an equilibrium since it entails negative market profits for high quality producers, which are avoided by shutting down (implying \( \kappa = 0 \)); and \( P > \bar{c} \) cannot be an equilibrium as an idle high-quality producer increases profit by producing and selling a unit of the good. Hence, for an equilibrium with mild adverse selection to occur, it must be that \( P = \bar{c} \).

At this price all low quality firms produce, yielding output of \( (1 - \gamma)n \). If a fraction \( \kappa \) of the \( \gamma n \) high quality firms produce, market output is thus \( (1 - \gamma + \kappa \gamma)n \) and the proportion of high quality is \( \frac{\kappa \gamma}{1 - \gamma + \kappa \gamma} \). Therefore the existence of a \( \kappa \) such that Equation 5 holds for some \( n > \bar{n} \) is a necessary and sufficient condition for the market to have an equilibrium with mild adverse selection. 

While we leave a more technical and detailed discussion of the occurrence of mild adverse selection for the end of this section, it is worth noting at this stage that if several values for \( \kappa \) that satisfy Equation 5 exist, then the Pareto equilibrium selection criterion eliminates all but the largest of these. However, it is important to note that conditions for mild adverse selection need not exist in a given market: While high quality producers are indifferent between producing and not producing when the price is \( \bar{c} \), their decisions affect average quality and thus consumers’ willingness to pay. This, in turn, affects the market price for given market output and firms are no longer indifferent between producing or not at prices that are different from \( \bar{c} \) so that firms’ production plans are adjusted. Thus, output and average quality must be determined simultaneously and must yield a price of \( \bar{c} \). Such balancing is not always possible. Indeed, in the initial example used in the introduction to the paper no such balancing is possible so that mild adverse selection cannot occur.

Having formalized the condition for mild adverse selection, we now consider markets in which such a trade-off between quantity and quality can sustain the production of some high quality in the market when entry beyond \( \bar{n} \) takes place and re-examine the case of low entry
Proposition 3 (Mild Adverse Selection and Positive Profits) Suppose that the market has the potential for mild adverse selection. Then there exists $\underline{\iota}' \in [0, \overline{\iota})$ such that for all $\iota \in [\underline{\iota}', \overline{\iota})$,

1. there is mild adverse selection with a fraction $\kappa$ of high quality producers still operating in the market, so that $\alpha \in (0, \gamma)$;

2. firms make positive expected profit, i.e., $E\pi - \iota > 0$; and

3. the equilibrium number of firms is $n^* > \overline{n}$.

Proof. As in the proof to Proposition 2, note that an implication of Proposition 1 is that since $\iota < \overline{\tau}$ entry assuredly takes place at least till $\overline{n}$. However, unlike in the proof of Proposition 2, from Lemma 1, given mild adverse selection there exists $n > \overline{n}$ such that the expected market price is $P = \overline{\iota}$; and, therefore, entry continues beyond $\overline{n}$; confirming the third claim in the proposition.

Now define $\overline{n}'$ as the largest number of firms such that mild adverse selection can be sustained, i.e., $P (\overline{n}') = \overline{\iota}$ and $P(n) < \overline{\iota}, \forall n > \overline{n}'$. Then the remainder of the proof follows the proof of Proposition 2 mutatis mutandis with $\overline{n}'$ replacing $\overline{n}$. In particular, if $P ((1 - \gamma)\overline{n}', 0) \leq \underline{c}$, let $\underline{\iota}' = 0$; and if $P ((1 - \gamma)\overline{n}', 0) > \underline{c}$, let $\underline{\iota}' = (1 - \gamma) (P ((1 - \gamma)\overline{n}', 0) - \underline{c})$.

When conditions on demand, the distribution of quality, and costs allow for mild adverse selection, this leads to entry beyond $\overline{n}$, whenever entry costs are not prohibitive, i.e., they are not above $\overline{\iota}$. However, as some high quality firms opt not to produce, average quality in the market deteriorates upon entry beyond $\overline{n}$. At some point continued entry leads to such a deterioration of average quality of the goods on offer that consumers are no longer willing to pay a price that covers the costs of producing high quality, at which point further entry results in classic adverse selection in the market.

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\(^{16}\)The analysis of high entry costs given in Proposition 1 is independent of whether the market has the potential for mild adverse selection.
A comparison between markets with the possibility of mild adverse selection with those where only classic adverse selection can occur is not directly possible, since these markets must differ in some aspects of demand, cost, or the exogenous quality parameter. However, it may nonetheless be worth noting that if the markets are sufficiently similar in the relevant aspects, then the threshold level identified under mild adverse selection, \( \zeta' \), may be smaller than that under classic adverse selection, \( \zeta \), since mild adverse selection permits entry beyond \( \pi \). In particular, if demand for low quality, the cost of producing low quality, and the exogenous probability of being of high quality are the same across markets and if demand for and costs of high quality are such that \( \pi \) is the same in both markets, then \( \zeta' \leq \zeta \) with equality only when \( \zeta' = \zeta = 0 \).

Thus, loosely speaking, while markets with mild adverse selection are more prone to exhibit adverse welfare effects of adverse selection, a complete market collapse—and hence the most drastic implication for welfare—is more likely to be deterred, since the critical threshold for entry costs is low. This observation naturally leads to a more detailed examination of welfare in these markets.

