Discussion Paper Series

2007 – 4

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Bookmaker and pari-mutuel betting: Is a (reverse) favourite-longshot bias built-in?*

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November 26, 2007

Abstract

A widely documented empirical regularity in gambling markets is that bets on high probability events (a race won by a "favourite") have higher expected returns than bets on low probability events (a "longshot" wins). Such favourite-longshot (FL) biases however appear to be more severe and persistent in bookmaker markets than in pari-mutuel markets; the latter sometimes exhibit no bias or a *reverse* FL bias. Our results help understand these differences: the odds grid in bookmaker markets leads to a built-in FL bias, whereas that used in pari-mutuel betting pushes these markets toward a *reverse* FL bias.

JEL classification: G13; L13; L83

Keywords:

Gambling; Favourite-Longshot Bias; Bookmaker Betting; Parimutuel Betting; Breakage; Tick Size

*This paper benefited from helpful comments from Michael Mandler, David Metcalf, David Paton, Jonathan Wadsworth, Justin Wolfers, and participants of the conference on "The Growth of Gambling and Prediction Markets: Economic and Financial Implications" in May 2007. We are grateful to Paul Davidson, Tony Lusardi, Chris Ralph, and Leighton Vaughan Williams for information about bookmaker odds and bookmaker software.

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1 Introduction

The favourite-longshot (FL) bias is a widely documented empirical regularity in gambling markets, according to which the expected returns from betting on competitors with low predicted chances of winning a race or a game ("longshots") are lower than those from betting on competitors with high predicted chances of winning ("favourites"). However, the phenomenon appears to differ across the two main forms of betting, which are bookmaker markets, where intermediaries offer fixed odds to the public, and pari-mutuel (totalisor or tote) markets, where punters' stakes are pooled and distributed to the holders of winning bets. Documented FL biases tend to be stronger in bookmaker betting than in pari-mutuel markets (Sauer 1998, Figure 1, Coleman 2004, Snowberg and Wolfers 2005, Figure 2), even when both markets operate in parallel (Gabriel and Marsden 1990, 1991, Bruce and Johnson 2000, Cain et al. 2001). Examples of gambling markets which exhibit no bias or a *reverse* FL bias are all from pari-mutuel betting[1] or from US money-line markets in hockey and baseball.[2] An exception is Cain et al. (2003), who find weak evidence of a reverse FL bias based on a tiny sample of extreme favourites in British soccer (10 quotes) and greyhound (4 quotes) bookmaker markets. While an extensive literature exists on the potential sources of a FL bias in each of these types of betting markets,[3] it is less well understood why the FL bias is less pronounced or even reversed in pari-mutuel betting. Our paper contributes toward answering this question by showing that the odds grid commonly employed in bookmaker markets leads to a built-in tendency to produce a FL bias, whereas that used in pari-mutuel betting pushes these markets toward a *reverse* FL bias.

Bookmaker markets usually operate with less than 100 different odds levels, whereas pari-mutuel rules lead to hundreds of thousands of different odds levels. To understand the significance of this for betting returns, note that a bet on a race entrant to win is a state-contingent security. The quoted odds can be thought of in terms of the price that they imply for a lottery with £1 payout if the competitor wins. If the bet is a fair one, this price is equal to the objective win probability of the entrant – or put differently, each odds level has an odds-implied win probability,[4] which we

[1] Busche and Hall (1988), Busche (1994), Swidler and Shaw (1995), Busche and Walls (2000), Gandar et al. (2001).

[2] See Woodland and Woodland (1994, 2001, 2003) and Gandar et al. (2002, 2004). Money-line markets are special in that odds come in pairs, i.e. the odds quoted on one team lock in the odds for the other team. In other bookmaker markets, odds quoted on individual entrants are not linked.

[3] For surveys see Thaler and Ziemba (1988), Sauer (1998), Vaughan Williams (1999), and Jullien and Salanié (2008).

[4] As explained in Section 2 the odds-implied win probability is equal to $1/[odds+1]$, where odds are to a £1 stake.

call the price of the bet. Hence, for such a bet the

$$\text{expected return} = \frac{(\text{objective win probability}) - \text{price}}{\text{price}}. \quad (1)$$

The bigger the discrepancy between objective win probability and price in relation to the price of the bet, the lower the return.

In bookmaker markets, the odds grid is relatively fine for low odds (with high odds-implied win probabilities) and relatively coarse for high odds (with low odds-implied win probabilities), leading to a built-in tendency toward a FL bias. For example, entrants with "fair" odds between 1/40 and 1/25 will have quoted odds of 1/40 in the absence of any other source of bias; which implies that the expected return on the bet is at worst - 1.4%. In contrast, for entrants with "fair" odds between 500/1 and 1000/1 quoted odds will be 500/1, which implies that the expected return on the bet can be as low as -50%.

Pari-mutuel markets subject payouts to *breakage*. In the UK, for example, winning payouts are rounded down to the nearest 10p. This leads to an equally spaced odds grid that has a built-in tendency toward a *reverse* FL bias. For entrants with high chances of winning (i.e., at the low odds end), the payout to a winning £1 stake will be very low. Say the "fair" odds imply a winning payout between £1.10 and £1.19. Then breakage will take away up to 9p, so the expected return may be as low as -7.6%. In contrast, breakage will hurt relatively less winning payouts for bets on entrants with low chances of winning (i.e., at the high odds end). Say the "fair" odds imply a winning payout between £100.10 and £100.19, then the expected return will be -0.09% at worst. Below we develop the intuition given here more fully and provide formal conditions for a FL bias or its reverse.

Our systematic analysis of the link between characteristics of the odds grid and betting returns develops themes from the following papers. Schnytzer and Shilony (2006, p.289) observe that the discrete nature of odds "seems to prevent complete removal of the bias" in bookmaker markets. For pari-mutuel betting, Coleman (2004) was the first to explicitly note the potential link between breakage and the reverse FL bias observed in some of these markets. His paper then investigates empirically the impact of breakage on this market, as do Busche and Walls (2001), Walls and Busche (2003a,b), and Gramm and Owens (2005) (see Section 4). Our results complement other explanations for why a FL bias arises in gambling markets.[5] Different degrees of bias in pari-mutuel betting have been attributed to heterogeneous information (e.g., Ali 1977, Ottaviani and Sørensen

[5]For a complete review see the surveys cited in footnote 3.

2006) and varying levels of sophistication of bettors (e.g., Sobel and Raines 2003).[6] Uninformed bettors cannot distinguish good bets from bad ones, and therefore tend to put too little money on favourites and too much money on longshots. Arbitrage opportunities are limited by the *take* (the tax on the betting pool levied by the pari-mutuel operators) and the size of the betting pool (Hurley and McDonough 1995, Terrell and Farmer 1996, Busche and Walls 2000, Gramm and Owens 2005). In bookmaker markets, variations in bias have been attributed to different exposure to insider information (Shin 1991, 1992, 1993, Vaughan Williams and Paton 1997). Ottaviani and Sørensen (2005) show that this can also lead to a FL bias in pari-mutuel betting, if insiders are cash constrained. Vaughan Williams and Paton (1998) and Ottaviani and Sørensen (2006) offer information based rationales for *reverse* FL biases. The relevance of our results is that any of the proposed forces toward a FL bias are likely to be exacerbated by the impact of the odds grid in bookmaker betting and attenuated or even reversed in pari-mutuel markets. This helps explain why observed FL biases in bookmaker betting are stronger and more persistent than those in pari-mutuel markets.