### 2.3 Welfare

The focus of the preceding analysis has been firms’ forward looking decisions based on their profit considerations. In this subsection we expand the analysis beyond profit in order to assess overall equilibrium welfare under endogenous entry in markets with adverse selection. To this end, let \( CS(n) \) denote consumer surplus in the (unique) market equilibrium with \( n \) firms and define \( W(n) := CS(n) + nE\pi(n) \) as the total welfare in the market when \( n \) firms have entered.

Proposition 1 establishes that when entry costs are above \( \tau \) the entry equilibrium yields the maximum welfare. This follows, because given that there is no adverse selection in the market, there is no welfare loss in the market; and given that firms’ entry decisions yield zero expected profit, any increase in the welfare in the market upon entry is insufficient to offset additional entry costs. This insight does not apply to the cases of Propositions 2 and 3 since these equilibrium configurations are characterized by positive profits. However, as these equilibrium configurations also do not exhibit classic adverse selection, positive profits
need not imply welfare losses compared to increased entry. Indeed, limited entry not only protects the above normal profits, but also protects consumer surplus in the market that arises because high quality is traded. Formally,

**Proposition 4 (Welfare Preservation)** When \( \iota \in (\underline{\iota}, \overline{\iota}) \) so that latent adverse selection serves as an entry barrier and \( n^* = \overline{n} \), overall welfare is preserved compared to market settings with an increased number of firms entering.

**Proof.** Note first that market welfare is increasing in \( n \) for \( n \leq \overline{n} \). Consider now welfare for \( n \geq \overline{n} \) and denote by \( \kappa \) the portion of high quality firms who produce in the market so that \( \kappa = 0 \) in markets with classic adverse selection and \( \kappa \in (0, 1) \) in markets with mild adverse selection. Market output is thus given by \( Q = (1 - \gamma + \kappa \gamma)n \) and the proportion of high quality in the market is given by \( \alpha = \frac{\kappa \gamma}{1 - \gamma + \kappa \gamma} < \gamma \).

Welfare at \( \overline{n} \) is given by

\[
W(\overline{n}) = CS(\overline{n}) + \overline{n} \times E\pi(\overline{n}) = \int_{0}^{\overline{n}} [\gamma \overline{P} + (1 - \gamma)\overline{P} - \overline{c}] dQ + \int_{0}^{\overline{n}} (1 - \gamma)(\overline{c} - \bar{c})dQ + \int_{0}^{\overline{n}} (1 - \gamma)(\overline{c} - \bar{c})dQ.
\]

The second and fourth integral are both positive, let their sum be denoted by \( A \); and the first and third can be combined to yield

\[
W(\overline{n}) = \int_{0}^{(1 - \alpha)\overline{n}} [\gamma \overline{P} + (1 - \gamma)\overline{P} - \overline{c}] dQ + A.
\]

When replacing \( \gamma \) with \( \alpha \) in the integral, the integral itself is the market welfare under incremental entry beyond \( \overline{n} \). However, as \( \gamma > \alpha, \overline{P} > P > 0 \) and \( A > 0 \), there is a discrete fall in welfare upon entry beyond \( \overline{n} \). Note lastly that for entry beyond that welfare decreases as average profit weakly decreases and output and average quality also decrease; with another discrete fall in welfare at \( \overline{n}' \) in the case of markets with mild adverse selection.

In other words, when entry costs are are such that \( \iota \in (\underline{\iota}, \overline{\iota}) \) (for classic adverse section and \( \iota \in (\underline{\iota}', \overline{\iota}) \) for mild adverse selection), entry beyond the entry equilibrium reduces market
welfare, as the market collapses and classic adverse selection occurs. Hence, the welfare losses associated with classic adverse selection are averted.

Nevertheless, the fact that profit is not competed away in the entry process suggests the potential for welfare improving policies. Indeed, it is still possible for a welfare-maximizing government to raise welfare, even when the quality of the individual firms is also unobservable to the government (i.e., second-best welfare maximization). To see this, note that while entry beyond \( n \) necessarily (weakly) reduces the market price, incremental entry beyond \( n \) coupled with a commitment to full production by all firms (including all high quality firms, who then produce at a loss) yields a price that is above expected production and entry costs (i.e., \( P(n + \epsilon, \gamma) > \gamma \tau + (1 - \gamma) \xi + \iota \), with \( \epsilon \) small, but positive). Such incremental entry increases welfare because the gain to consumers, due to increased production and increased average quality, is greater than the loss to high quality firms from producing without being able to cover production costs. Hence, the equilibrium entry level is less than the second best welfare optimum.

It may seem counter-intuitive that it is socially optimal to have a high quality firm sell in the market in which it earns negative economic profits, but the high quality firm creates a positive externality by increasing the average quality in the market. Thus, the constrained welfare-optimal amount of entry, denoted by \( n^{**} \), is obtained when entry costs are just offset by market profit under (forced) full production, i.e., \( P(n^{**}, \gamma) = \gamma \tau + (1 - \gamma) \xi + \iota \).

Thus, while forward-looking firms refrain from entering and thereby prevent the welfare losses associated with adverse selection, the entry level is inefficient compared to the second-best welfare optimum. Despite investment-entry being socially insufficient, the traditional solution to increase entry—i.e., subsidizing investment-entry—does not work. This is because the additional entry that the subsidy induces does not result in the positive market externality of high quality output, since at the point of the production decision, the subsidy is sunk and high quality producers are better off refraining from production. That is, the negative welfare effects of limited entry are not curbed by the introduction of an investment-entry subsidy. Indeed, this suggests a further indirect test for the presence of adverse selection in markets, namely that investment-entry subsidies (short of the remaining above-normal profit) do not affect the market equilibrium.
Although an investment-entry subsidy does not move the market towards the second-best welfare maximum, the classic solution of offering a production subsidy for firms in the market does so, provided that this policy is announced before investment-entry occurs. Specifically, a production subsidy that covers the high quality firm’s short-fall of revenue over costs, i.e., $\overline{c} - P(n^{**}, \gamma)$, results in the second best welfare optimum: all high quality firms are able to cover their costs of production when $n^{**}$ firms enter, as they sell their product at the price of $P(n^{**}, \gamma)$ and obtain the subsidy. This outcome, however, continues to result in positive ex ante (expected) profits, since high quality firms break even and low quality firms make positive profit. Nevertheless, despite the positive profits at this new level of entry, additional firms do not enter, as otherwise this added entry again reduces prices to a level where adverse selection sets in, which renders investment costs unrecoverable.

It should be noted that the welfare-increasing policy can be made revenue-neutral. This is done by imposing an investment-entry tax in the first period equal to the value of the subsidy. With this tax and the production subsidy, expected profits for the $n^{**}$ firms that enter are zero. Consequently, such a revenue neutral policy is welfare enhancing even if the subsidy and tax fall short of the optimal level, since the increased entry coupled with the positive market externality from sustained production of high quality raises welfare. If, instead, the tax and subsidy is set above the optimum, then the optimal level of investment-entry (i.e., $n^{**}$) followed by full production still results, because investment-entry greater than $n^{**}$ generates negative profits and so entry beyond $n^{**}$ does not occur. We summarize this discussion in the following proposition.

**Proposition 5 (Revenue-Neutral Welfare Optimizing Policy)** The second best social welfare optimum can be achieved with a period-two production subsidy and a revenue-neutralizing period-one investment tax. Moreover, even if the government sets the wrong subsidy level, as long as there is a revenue-neutralizing investment tax, welfare increases.

An advantage of such a combined policy in which firms are first taxed and later subsidized is that the policy is easy to implement. In contrast to previous suggestions that restrict the subsidy to high-quality producers (see, e.g., Bagwell and Staiger, 1989, or Grossman and Horn, 1988), there is no need for the verification of a firm’s quality as all firms receive
the subsidy. Hence, firms need not worry about the possibility of an erroneous or faulty application of the subsidy rule, which otherwise might lead high-quality producers to refrain from producing.

Despite the fact that the proposed policy in Proposition 5 does not require verification of quality since the subsidy applies indiscriminately to all firms, the policy is costless due to its revenue-neutrality. Hence the government need not know if there is latent adverse selection, that is, if there is no latent adverse selection, then the investment-entry decision is unaffected. A final advantage of the proposed policy is that, since the policy is revenue neutral, an industry will only lobby for it when the policy increases overall welfare.

2.4 Conditions For Classic Adverse Selection

Since the notion of mild adverse selection as opposed to classic adverse selection is novel to this paper and since the two types demonstrate some slight differences in the properties of the entry and market equilibrium, we briefly delineate what distinguishes the one form from the other. As a matter of nomenclature, we refer to a market in which classic adverse selection may occur, but mild adverse selection cannot happen, as a market with “classic adverse selection” (even though in equilibrium there is no adverse selection when \( \iota > \iota' \)). We otherwise speak of a market with “mild adverse selection” (even though this market also does not exhibit any adverse selection when \( \iota > \iota' \) and exhibits classic adverse selection when \( \iota < \iota' \)).

In line with Lemma 1 let \( \kappa \) denote the proportion of high quality firms in the market that actually produce (so that the proportion \((1 - \kappa)\) of high quality producers shut down). Then, for a given number of firms in the market, \( n \), market output is given by \( Q(\kappa|n) := (1 - \gamma + \kappa \gamma)n \); and the proportion of high quality in the market is given by \( \alpha(\kappa) := \frac{\kappa \gamma}{1 - \gamma + \kappa \gamma} \). Define the market price (i.e., a firm’s revenue) for given \( n \) and given \( \kappa \) by

\[
R(\kappa|n) := P(Q(\kappa|n), \alpha(\kappa)) = \alpha(\kappa) P(Q(\kappa|n)) + (1 - \alpha(\kappa)) P(Q(\kappa|n)).
\] (6)

This representation allows one to consider how the market price varies with incremental
changes in the proportion of high quality output in the market. In particular,

\[
\mathcal{R}'(\kappa|n) = \frac{dR}{d\kappa} = \frac{\partial P}{\partial Q} \frac{dQ}{d\kappa} + \frac{\partial P}{\partial \alpha} \frac{d\alpha}{d\kappa} = \left( (1 - \alpha)P' + \alpha P' \right) \gamma n + (P - P) \frac{\gamma (1 - \gamma)}{(1 - \gamma + \kappa \gamma)^2}.
\]