The paper is organised as follows. Section 2 discusses the characteristics of bookmaker odds. Section 3 introduces the model and derives conditions for the bookmaker odds grid to produce a FL bias. The model is then applied to the UK bookmaker odds, comparing the predicted pattern of betting returns from simulations with that present in actual race data. Section 4 derives implications for pari-mutuel markets and simulates the pattern of betting returns predicted by the model for the UK Tote odds grid. The final section discusses our findings and their relation to the empirical evidence on differences in biases across gambling markets.

2 Bookmaker odds

Bets in bookmaker markets are struck using a limited list of odds. In some cases, this is simply a consequence of regulations. For example, South Australia imposed a list of 59 quotable odds levels between 1/10 and 500/1 (see Table 1 and Independent Gambling Authority 2000, part 5, rule 27). However, similar grids prevail in Ireland and the UK, where no regulatory body specifies an official odds grid that must be adhered to. Table 1 lists the 88 odds levels from UK starting prices[7] in

[6] Henery (1985), Vaughan Williams and Paton (1998), and Levitt (2004) propose related informational explanations for biases in bookmaker prices.

[7] Starting prices are the odds at which a "sizeable" bet could have been placed with on-course bookmakers at the off, independently determined by official starting price reporters. In horse and greyhound racing, punters on- and off-course have the option to bet at the starting price or at the offered fixed odds.

Odds no.[a]	Fractional odds[b]	Net decimal odds (ω_i)	Implied win probability $\left(p_i = \frac{1}{\omega_i+1}\right)$	Odds no.	Fract. odds	ω_i	p_i	Odds no.	Fract. odds	ω_i	p_i	Odds no.	Fract. odds	ω_i	p_i
1	1/40$^\diamond$	0.025	0.976	***23***	4/9*	0.444	0.692	***45***	85/40	2.125	0.32	***67***	15/1*	15	0.063
2	1/25$^\diamond$	0.04	0.962	***24***	40/85	0.471	0.68	***46***	9/4	2.25	0.308	***68***	16/1*	16	0.059
3	1/20$^\diamond$	0.05	0.952	***25***	1/2*	0.5	0.667	***47***	5/2*	2.5	0.286	***69***	18/1	18	0.053
4	1/16$^\diamond$	0.063	0.941	***26***	8/15	0.533	0.652	***48***	11/4	2.75	0.267	***70***	20/1*	20	0.048
5	1/14$^\diamond$	0.071	0.933	***27***	4/7	0.571	0.636	***49***	3/1*	3	0.250	***71***	22/1	22	0.043
6	1/12$^\diamond$	0.083	0.923	***28***	8/13	0.615	0.619	***50***	10/3	3.333	0.231	***72***	25/1*	25	0.038
7	1/10*	0.1	0.909	***29***	4/6	0.667	0.600	***51***	7/2*	3.5	0.222	***73***	28/1	28	0.034
8	1/9	0.111	0.9	***30***	8/11	0.727	0.579	***52***	4/1*	4	0.200	***74***	33/1*	33	0.029
9	2/17	0.118	0.895	***31***	4/5*	0.8	0.556	***53***	9/2*	4.5	0.182	***75***	40/1*	40	0.024
10	1/8*	0.125	0.889	***32***	5/6	0.833	0.545	***54***	5/1*	5	0.167	***76***	50/1*	50	0.020
11	1/7*	0.143	0.875	***33***	9/10*	0.9	0.526	***55***	11/2*	5.5	0.154	***77***	66/1	66	0.015
12	2/13	0.154	0.867	***34***	10/11	0.909	0.524	***56***	6/1*	6	0.143	***78***	80/1*	80	0.012
13	1/6*	0.167	0.857	***35***	1/1*	1	0.500	***57***	13/2*	6.5	0.133	***79***	100/1*	100	0.010
14	2/11	0.182	0.846	***36***	11/10*	1.1	0.476	***58***	7/1*	7	0.125	***80***	125/1	125	0.008
15	1/5*	0.2	0.833	***37***	6/5*	1.2	0.455	***59***	15/2	7.5	0.118	***81***	140/1	140	0.007
16	2/9	0.222	0.818	***38***	5/4	1.25	0.444	***60***	8/1*	8	0.111	***82***	150/1*	150	0.007
17	1/4*	0.25	0.8	***39***	11/8	1.375	0.421	***61***	17/2	8.5	0.105	***83***	200/1*	200	0.005
18	2/7*	0.286	0.778	***40***	6/4*	1.5	0.400	***62***	9/1*	9	0.100	***84***	250/1*	250	0.004
19	3/10	0.3	0.769	***41***	13/8	1.625	0.381	***63***	10/1*	10	0.091	***85***	300/1*	300	0.003
20	1/3*	0.333	0.75	***42***	7/4	1.75	0.364	***64***	11/1*	11	0.083	***86***	400/1*	400	0.002
21	4/11	0.364	0.733	***43***	15/8	1.875	0.348	***65***	12/1*	12	0.077	***87***	500/1*	500	0.002
22	2/5*	0.4	0.714	***44***	2/1*	2	0.333	***66***	14/1*	14	0.067	***88***	1000/1$^\diamond$	1000	0.001

[a] Starting (fractional) odds observed in UK horse races 2001-2006 (source: Raceform). [b] An * denotes the 59 allowed odds in Southern Australia (pre-2003), which also include 6/10, 7/10, 14/10, 16/10, 18/10, 22/10, 28/10, 32/10, 30/1, 60/1; a $^\diamond$ denotes the unregulated range of odds in Southern Australia, shorter than 1/10 or longer than 500/1 (source: Independent Gambling Authority 2000, Part 5, rule 27).

Table 1: British and Australian bookmaker odds in horse racing

horse races between 2001-2006; a list that appears to be stable over time (e.g., see the odds reported in Dowie 1976). Appendix B gives further UK race statistics. British starting prices are quoted as *fractional* (traditional, or imperial) odds: odds 8/11 mean that a punter who bets £ 11 gets paid £ 8 plus the stake when the horse wins and loses the stake otherwise. The odds-implied win probability for the bet would be $\frac{1}{8/11+1}$, or 57.9 percent.

It is unlikely that the 693,554 horses from the UK races in Table 1 could all be mapped exactly to one of 88 win probabilities in the continuum $[0, 1]$. We will briefly review potential explanations for why observed betting odds lie on a coarse grid before turning to our main question of what the implications of this phenomenon are for betting returns.

Fractional odds are relatively difficult to compare. For example, consider odds 8/11 versus 5/6. Computing the gross winning payouts reveals that the former pays £ 1.72 to a £ 1 bet, and therefore is less generous than the £ 1.83 payout of the latter. This suggests that one possible explanation for a coarse odds grid is that it is optimal to reduce complexity by having fewer odds categories, because this lowers the cost of completing betting transactions (Harris 1991). If this explanation held, displaying odds in the easier to understand gross or net payout format, so-called *decimal* odds, should lead to a finer grid. Both types of odds representations are simultaneously available on betting exchanges, such as BackAndLay.com, which indeed has a richer "odds ladder" with 517 entries.[8]

For an alternative explanation for the limited set of odds, it is instructive to look at financial markets more generally. Coarse price grids often arise endogenously in asset markets (e.g., Grossman et al. 1997 and Gwilym et al. 1998). Sometimes this is supported by market rules about minimum price increments (tick size), which allow dealers to recover fixed costs from their operations and provide them with incentives to make the market in a stock (Huang and Stoll 2001). Pricing grids can over time become a convention from which it is difficult to deviate. A well-studied case is that of NASDAQ dealers, who faced an official tick size of 1/8 dollar but in practice avoided odd-eighth quotes, so that the observed price grid had a 1/4 dollar tick size. Market participants explained this in terms such as "It has a lot to do with tradition and the avoidance of strange-looking quotations" (Christie and Schultz 1995, p.205). However, after Christie et al.'s press release on odd-eight avoidance, trading suddenly started at odd-eight quotes, making this a classic case study of a tacit agreement in a repeated game (Christie et al. 1994, Christie and Schultz 1994, 1995, and Cason 2000). Dealers are involved in a repeated game and deviations from the agreed price grid are easily observable. A simple trigger strategy mechanism with the threat of reverting to the finest possible

[8]See `http://www.backandlay.com/help.jsp` (last accessed February 2007).

grid if a dealer breaks the agreement, can sustain a coarse price grid as a Nash equilibrium: the profits from a unilateral defection to a price off the agreed grid must be less than the expected future profits from sticking to the agreement. In line with this, Christie and Schultz (1994, p.1816) find that the "important factor in predicting the use of odd-eight quotes is not the economic costs and risks of market making, but whether a practice of avoiding odd-eight quotes is already established."

Bookmaker markets and the pre-reform NASDAQ market[9] share several of the structural features which, according to Christie and Schultz (1994), facilitate sustaining a price grid. First, quotes are transparent to all market participants, so deviations from the price grid are easy to detect. Bookmakers have their stands side by side in the betting ring and their quotes are easily visible from their boards. Second, there is a stable set of market participants who interact frequently and over a long period of time. On-course bookmaking requires a license and a pitch on a racecourse. There currently are 680 authorized bookmakers in the UK (NJPC 2006). These can bid for pitches which have become tradeable in 1998. For example, in the September 2006 auction for pitches in Ascot flat races, lot number 25 went for £4,500 and lot number 160 for only £100.[10] Such price differentials reflect different access to race events and differences in demand at various positions in the betting ring. Pitch positions are chosen in order of lot numbers of attending bookmakers. For example, the limited places at the rail to the members' section are highly attractive because only these bookmakers can directly serve the punters who were willing to pay the considerably higher admission fee to that section. This spatial component presumably limits the extra order flow a bookmaker can achieve by improving odds relative to his colleagues. There is a maximum number of bookmakers admitted per fixture (*designated number*), set by the National Joint Pitch Council (NJPC). For example, in 2005 there were around 1,300 fixtures with a total of 63,432 pitch positions, of which 78 per cent were occupied (NJPC 2006). Thus, on average roughly 40 bookies are present at each meeting.[11]

Coordination on a common odds grid is facilitated by additional factors in bookmaker markets. From 1947 on, Australian race tracks introduced standardised bookmaker indicator boards. These had slots for thirty runners and preprinted labels with odds that could be changed by the turn of

[9] Christie et al.'s study sparked class action lawsuits, antitrust and SEC investigations, and lead to a reform of the trading system on NASDAQ (Christie and Schultz 1995, Securities and Exchange Commission 1996).

[10] `http://www.njpc-ltd.co.uk/pages/auctions/results/`, accessed in November 2006.

[11] Using data from the Horse Race Levy Board (`http://www.hblb.org.uk`), the average number of bookies is found to be 42 for our sample period.

a knob.[12] Modern on-course bookmaker odds-display systems have pre-programmed the standard odds grid and a fitting keyboard (the main UK systems are Exante and WinningOdds).

3 The impact of the odds grid on returns in bookmaker markets

3.1 Model

There are $n \geq 2$ risk-neutral bookmakers who know the true win probability for each competitor in an event, say a horse race. This probability $p \in [0, 1]$ for a particular horse implies *fair* odds on that entrant of $\omega^f = \frac{1-p}{p} \in \Omega^f = [0, \infty)$. Bookmakers use a common odds grid: the odds offered to bettors must be in the set of *quotable* odds, $\Omega = \{\omega_1, \omega_2, \dots, \omega_M\}$, where $\omega_1 < \omega_2 < \cdots < \omega_M$. We assume that there is a positive probability of bets being placed on each horse in a race.[13] Bettors place their stake with the bookmaker who offers the highest odds on the horse of their interest, randomizing in case of ties. As a consequence, competition leads bookmakers to offer the fair odds on a horse if the odds grid permits, otherwise they round down to the nearest quotable odd. That is, the quoted odds on a horse with true win probability $p = \sup \left\{\omega_i \in \Omega : \omega_i \leq \omega^f\right\}$.

Typically, studies on betting markets collect a large sample of races, estimate for each entrant the fair odds based on the sample average win probability computed for the relevant odds range (or favourite position) and compare this with the entrant's quoted odds. A favourite-longshot bias (FL bias) is said to exist if there is a threshold level of odds (a threshold favourite position) so that the average return on a bet struck above the threshold (the range of longshots) is lower than that for the odds below the threshold (the range of favourites). We say that a *strict FL bias* arises if average returns per unit stake decrease monotonically with quoted odds.

We will now derive conditions under which a strict FL bias occurs in the model. Assume that the fair odds for a given horse in a sample of horse races has a density function $f(\omega^f)$.[14] A punter who strikes a £ 1 bet on a horse with quoted odds ω_i receives the stake back in addition to ω_i if the

[12] One such board is in the Powerhouse Museum in Sydney and described in their online catalogue (`http://www.powerhousemuseum.com/collection/database/?irn=319886`).

[13] For example, bettors may derive a consumption value from gambling and have different preferences regarding what horse names to bet on or how to distribute their stakes across the available odds in a race. Estimates of the consumption value of gambling are given in Johnson *et al.* (1999). For surveys of modeling and estimating preferences that lead to gambling even with negative expected returns see Jullien and Salanié (2000, 2008) and Hartley and Farrell (2002).

[14] Section 3.2.1 shows how such a distribution can be derived from primitives.

horse wins and loses the stake otherwise. Thus, the punter's expected profit on such a bet is

$$E[\pi|\omega_i] = E[p|\omega_i]\,(\omega_i+1) - 1 = \frac{\omega_i+1}{E\left[\omega^f|\omega_i\right]+1} - 1. \tag{2}$$

The evolution of betting profits is governed by the properties of the *grid size*, $g_i = \omega_{i+1} - \omega_i$, and the properties of the conditional expectations $E\left[\omega^f\middle|\,\omega_i\right]$. The latter can be conveniently described in terms of parameters a_i linked to the distribution of fair odds, and which are defined as follows:

$$E\left[\omega^f\middle|\,\omega_i\right] = E\left[\omega^f\middle|\,\omega_i \le \omega^f < \omega_{i+1}\right] = \omega_i + a_i\,g_i. \tag{3}$$

That is, the expected value of the fair odds of a horse exceeds the quoted odds ω_i by a fraction a_i of the grid size at that point. One can therefore think of $a_i\,g_i$ as the *effective* grid size, measuring how much value is taken away due to the discrete nature of quoted odds. Substituting (3) into (2) yields

$$E[\pi|\omega_i] = -\frac{a_i\,g_i}{\omega_i + a_i\,g_i + 1}. \tag{4}$$

For a given odds level ω_i, the discrepancy between the quoted odds and the fair odds increases with the effective grid size $a_i\,g_i$. From this we directly obtain the condition for a strict FL bias.

Proposition 1

A strict FL bias exists if and only if for all odds intervals $i = 1, \ldots, M-1$

$$\frac{a_{i+1}\,g_{i+1}}{a_i\,g_i} \quad > \quad \frac{\omega_{i+1}+1}{\omega_i+1} = \frac{\omega_i+g_i+1}{\omega_i+1}. \tag{5}$$

Proof.