When considering a reduction in \( \kappa \), the first term is the slope of the demand curve for a given quality composition of output, so this term captures the positive price effect of a reduction in output. This term is weighted by \( \gamma \), since we are only considering variations in output from high quality producers alone, not all firms in the market. The second term measures the (negative) effect on the price premium that consumers are willing to pay for high quality over low quality, weighted by the marginal impact of decreases in average quality, due to a reduction in \( \kappa \). Whether these two effects can offset each other in such a way to establish a market price that leaves high quality firms indifferent about their production decision, i.e., \( \mathcal{R}(\kappa) = \bar{c} \), determines whether a market can exhibit “mild adverse selection” (see Lemma 1). In particular then, a market exhibits classic adverse whenever

\[
\mathcal{R}(\kappa|n) < \mathcal{R}(1|\bar{n}) = \bar{c}, \quad \forall \kappa \in [0, 1] \text{ and } n > \bar{n}.
\]

In order to better understand the condition, consider a market in which there are currently \( \bar{n} \) firms so that the market price is just sufficient to cover the cost of high quality production, i.e., \( \mathcal{R}(1|\bar{n}) = P(\bar{n}) = \bar{c} \). At this point, for classic adverse selection to not occur, the negative price effects of incremental entry of average quality must be offset by the positive price effects of incremental exit of high quality, when taking account of the negative price effect of deterioration of quality in the market as average quality enters and high quality exits. Formally, suppose that \( \mathcal{R}'(1|\bar{n}) < 0 \) (which is a sufficient condition for a market with mild adverse selection). This states that a marginal reduction in high quality leads to an increase in the price when the market is at \( \bar{n} \). Note that \( \mathcal{R}' \) is continuous in \( n \). Therefore a marginal change in \( n \) does not change the sign of \( \mathcal{R}' \), implying that an increase in price, due to incremental entry beyond \( \bar{n} \), can be offset by an incremental reduction in high quality output. If this is not the case, then the possibility that an incremental reduction in high quality can yield mild adverse selection is precluded. This yields,
Lemma 2 (Necessary Condition for Classic Adverse Selection) A necessary condition for a market with classic adverse selection (i.e., no mild adverse selection) is that

\[ R'(1|\pi) \geq 0. \]

The condition given in Lemma 2 is necessary, but not sufficient to assure that Equation 7 holds, since mild adverse selection need not be the result of a marginal adjustment process. In particular, there are market constellations in which upon incremental entry beyond \(\pi\) mild adverse selection emerges due to a (potentially large) positive measure of high quality firms ceasing production. Indeed, in the example given in the introduction, if the cost of producing high quality is given by 2.15, rather than 2.20, then upon entry of the eleventh firm in the market, high quality cannot cover its cost even after the exit of one high-quality producer as the price drops to 2.13. However, if two high quality producers simultaneously exit, the price increases to 2.17, which is sufficient to cover high quality costs. That is, while a marginal reduction in high quality output may not suffice to restore an equilibrium, a large reduction (falling short of complete shut-down of high quality) may yield an equilibrium with mild adverse selection.

We now consider conditions that render Lemma 2 sufficient for a market with classic adverse selection.

Lemma 3 (Sufficient Condition for Classic Adverse Selection) A sufficient condition for a market with classic adverse selection (given Lemma 2) is that

\[ R''(\kappa|\pi) \neq 0, \]

i.e., \(R(\kappa)\) is either strictly concave or strictly convex when evaluated at \(\pi\).

Proof. If \(R(\kappa|\pi)\) is convex, then it lies below any of its secant lines. Consider the secant line constructed from the points \(\kappa = 0\) and \(\kappa = 1\), i.e., \(S(\kappa) := R(0|\pi) + [R(1|\pi) - R(0|\pi)] \kappa\); or \(S(\kappa) = (1 - \kappa)P((1 - \gamma)\pi, 0) + \kappa c\), since \(R(0|\pi) = P((1 - \gamma)\pi, 0)\) and \(R(1|\pi) = c\). Notice that \(P((1 - \gamma)\pi, 0) < P(0, 0)\), since \(P\) is decreasing in its first argument. Since \(P(0, 0) < c\), it follows that \(R(\kappa|\pi) < S(\kappa) < (1 - \kappa)P(0, 0) + \kappa c < c\), \(\forall \kappa \in (0, 1)\).

Now suppose that \(R(\kappa|\pi)\) is concave. Then the function lies below any of its tangent lines. Since \(R'(1|\pi) > 0\), it therefore follows that \(R(\kappa|\pi) < R(1|\pi) = c\), \(\forall \kappa < 1\). Hence, regardless
of whether $R(\kappa|\bar{n})$ is concave or convex, $R(\kappa|\bar{n}) < \bar{c}$, $\forall \kappa \in (0, 1)$ so that incremental entry beyond $\bar{n}$ does not result in mild adverse selection.

Note finally that $\frac{d}{dn} R = (1 - \alpha)P' + \alpha \bar{P}' (1 - \gamma + \gamma \kappa) < 0$ so that $R(\kappa|n) < \bar{c}$, $\forall n > \bar{n}$, violating Lemma 1 and, thus, ruling out mild adverse selection for any $n \geq \bar{n}$. 