Use (4) twice to compute the left hand side of $E[\pi|\omega_{i+1}] - E[\pi|\omega_i] < 0$ and rearrange. ■

Expected profits decrease with each increment in quoted odds if the effective grid size $a_i\,g_i$ grows faster than the gross quotable odds $\omega_i + 1$. To gain some intuition for the driving forces, suppose that the a_is are constant, which isolates the *grid size effect*: profits decrease if the grid size grows faster than the gross quotable odds, i.e.,

$$\frac{g_{i+1}}{g_i} \quad > \quad \frac{\omega_{i+1}+1}{\omega_i+1} \quad \text{(grid size effect).} \tag{6}$$

Now suppose that the a_is are not constant and condition (6) holds. Then, the *grid size effect* could only be reversed if the distribution of fair odds over the interval $[\omega_i, \omega_{i+1}]$ was more right skewed than that over the adjacent interval $[\omega_{i+1}, \omega_{i+2}]$, so that $a_i > a_{i+1}$ (*distribution of odds effect*). As we will see in the empirical application of the model in Section 3.2, the grid size effect tends to

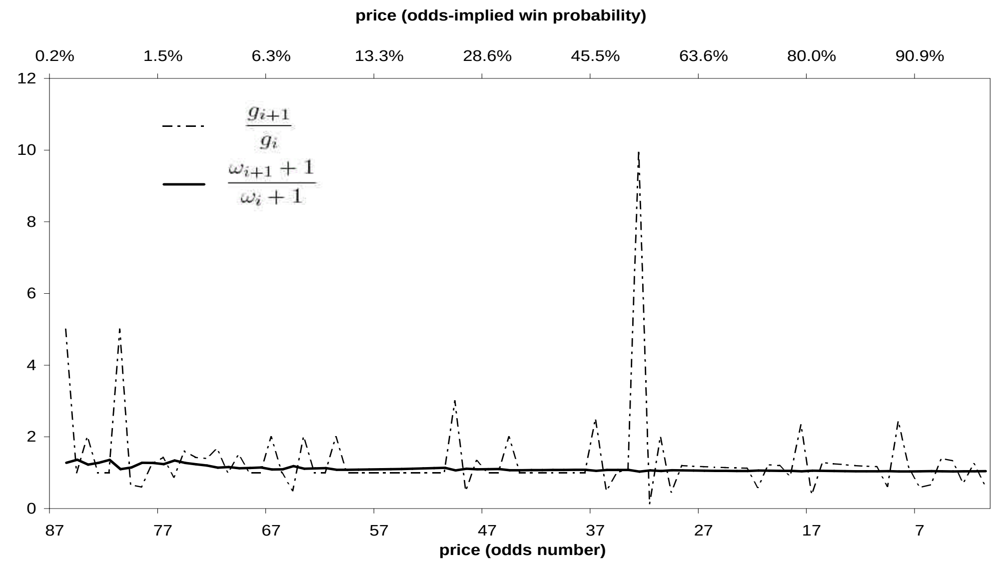


Figure 1: Characteristics of the UK odds grid

dominate. Appendix A gives the corresponding condition for a strict FL bias if one uses, instead of odds, the price (odds-implied win probability) representation.

Returning to the UK bookmaker odds grid given in Table 1, the grid size effect condition (6) is satisfied for roughly half of the adjacent odds bins. Figure 1 shows that when the condition holds, the grid size growth exceeds that of the gross odds by more than it falls short of the gross odds growth when the condition fails. Does this lead to a FL bias under plausible assumptions regarding the (unobservable) distribution of fair odds? How do the model's predictions relate to the actual pattern of returns observed in the UK race data? These questions will be explored in the next section.

3.2 Empirical application to the UK bookmaker odds grid

As a first step, we model the odds of entrants in a particular race to derive reasonable candidate distributions of fair odds. This allows us to gauge the importance of the *distribution of odds effect* for the evolution of returns under the UK bookmaker odds grid: we compare for different distributions of fair odds the predictions from the model in Section 3.1. As a final step, we relate the pattern of returns generated by the model to that estimated from the actual UK race data.

3.2.1 The distribution of fair odds

The winning odds depend on the relative strength of the entrants in a race. Consider a race with H horses and suppose that the strength of each horse s_h has a distribution with density g on the support $[0, 1]$. Following the literature on sports contests (e.g., Szymanski 2003), we model the winning probabilities using a contest success function,[15] where

$$p_h = \frac{s_h^m}{\sum_{j=1}^{H} s_j^m}, \quad m \geq 0. \tag{7}$$

The parameter m measures the discriminatory power of a contest. If $m = 0$, all entrants have equal chance of winning, regardless of their differences in strength. If $m = 1$, the chance of winning corresponds to an entrant's strength relative to the overall strength of the field. For m large, even a slightly higher strength than that of the competitors ensures a high probability of winning the race. To apply this framework, we start with four very different distributions of underlying strengths of horses (see panel (a) of Figure 2): the uniform distribution; a bimodal distribution, beta(0.5,0.5); a unimodal symmetric distribution, beta(2,2); and a unimodal left-skewed distribution, beta(2,5). For each of these, we generate the fair odds distribution for races with ten entrants.[16] Panel (b) of Figure 2 gives kernel density estimates for the "fair" prices, derived using a logit contest success function ($m = 1$) – the most common form in applications – and one example with parameter $m = 2$.[17] For the plots it is more convenient to use the price (odds-implied win probability) representation explained in Appendix A. It is interesting to note that many of the differences in initial distributions of strengths disappear. All resulting distributions of fair odds are concentrated around 1/10, the average win probability of an entrant. Intuitively, for any of the initial strength distributions, it is quite rare that one of the 10 entrants will dominate the field by a very large margin. Therefore, differences in strength distributions do not translate into dramatically different distributions of winning probabilities. The latter thus appear to depend largely on the number of competitors in a race, and only to a lesser extent on the details of the strength distribution in the population.

In the final step, we apply the model from Section 3.1 to determine for each horse in the simulation the quoted price according to the UK price grid. The expected return of each horse is given by (fair

[15] This was introduced by Tullock (1980) and axiomatised by Skaperdas (1996) and Clark and Riis (1997).

[16] This is the mode of the distribution of runners in the UK race data (5,240 races or roughly 9% of all races).

[17] Each density estimate is based on a simulation of 52,400 fair odds (5,240 random samples of 10 runners), where the same seed for the random number generator is used across distributions.

price - quoted price)/(quoted price). This leads to the average returns in each price bin plotted in panel (c) of Figure 2. Two features of the return distributions are striking: they all exhibit a pronounced FL bias and the return patterns are almost identical across distributions. Thus, the evolution of expected betting returns appears to be driven mostly by the *grid size effect*, i.e., whether condition (6) holds or not, with little impact of the *distribution of odds effect*.

3.2.2 Comparison with the FL bias present in UK horse race data

The previous section showed that the model predicts a FL bias for the UK odds grid. This section explores how much of the actual FL bias present in horse race data can be attributed to the coarseness of the odds grid.