Lemma 3 essentially imposes a simple smoothness condition on the price adjustment process as quantity and quality vary. The condition can be made weaker, since high quality being driven entirely off the market only requires that once—for a fixed number of firms in the market—the price reaches an extremum under variation in the quality make-up of supply, then this extremum is not just local, but also global. For instance, either quasi-concavity or quasi-convexity of $R$ is also sufficient to guarantee the desired result.

We close this section with two final observations. First, while the primary argument made is applied to conditions when there are $\bar{n}$ firms in the market, the final argument in the proof of Lemma 3 establishes that mild adverse selection can be ruled out for measurable entry beyond $\bar{n}$ (i.e., a coordinated simultaneous entry of several firms). Second, it is straightforward to show that Lemma 3 always holds when demand is not too convex (e.g., linear) and the price premium function (i.e., $\bar{P} - P$) is either decreasing or elastic.

### 3 Robustness, Generalizations and Extensions

In this section we offer some results on the robustness of the insights by considering some generalizations and extensions. First, we consider a version of the model with a continuous distribution of types which yields similar insights to those already established. We then briefly illustrate the main results for imperfectly competitive markets by considering a second period game in which firms are Cournot competitors. This is followed by a sketch of how in monopoly settings in which the firm invests in order to produce a good of uncertain quality the result of diminished entry (viz. reduced capacity) also occurs.
3.1 Continuous Distribution of Quality

Though we focused on the case of two distinct levels of quality in the main body of the paper, all the insights essentially carry over to cases in which quality is distributed continuously. We now suppose that quality, which is indexed by \( s \), is distributed \textit{ex ante} according to the strictly increasing and twice differentiable distribution function \( F(s) \) on \([s, \bar{s}]\). The cost associated with producing a unit of the good with quality index \( s \) is given by the strictly increasing twice differentiable function \( C(s) \). Given the Pareto selection criterion in conjunction with the law of one price, if it is profitable for a firm of quality index \( \sigma \in [s, \bar{s}] \) to produce, it is also profitable for all firms with quality index \( s \leq \sigma \) to produce. All other assumptions on firms remain the same. In particular, we consider a continuum of firms who each observe an independent draw from the distribution of quality parameters \( F(s) \) upon entry. Consequently there is no aggregate uncertainty and the distribution of quality among the firms in the market is also characterized by \( F(s) \).

Demand for quality \( s \) is given by \( p(Q, s) \), which is is twice differentiable and decreasing in market output \( Q \) and increasing in quality \( s \). Define demand for the case that \( \sigma \) is the highest level of quality on offer by \( P(Q, \sigma) := \int_{s}^{\sigma} \frac{p(Q,s)\,dF(s)}{F(\sigma)} \) and it follows that \( P(Q, \sigma) \) is also twice differentiable, decreasing in \( Q \), and increasing in \( \sigma \). Assume that the lowest quality alone cannot support efficient market transactions, i.e., \( P(0, s) \leq C(s) \); whereas there is potential for trade given the \textit{ex ante} average quality, i.e., \( P(0, \bar{s}) > C(\bar{s}) \). Hence, \( \bar{n} \) is implied by \( P(\bar{n}, \bar{s}) = C(\bar{s}) \).

It readily follows that the analogue to Proposition 1 holds with \( \tau = C(\bar{s}) - Ec \), where \( Ec := \int_{s}^{\bar{s}} C(y)\,dF(y) \) is the expected cost of a firm under the prior distribution of quality. For this case \( n^* \) is then implied by \( P(n^*, \bar{s}) = \iota + Ec \) with \( \iota \geq \tau \).

In order to distinguish the cases of classic adverse selection from mild adverse selection define similarly to Equation 6,

\[
\mathcal{P}(\sigma|n) := P(nF(\sigma), \sigma) - C(\sigma). \tag{8}
\]

That is, \( \mathcal{P}(\sigma|n) \) is the equilibrium market profit of the marginal producer with quality index \( \sigma \), given that \( n \) firms are in the market.\(^{17}\)

\(^{17}\)Because in the two-type case there is only one cost-type (the high cost firm) who makes the marginal
We define classic adverse selection in this context as a case where a marginal deterioration of quality leads to a market collapse, i.e., a discrete drop in average quality and market price so that no firms continue to produce. In contrast, mild adverse selection entails marginal exit of high quality in such a way that prices adjust smoothly to the altered conditions in the composition of supply. Hence, analogous to Equation 7, the condition that characterizes markets with classic adverse selection is given by

\[ P(\sigma|n) < P(\bar{s}|\bar{n}) = 0, \quad \forall \sigma \in [\underline{s}, \bar{s}] \text{ and } n > \bar{n}. \]

The necessary and sufficient conditions for a market to exhibit classic adverse selection are

\[ P'(\bar{s}|\bar{n}) \geq 0, \]
\[ P''(\sigma|\bar{n}) \neq 0. \]

The proofs follow mutatis mutandis those of Lemmata 2 and 3 from the two-type case and are therefore omitted.

Intuitively speaking the necessary condition assures that if entry beyond \( \bar{n} \) takes place so that the price decreases due to the increased supply, profit of firms at the upper end of the quality support decrease, which implies that a positive measure of high quality firms must cease production. The sufficient condition then guarantees that not only do a positive measure of firms near the upper end of the quality support exit, but so do in fact firms of all quality types.

Given these conditions, the results of positive profits and no adverse selection in the market (Proposition 2) and positive profits with mild adverse selection (Proposition 3) carry over with only minor qualifications to the current setting as illustrated in the following two examples.

**Positive Profits and No Adverse Selection** Let quality be distributed uniformly on the unit interval, i.e., \( F(s) = s \) on \([0, 1]\) and let costs be given by \( C(s) = \sqrt{s}/3 \). Demand for given quality is \( p(Q, s) = s(1 - Q) \), so \( P(Q, \sigma) = \int_0^\sigma \frac{s(1-Q)}{\sigma} ds = \frac{\sigma}{2}(1 - Q) \).
Given these parameters, \( P(\sigma|n) = \frac{\sigma}{2}(1-n\sigma) - \sqrt{\sigma}/3 \) and \( P'(\sigma|n) = 1/2 - n\sigma - 1/(6\sqrt{\sigma}) \).

The full production threshold \( \bar{\pi} \) is implied by \( P(\bar{\pi}, 1) = C(1) \), i.e., \( \frac{1}{2}(1 - \bar{\pi}) = 1/3 \), so \( \bar{\pi} = 1/3 \). Thus, \( P'(\sigma = 1|n = 1/3) = 1/2 - 1/3 - \frac{1}{6} = 0 \), so the necessary condition for classic adverse selection is met. Note that when entry is at \( \bar{\pi} = 1/3 \) average market profits are given by \( P(\bar{\pi}, 1) = \int_{\frac{1}{2}}^{\bar{\pi}} C(s)dF(s) = P(1/3, 1) - \int_0^{1/3} (\sqrt{s}/3)ds = \frac{1}{2} (1 - \frac{1}{3}) - \frac{1}{9} = \frac{1}{9} \). Hence, \( \tau = \frac{1}{9} \).

Now consider \( n > \bar{\pi} = 1/3 \) and note that the highest quality producer’s market profit must be zero. From Equation 8 we have

\[
P(\sigma|n > \bar{\pi}) = P(nF(\sigma), \sigma) - C(\sigma) = \frac{\sigma}{2}(1-n\sigma) - \frac{1}{3} = 0.
\] (9)

However, for all \( n > \bar{\pi} = 1/3 \) Equation 9 does not have a non-negative root in \( \sigma \) so there exists no market equilibrium with production for \( n > \bar{\pi} \) and therefore \( \tau = 0 \), \( n^* = \bar{\pi} = 1/3 \) and in the long-run equilibrium firms make an average profit of \( 1/9 - \tau > 0 \).

**Positive Profits with Mild Adverse Selection** Consider the above example, now with costs given by \( C(s) = \sqrt{s}/4 \). Then \( \bar{\pi} = 1/2 \), since \( \frac{1}{2}(1 - \bar{\pi}) = 1/4 \); and \( P'(\sigma = 1|n = 1/2) = 1/2 - 1/2 - 1/8 = -1/8 < 0 \), so the necessary condition for classic adverse selection is violated (i.e., the sufficient condition for mild adverse selection is met).

Note that when \( \bar{\pi} = 1/2 \) firms enter, average market profits are \( P(1/2, 1) - \int_{0}^{1/2} (\sqrt{s}/4)ds = \frac{1}{2} (1 - \frac{1}{2}) - \frac{1}{6} = \frac{1}{12} \), so \( \tau = \frac{1}{12} \).

Now, analogous to Equation 9, the equilibrium condition for the highest level of quality for entry beyond \( \bar{\pi} = 1/2 \) is given by

\[
P(\sigma|n > \bar{\pi}) = \frac{\sigma}{2}(1-n\sigma) - \frac{\sqrt{\sigma}}{4} = 0.
\] (10)

This equation does have a root in \( \sigma \) provided that \( n \leq 16/27 \), but not for entry beyond that, so \( \bar{\pi}' = 16/27 \). At \( \bar{\pi}' \) Equation 10 reveals that \( \sigma = 9/16 \). A firm’s expected market profit (after entry, but before quality and costs are realized) at this point is given by \( F(\sigma)P(\sigma|\bar{\pi}') = 9/256 \). So for \( \tau \in [0, 9/256) \), \( n^* = 16/27 \) and long-run equilibrium profit is \( 9/256 - \tau > 0 \).

The main distinction between Proposition 3 for the two-type case and the current example for a continuous distribution of quality concerns firms’ profits under mild adverse selection. In particular, where in Proposition 3 firms retain positive long-run profit for any entry
cost between $\ell'$ and $\bar{\ell}$, this is not the case in the present example. Specifically, the entry equilibrium configuration for $\ell \in [9/256, 1/12 = \bar{\ell}]$ entails zero expected profit as firms enter beyond $\bar{n} = 1/2$ and quality gradually adjusts with the implied price decline. However, such gradual adjustment is not possible beyond $n^* = 16/27$ at which point a positive profit equilibrium emerges when entry costs are below $9/256$.