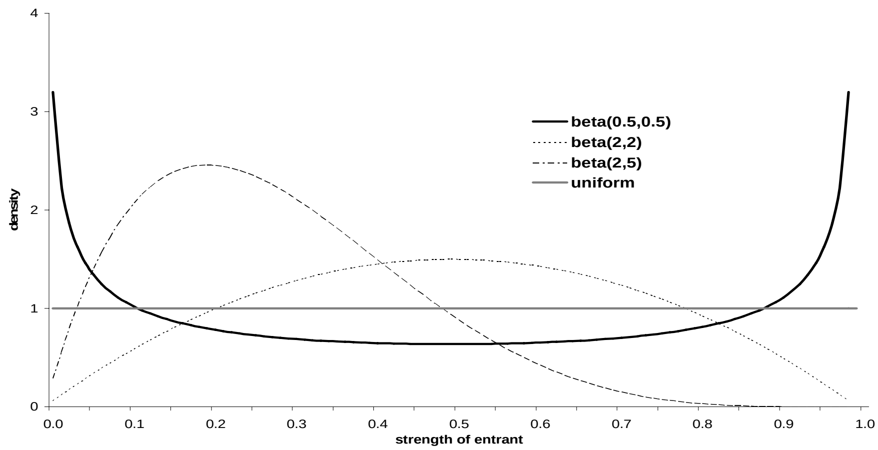
The data set covers all UK horse races from 1 January 2001 to 31 November 2006 and was obtained from Dataform, who publish the annual form books. After eliminating 149 races with deadheats, the sample contains 60,487 races with a total of 693,554 horses. To estimate betting returns, horses are grouped by quoted starting odds ω_i, which translate into *prices* (odds-implied win probabilities) $p_i = \frac{1}{\omega_i+1}$. For each of the 88 different quoted odds/prices the actual probability of winning is estimated as $\hat{p}_i =$($\sharp$ winners in odds bin)/($\sharp$ runners in odds bin). Thus, the expected return for each quoted odds category is given by

$$\text{expected return on a bet at odds } \omega_i = \frac{\hat{p}_i - p_i}{p_i}. \tag{8}$$

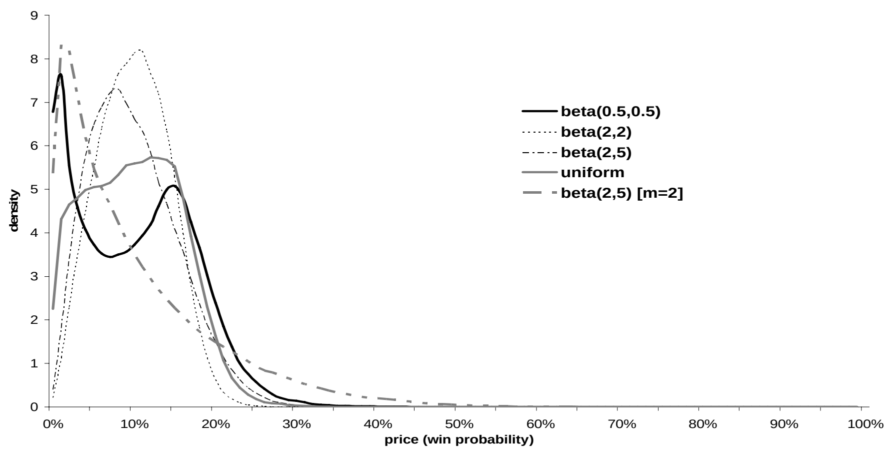
Figure 3 plots the expected returns for each quoted price with 95% confidence intervals. Note that these are not symmetric because they are based on exact binomial distribution confidence intervals for $\hat{p}$ (computed using Duembgen's (2001) Matlab routine) rather than a normal distribution approximation, which ignores the bounds 0 and 1 on the win probability.[18] The picture is in line with earlier estimates of the FL bias in UK data (e.g., Dowie 1976, Jullien and Salanié 2000, Vaughan Williams and Paton 1997). These however are based on much smaller samples and much coarser price intervals; our figure is at the finest possible resolution – the actual price grid.

To derive the relevant set of predictions from the model, we adopt the following procedure. First, we replicate the exact distribution of runners in the UK race data (see Figure 4). Specifically, for each race in the data set, a corresponding race is generated with win probabilities determined by applying

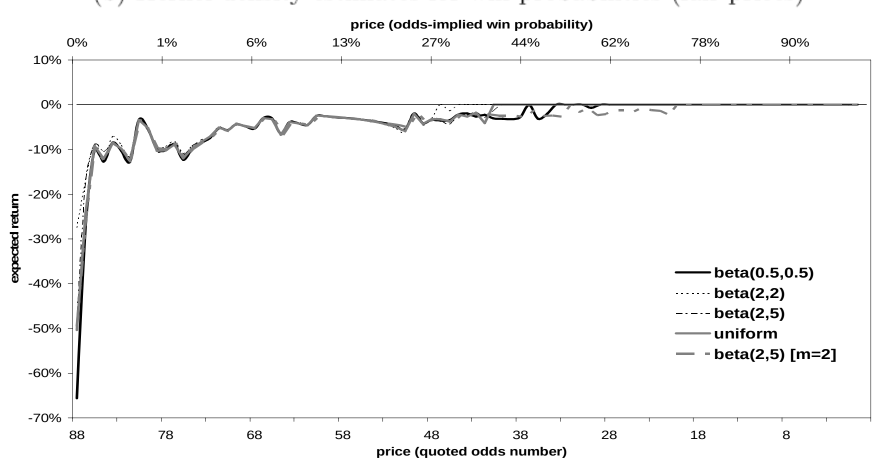
[18] A correction is made to account for the fact that in many price bins there are several horses from the same race: a bin with w winners among n horses from r races is treated as r independent draws from a binomial distribution with w successes (details are available from the authors). The estimator $\hat{p}$ is not biased by such multiple entrants in a bin from a single race (see Ali 1977).



(a) Probability density functions for the strength of entrants



(b) Kernel density estimates for win probabilities (fair prices)



(c) Expected returns by price (odds-implied win probability)

Figure 2: Distribution of odds effect (simulation based on 10 runners per race)