### 3.2 Cournot Competition

In the base model we have followed the canonical example of adverse selection in Walrasian markets. Here we consider the case of adverse selection in markets with Cournot competitors and demonstrate how our main insights apply to this setting as well. Indeed, to our knowledge there are no models of adverse selection in imperfectly competitive markets in the literature.

Firms sequentially enter the market making an investment outlay of $\ell$ in order to obtain an (unlimited) capacity to produce the good, which is of high quality with probability $\gamma$ and of low quality with probability $1 - \gamma$. Either quality of the good is produced at a constant marginal cost, which we normalize to zero; however, high quality firms incur a fixed cost of production of $C$, whereas low quality producers have fixed costs of $C$, with $C > C \geq 0$.\(^{18}\) Demand is as before in the base model.

Since all firms have the same constant marginal cost, all active firms produce the same amount of output in equilibrium. As before, let $\kappa$ denote the proportion of high quality producers that are active in the market so that the number of active firms is given by $(1 - \gamma + \kappa \gamma) n$. The proportion of high quality in the market is again given by $\alpha(\kappa) = \frac{\kappa \gamma}{1 - \gamma + \kappa \gamma}$. Then, letting $q(\kappa|n)$ denote an active firm’s output, its equilibrium revenue is, analogous to Equation 6, given by

$$R(\kappa|n) = P((1 - \gamma + \kappa \gamma) q(\kappa|n), \alpha(\kappa)) q(\kappa|n).$$

\(^{18}\)Allowing for differential marginal costs is also possible, but comes at the expense of more cumbersome derivations which we wish to avoid in this extension so as to demonstrate the primary issue at hand.
Yielding the two familiar conditions for classic adverse selection,

\[ R'(1|\pi) \geq 0, \]

\[ R''(\kappa|\pi) \neq 0, \]

where \( \pi \) is implied by \( R(1|\pi) = P(\pi q(1|\pi), \gamma)q(1|\pi) = \bar{C} \), since in order for all high quality firms in the market to produce, revenue must cover their costs. We briefly illustrate this market with an example.

**Positive Profit and No Adverse Selection** Demand for goods of (known) high quality is given by \( P = 1 - Q \) and we assume that low quality alone cannot sustain the market, i.e., demand for low quality is naught. Hence,

\[ P(Q, \alpha) = \alpha(1 - Q). \]

The standard linear-demand Cournot framework yields that with \( n \) firms in the market, each active firm produces

\[ q(\kappa|n) = \frac{1}{(1 - \gamma + \kappa \gamma)n + 1}, \]

implying

\[ Q(\kappa|n) = \frac{(1 - \gamma + \kappa \gamma)n}{(1 - \gamma + \kappa \gamma)n + 1}. \]

A firm’s revenue is thus

\[ R(\kappa|n) = \alpha(\kappa)(1 - Q(\kappa|n))q(\kappa|n) = \alpha(\kappa)q(\kappa|n)^2, \]

so that \( \pi = \sqrt{\gamma/\bar{C} - 1} \), since \( R(1|\pi) = \bar{C} \). At \( \pi \) the firms’ expected market profits are \( (1 - \gamma)(\bar{C} - \bar{C}) \), which fixes \( \tau \).

Note that

\[ R'(1|\pi) = \alpha'(1)q(1|\pi)^2 + \alpha(1)2q(1|\pi)q'(1|\pi) = [\alpha'(1) - \alpha2\gamma\pi q(1|\pi)]q(1|\pi)^2 \]

\[ = \left[ 1 - 3\gamma + 2\gamma\sqrt{\bar{C}/\gamma} \right] \bar{C}, \]

so, analogous to Lemma 2, for sufficiently small \( \gamma \) the necessary condition for classic adverse selection is met.
Moreover,

\[ R''(\kappa|\pi) = \alpha''(\kappa)q(\kappa|\pi)^2 - 4\alpha'(\kappa)\gamma\pi q(\kappa|\pi)^3 + 6\alpha(\kappa)\gamma^2\pi^2 q(\kappa|\pi)^4 \]

\[ = \alpha''(\kappa)q(\kappa|\pi)^2 - \alpha'(\kappa)\gamma\pi q(\kappa|\pi)^3 - (3\alpha'(\kappa)q(\kappa|\pi)^2 - 6\alpha(\kappa)\gamma\pi q(\kappa|\pi)^3)\gamma\pi q(\kappa|\pi). \]

It can be shown that the sufficient condition always holds whenever the necessary condition holds. That is, Equation 11 is both necessary and sufficient. Thus, we let \( \gamma = 2/5, C = 1/36 \) and \( \underline{C} = 0 \). Then the conditions are met and \( \pi \approx 2.8 \). Entry beyond \( \pi \) leads to a market collapse with no firms earning any revenue, hence \( \underline{\pi} = 0 \). Therefore \( n^* = \pi \) and the long-run equilibrium average profit is \( (1 - \gamma)(\underline{C} - \underline{C}) = 1/60 \) even with costless entry.