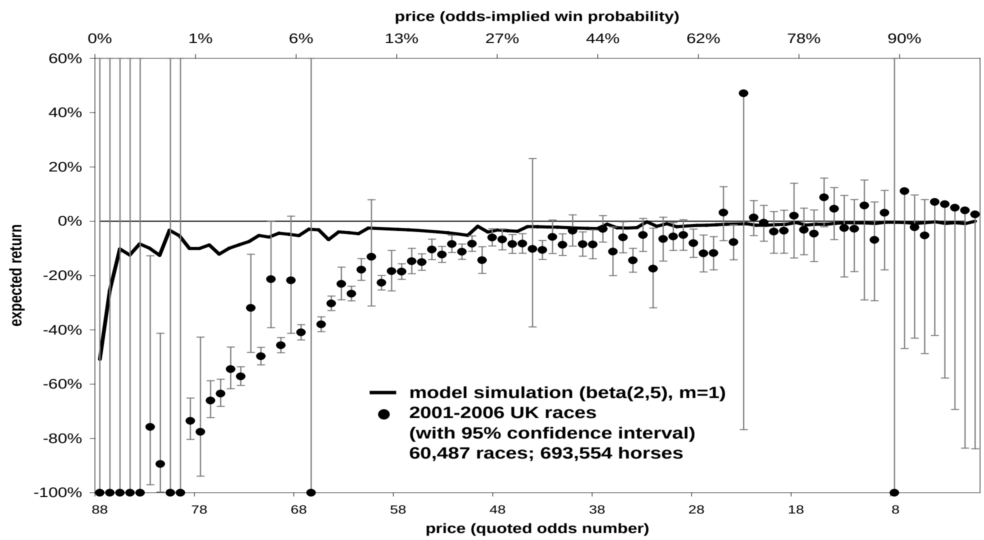


Figure 3: Actual FL bias present in UK horse races vs model-based simulation

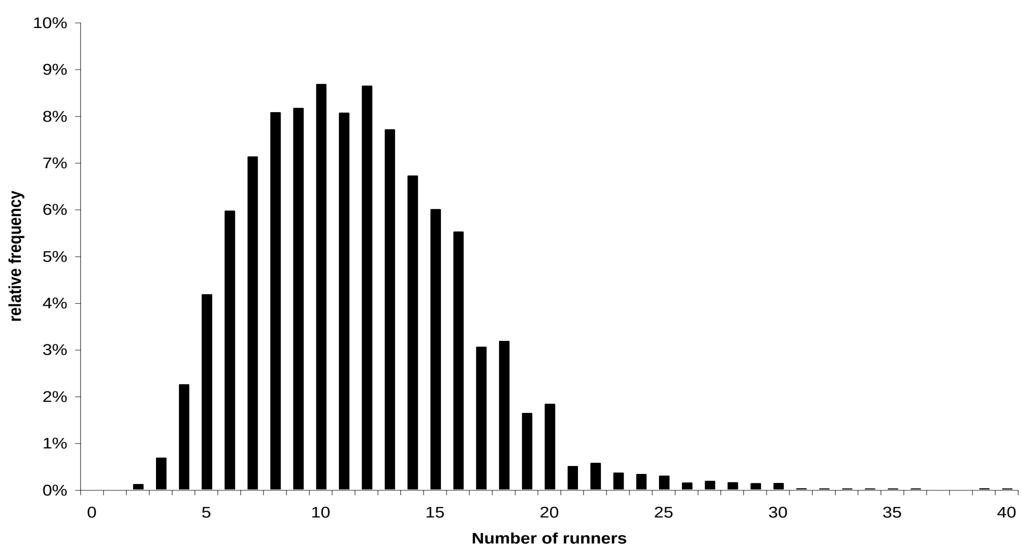


Figure 4: Distribution of number of runners in UK horse races 2001-2006

the contest success function to a vector of random draws from the distribution of strengths. As the preceding section showed, the exact shape of the strength distribution does not significantly impact the predictions. So for clarity we present below only the results using a logit contest success function (m=1) with a beta(2,5) strength distribution.[19] In the second step, the average win probability $\bar{p}_i$ is computed for the horses falling into each price range $(p_{i+1}, p_i]$; in other words, for the simulated data we have a precise measure rather than an estimate $\hat{p}_i$. From this we obtain the expected return for all odds/prices as $\frac{\bar{p}_i - p_i}{p_i}$, plotted as solid line in Figure 3.

The simulation shows that the UK odds grid has a pronounced built-in FL bias, which can explain part of the observed bias in the real race data. It is important to stress that our model made no behavioral assumptions that would lead to a bias in betting returns, other than introducing a fixed odds grid. Thus, it complements extant explanations for why a FL bias arises in bookmaker betting: these markets have an inherent tendency to display a FL bias owing to the nature of the odds used in practice.

What proportion of the overall bias observed in betting returns on UK horse races can potentially be explained solely by the effects of the odds grid? As can be seen from Figure 3, the returns predicted by the model at a given odds level i ($\bar{r}_i^{sim}$) are higher than the empirical returns ($\bar{r}_i^{UK}$): for 99.81% of all quoted horses $0 > \bar{r}_i^{sim} > \bar{r}_i^{UK}$ (see Appendix B). To measure the proportion of the overall bias that can potentially be attributed to the odds grid, we thus compute at each of these odds levels the ratio of bias explained over total expected bias. These ratios are then weighted by the relative frequencies of the respective odds:[20]

$$\text{proportion of bias explained} = \sum_i \left(\frac{\sharp \text{ odds i quoted}}{\text{total } \sharp \text{ of horses in all races}} \times \frac{\bar{r}_i^{sim}}{\bar{r}_i^{UK}} \right). \quad (9)$$

The measure suggests that the odds grid can explain around 20.85 % of the average observed bias in UK betting returns. To account for the remainder, other explanations are needed, such Henery's (1985) market segmentation hypothesis and insider information (Shin 1991, 1992, 1993).

It turns out that our framework also helps explain why pari-mutuel markets sometimes display no bias or one opposite to that found in bookmaker markets: the next section shows that the odds grid employed in pari-mutuel betting pushes these markets toward a *reverse* FL bias.

[19] Results for other distributions and parameters m are very similar and available from the authors.

[20] The remaining 0.19% of quoted odds, where $\bar{r}_i^{UK} > \bar{r}_i^{sim}$, have no perceivable impact on the model's explanatory power.

4 Implications for pari-mutuel betting markets

In contrast to fixed-odds betting where bookmakers compete for bets, there is a monopoly in pari-mutuel (totaliser or tote) betting. The tote runs alongside bookmaker markets in the UK and Australia, but in other countries, punters usually only have access to pari-mutuel betting. In tote betting, all bettors' stakes are pooled and this pool is distributed among the winning tickets after two types of deductions by the pari-mutuel operator. The first is a proportionate tax on the betting pool before payouts are determined (e.g., 13.5% for the UK Tote), the *take*. The second is *breakage* (or *fractions* in Australia), which refers to deductions resulting from the rounding down of payouts. For example, the UK Tote fixes so-called *dividends* per pound bet by rounding down to the nearest 10 pence for dividends above £ 1.1 and to the minimum dividend of £ 1.02 otherwise; payouts to a single bettor are capped at £ 50,000.[21]

Almost all pari-mutuel studies aggregate the take and an average breakage cost into a total cost of betting (see for example Ali 1977, p.805). But these two components affect betting returns differently. The take uniformly reduces returns and therefore cannot explain systematic changes across different price ranges, such as a FL bias. To see this, suppose that the gross dividends on a horse j correspond to the fair gross odds[22] $1+\omega_j^f$, i.e.,

$$d = \frac{\sum_{h=1}^{H} b_h}{b_j} = 1 + \omega_j^f, \tag{10}$$

where H is the number of horses in the race and b_j are the total bets placed on horse j. Thus, in the absence of breakage, a track take of t implies an expected return of

$$\frac{1}{1+\omega_j^f}\, d\,(1-t) - 1 = -t. \tag{11}$$

In other words, the expected return on all horses is equal to minus the track take.[23] In contrast, breakage imposes a proportionately larger cost on favourites than on longshots, as pointed out by Busche and Walls (2001): for example, the loss due to rounding of between 1 and 10 p corresponds

[21] See www.totesport.com. Deductions vary across countries and race tracks: the take is around 16% in New Zealand, 18% in the US, and 26% in Japan; normalised to a 1 dollar bet, rounding down is to the nearest 10 cents in the US, to the nearest 10 cents in Australia, to the nearest 5 cents in Canada and New Zealand, and to the nearest cent in Macau (Busche and Walls 2000, Gandar et al. 2001, Coleman and McGrath 2006).

[22] Using exact betting volumes rather than post-breakage odds, several studies find gross dividends to indeed correspond to the fair gross odds estimated from observed win frequencies (Busche and Hall 1988, Busche 1994, Bruce and Johnson 2000, Walls and Busche 2003b).

[23] In line with this, the track take does not affect the bias in experimental betting markets conducted by Hurley and McDonough (1995). See also, Coleman (2004).