### 3.3 Monopolistic Markets

Having shown that limited entry and above normal profit can occur even under costless entry in Walrasian and Cournot markets due to latent adverse selection, we now briefly consider the case of monopolistic markets. As the monopoly market implies restricted entry, it is clear that profits are expected to occur in equilibrium and therefore the point of this section is to demonstrate that latent adverse selection nonetheless affects the market equilibrium. In particular, the potential for adverse selection leads to “limited entry” in terms of a reduced capacity choice by the monopolistic firm. Coupled with the result is, similar to the other models, that the market equilibrium exhibits no adverse selection.

Formally, we suppose that the firm incurs an investment outlay of \( \iota \) in order to obtain an observable production capacity which, for consistency of notation with the base model, we denote by \( n \). After the capacity decision, the firm observes the quality of its product as being high with probability \( \gamma \) or otherwise low. In order to not distract from the point at hand, we preclude signalling equilibrium configurations by assuming that low quality alone cannot sustain sales, which implies that a low quality-producer will always mimic the strategy of the high-quality producer, thus, eliminating any separating equilibrium. Once the firm knows its quality and costs, it chooses a price and then produces output \( Q \leq n \).

Suppose \( \gamma = 2/3 \); demand for known high quality is given by \( P = 6(1 - 0.05Q) \), whereas there is no demand for low quality. Hence demand for average quality is \( P = 4(1 - 0.05Q) \). Unit cost of high quality is \( c = 3 \) and \( \underline{c} = 0 \). Consumers (rationally) anticipate that a high
quality producer would leave his capacity unused, if the price is below $\bar{c}$, since this price is below the unit cost of production. However, above this price, as either type would in fact sell (and the low quality producer would sell whatever the high quality producer sells at these prices), demand follows the demand for expected quality. In sum

$$P = \begin{cases} 4(1 - 0.05Q) & \text{if } Q < 5 \\ 0 & \text{if } Q \geq 5. \end{cases}$$

Thus, the firm will only be able to sell when facing demand of $P = 4(1 - 0.05Q)$. If the firm has high costs, it produces $Q^* = 2.5$, which is also produced if the firm has low cost in order to mimic the high cost firm. Hence $n^* = 2.5$.

In contrast, if the firm were to be known to produce high quality its output is 5 and it is 0 if it is known to produce low quality, yielding an average output of $n^{FI} := (2/3)5 + (1/3)0 = 10/3 > 2.5 = n^*$. And optimal output under average quality produced at average costs is $n^{Avg} := 5 > 2.5 = n^*$.\(^{19}\) Thus, one obtains reduced capacity compared to either benchmark with $\iota = 0$.

Since positive profit naturally occurs in the monopolistic setting, this cannot be used to empirically detect the impact of the potential of adverse selection on the market. Notice, however, that if data can be obtained on the expectation of marginal costs, then if this average is below marginal revenue this is an empirical indication of lower than expected capacity, due to latent adverse selection.

4 Conclusion

This paper develops an explanation of how markets with the salient features of adverse selection arise by having the heretofore exogenous number of firms made endogenous. Firms enter through a fixed investment after which nature chooses the quality of the firm’s product that is unobservable to consumers. It is found that the potential for adverse selection—low quality producers driving out high quality ones—affects the market even though adverse

\(^{19}\)The fact that $n^{FI} < n^{Avg}$ is superficially similar to the insight that under demand uncertainty firms may under-invest in capacity—a point raised elsewhere in the literature, e.g., Pindyck (1988). The fact the effects of latent adverse selection are distinct from this is demonstrated by the fact that $n^* < n^{FI}$. 

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selection does not arise in equilibrium. Further, this latent adverse selection can lead to entry equilibrium configurations with positive profits, even under the assumption of costless entry.

The unobserved adverse selection and positive profits result from the interaction of two classic mechanisms. First, demand slopes downward, so less entry results in higher prices. Second, if the market price is high enough, then high quality producers are willing to produce. Hence, zero profits may no longer define the entry equilibrium. Instead the entry equilibrium is defined by the greatest level of entry under which adverse selection does not occur \textit{ex post}. That is, latent adverse selection is an entry barrier, and whenever it defines the entry equilibrium then equilibrium profits are positive even under costless entry.

The analysis has some additional insights. First, the role of downward sloping demand suggests it may play an important role in models of endogenous quality that heretofore have used unit demand—indeed, in our setting downward sloping demand gives rise to a form of “mild” adverse selection in which only some high quality producers exit the market. Secondly, it is found that the limited entry prevents welfare losses stemming from adverse selection, so that overall welfare is greater despite profits not dissipating. Nevertheless, welfare can be raised further through a revenue-neutral policy of an investment tax and a production subsidy. The revenue neutrality implies that even an incorrectly set tax and subsidy raises welfare. Finally, the findings suggest that in industries with high entry costs one would not find either direct or indirect empirical evidence of adverse selection, even though the market exhibits the characteristics for adverse selection. In cases of lower entry costs, indirect empirical evidence for latent adverse selection can be found in the absence of actual adverse selection coupled with positive profits that are not competed away.

While the main focus is on Walrasian markets, it is shown that the principle insights—in particular limited entry, positive long-run profits and the absence of observed adverse selection—hold also for Cournot markets with endogenous entry, and the result of latent adverse selection as an entry barrier carries over to the monopoly setting in the form of reduced capacity.
References