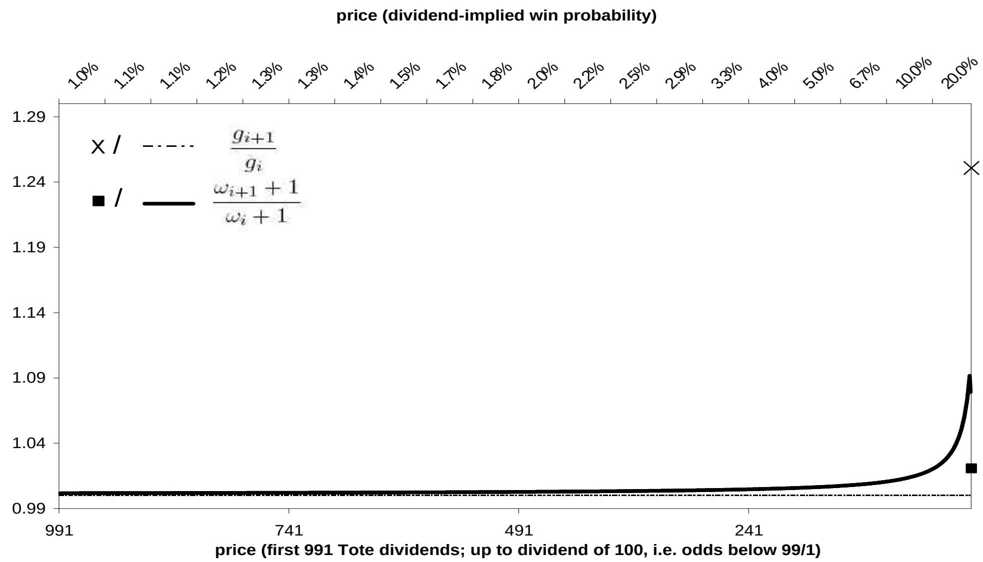


Figure 5: Characteristics of the UK Tote odds/dividend grid

to up to 5 percent of the pre-breakage payout for a £2 dividend but less than 1 percent for a £20 dividend. To analyse in more detail the impact of breakage on relative returns across price ranges, suppose for simplicity that the track take is zero and consider the breakage applied by the UK Tote. This gives rise to an odds grid $\Omega_{tote} = \{1.02, 1.1, 1.2, \ldots, 50000\}$ with a constant grid size $g_i = 0.1$ for $i > 1$.[24] Favourites are grouped into much larger price (odds/dividend-implied win probability) bins relative to the bookmaker grid in Table 1: there are only 10 Tote prices with implied win probabilities above 50%, versus 35 *odds-on* bookmaker prices. In contrast, longshots are grouped into much finer price bins: there are almost half a million Tote prices with implied win probabilities below 50%, versus 33 *odds-against* bookmaker prices. The fact that the odds grid is very coarse at the low odds end and becomes almost continuous for higher odds suggests that it will affect favourites in a much stronger way than longshots. Indeed, with constant odds increments g, we have $\frac{g_{i+1}}{g_i} = 1$ so that

$$\frac{g_{i+1}}{g_i} < \frac{\omega_i + g_i + 1}{\omega_i + 1}, \tag{12}$$

i.e., comparing with (6), the grid size effect pushes toward a *reverse* FL bias unless the distribution

[24] A feature specifically accounted for in the simulations below is that the minimum dividend of £1.02 is paid even if the pre-breakage dividend is below this value.

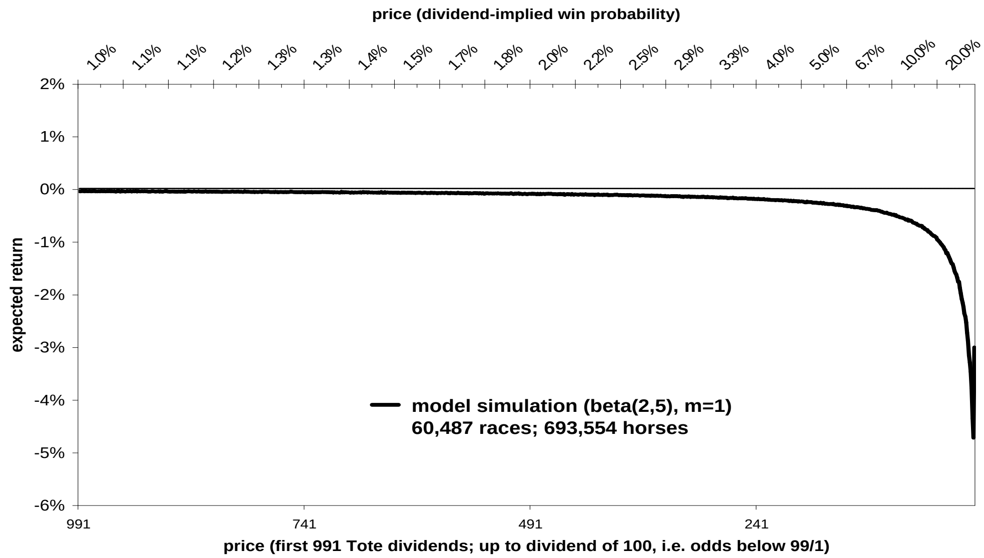


Figure 6: Model-based simulation for UK Tote grid

of odds effect is sufficiently strong to reverse it. More formally:

Corollary 1

With a constant grid size $g_i = g$, a strict reverse FL bias exists if and only if for all odds intervals $i = 1, \ldots, M-1$

$$\frac{a_{i+1}}{a_i} < \frac{\omega_i + g + 1}{\omega_i + 1}. \tag{13}$$

For the UK Tote odds grid, condition (12) is satisfied for all but the first of the almost half a million dividend increments (see Figure 5). Applying the grid to our sample of simulated races produces the returns plotted in Figure 6. There is a reverse FL bias which is *strict* but for one reversal when moving from the minimum dividend £1.02 to £1.1. This means that the shape of the return distribution corresponds to that predicted if the grid size effect prevails over the distribution of odds effect – as was also the case in the bookmaker market setting. There are two striking differences relative to bookmaker markets: the FL bias is *reversed* and the bias introduced by the odds grid is much less extreme.

We do not have available a UK Tote data set to compare the simulations with. However, Bruce and Johnson (2000) provide a comparison of UK Tote and bookmaker data for the period June to August

1996. They report Tote odds based on exact betting fractions (i.e., pre-breakage) for 2,109 races with 19,396 horses, and relate these to the corresponding odds from the bookmaking organisation Ladbroke plc. (p.417, Exhibit 1). Average Tote odds are significantly higher than bookmaker starting prices for odds below 5/2 (implied win probability greater than 29%) and significantly lower for odds above 5/1 (implied win probability less than 17%), all at the one percent significance level. This is in line with our model, when accounting for the take by uniformly shifting downward the returns. Applying a regression framework, they find strong evidence for a FL bias in bookmaker odds with a corresponding absence of any bias in the (pre-breakage) Tote odds. This suggests that when breakage is applied, Tote returns should exhibit a reverse FL bias – as predicted by our model.

Our analysis in this section provides a systematic treatment of the impact that breakage has on pari-mutuel betting returns and contributes to a small literature concerned with this issue. Busche and Walls (2001) were the first to empirically examine the impact of breakage for the data from Ali (1977), constructing an index of breakage which attributes a higher score the lower the odds. They conclude that failure to account for breakage biases tests of betting market efficiency away from the null hypothesis of efficiency. Gramm and Owens (2005) incorporate a measure of expected breakage costs in their regression analysis of 5,020 US races, but fail to find a significant effect. Coleman (2004) was the first to explicitly note the potential link between breakage and the reverse FL bias observed in some pari-mutuel markets. He re-estimates the expected returns from Busche and Hall's (1988) data (which exhibit a reverse FL bias) for two scenarios: after rounding the (pre-breakage) dividends down to the nearest 10 and 20 cents. He finds that the greater the amount of hypothetical breakage, the stronger the reverse FL bias is. While Coleman's (2004) analysis is directly comparable to the predictions of our model, the studies by Busche and Walls (2001), Walls and Busche (2003a,b) are not. Instead of betting returns, they examine the impact that breakage has on estimates of betting fractions, i.e., the relative proportions of bettors' stakes placed on horses in a race. They obtain the bet fractions from pre- and post-breakage prices (the dividend-implied win probabilities) by normalising them so that they sum up to one in *each* individual race, to then show that breakage leads to a bias in Ali (1977)-type z-statistics. The bias in bet fractions however is not identical to a bias in realised betting returns: the latter are given by (true win probability - price)/price; the distribution of returns has no straightforward relation to that of normalised bet fractions because these are derived using race-specific normalisation factors.

5 Discussion and concluding remarks

Favourite-longshot (FL) biases in gambling markets have come to be accepted as a robust empirical regularity (e.g., Thaler and Ziemba 1988). Nevertheless, several studies find no bias or even a reverse FL bias in betting markets, prompting Vaughan Williams and Paton's (1998) question "Why are some favourite-longshot biases positive and others negative?" Our analysis shows that differences in the odds grids employed across betting markets may provide part of the answer. In the absence of any other forces that cause biases in betting returns, there is a built-in tendency of bookmaker odds to produce a FL bias. In contrast, the constant odds increments in pari-mutuel betting lead to a built-in tendency toward a (mild) reverse FL bias. In other words, any factor that pushes betting returns toward displaying a FL bias is likely to be exacerbated in bookmaker markets, whereas it is likely to be attenuated or even reversed in pari-mutuel betting.

The empirical evidence on betting returns appears to be consistent with this. Sauer (1998, Figure 1) shows that the FL bias in UK bookmaker data (from Dowie 1976) is much stronger than that in US pari-mutuel data (from Snyder 1978).[25] When both types of markets operate simultaneously, bookmaker returns tend to be better on favourites and worse on longshots than those from the parallel pari-mutuel market, which reduces or even eliminates the FL bias in the latter (Gabriel and Marsden 1990, 1991, Bruce and Johnson 2000, Cain et al. 2001). Betting exchanges have created other parallel gambling markets in recent years. These have a much finer odds grid than the traditional bookmaker markets. For example, the betting exchange BackAndLay.com has an "odds ladder" with 517 odds levels (127 of which *odds on*, i.e., with implied win probability greater than 50%, and 490 *odds against*).[26] This contrasts with the 88 odds levels (35 odds on and 33 odds against) observed in UK bookmaker markets. In line with our model, the FL bias in bookmaker betting appears to be significantly stronger than that observed in simultaneously run betting exchanges (Smith et al. 2006).[27] Furthermore, documented cases where gambling markets exhibit no bias or a reverse FL bias are all from pari-mutuel betting (Busche and Hall 1988, Busche 1994, Swidler and Shaw 1995, Busche and Walls 2000, Gandar et al. 2001).[28]

[25] See also Coleman (2004) and Snowberg and Wolfers (2005, Figure 2).

[26] See `http://www.backandlay.com/help.jsp` (last accessed February 2007).

[27] More generally, trade execution costs (e.g. bid-ask spreads) tend to be lower in financial markets with electronic trading than in those with floor trading (e.g. Madhavan 1992, Pagano and Roell 1992, De Jong et al. 1995, and Huang and Stoll 1996).

[28] Reverse FL biases are also reported for US money-line markets in hockey (Woodland and Woodland 2001, Gandar et al. 2004) and, with some controversy, in baseball (Woodland and Woodland 1994, 2003, Gandar et al. 2002). These markets differ from the bookmaker markets that we study as their odds grid consists of fixed combinations of odds

Undoubtedly, our framework does not provide the complete picture, and therefore complements other explanations for why FL biases arise and what factors influence their severity. While recent theoretical contributions and empirical analyses have advanced our understanding of betting returns, Sauer's (1998, p.2048) appeal remains topical that "Work documenting the source of variation in the favourite-long shot bias would be particularly useful."

Appendix

A Price (odds-implied win probability) representation

It is often more convenient to work with the odds-implied win probabilities, $p_i = \frac{1}{\omega_i+1}$. These can be interpreted as prices for contingent claims that pay out £ 1 if the horse bet on wins and nothing otherwise. Thus, the odds grid implies a price grid $\mathcal{P} = \{p_1, \ldots, p_M\}$, where $1 \geq p_1 > p_2 > \cdots > p_M \geq 0$, because prices are ordered inversely to odds. The analogue of g_i in the price dimension is the *tick size* $\gamma_i = p_i - p_{i+1}$, and the analogue of a_i is the parameter α_i linked to the distribution of true win probabilities p as follows:[29]

$$E\left[p|p_i\right] = E\left[p|p_{i+1} < p \leq p_i\right] = p_i - \alpha_i\, \gamma_i. \tag{14}$$

With these definitions, the expected profit from placing a £ 1 bet on a horse at price p_i is given by $E[\pi|p_i] = -\frac{\alpha_i\,\gamma_i}{p_i}$, and we obtain the following corollary to Proposition 1:

Corollary 2

A strict FL bias exists if and only if for all price intervals $i = 1, \ldots, M-1$

$$\frac{\alpha_{i+1}\,\gamma_{i+1}}{\alpha_i\,\gamma_i} \quad > \quad \frac{p_{i+1}}{p_i}. \tag{15}$$

B Returns on UK horse betting: 2001-2006 data vs model-based simulation

Table 2 provides data on historical betting returns in UK horse betting from 2001-2006 and complements Figure 3. It gives the frequency distribution of quoted starting prices for each odds level i (N_i), the number of horses quoted at odds level i that won a race (W_i) and the average betting returns for bets struck at odds level i ($\bar{r}_i^{UK}$). This can be compared with the average return from the model-based simulation which uses parameters beta(2,5) and m=1 ($\bar{r}_i^{sim}$).

on both competing teams rather than unlinked individual odds.

[29]The precise relation between the parameters is $\gamma_i = \frac{g_i}{(\omega_i+1)(\omega_{i+1}+1)}$ and $\alpha_i = \frac{a_i\,(\omega_{i+1}+1)}{\omega_i+1+a_i\,g_i}$.

Fract. odds[a]	N_i	W_i	$\bar{r}_i^{UK}$ [%]	$\bar{r}_i^{sim}$ [%]	Fract. odds	N_i	W_i	$\bar{r}_i^{UK}$ [%]	$\bar{r}_i^{sim}$ [%]	Fract. odds	N_i	W_i	$\bar{r}_i^{UK}$ [%]	$\bar{r}_i^{sim}$ [%]	Fract. odds	N_i	W_i	$\bar{r}_i^{UK}$ [%]	$\bar{r}_i^{sim}$ [%]
1/40	2	2	2.5	-0.8	**4/9**	426	299	1.4	-0.9	**85/40**	87	25	-10.2	-2.0	**15/1**	1	0	-100	-2.9
1/25	2	2	4.0	-0.6	**40/85**	2	2	47.1	-1.0	**9/4**	6,331	1,788	-8.2	-3.7	**16/1**	46,696	1,622	-40.9	-5.3
1/20	3	3	5.0	-0.8	**1/2**	515	317	-7.7	-1.1	**5/2**	7,424	1,943	-8.4	-3.5	**18/1**	1,263	52	-21.8	-4.8
1/16	4	4	6.3	-0.2	**8/15**	214	144	3.2	-1.3	**11/4**	6,758	1,681	-6.7	-3.2	**20/1**	54,517	1,410	-45.7	-4.4
1/14	6	6	7.1	-0.6	**4/7**	648	364	-11.8	-1.5	**3/1**	11,018	2,590	-6.0	-4.0	**22/1**	1,871	64	-21.3	-5.9
1/12	8	7	-5.2	-0.8	**8/13**	555	303	-11.8	-1.5	**10/3**	4,792	947	-14.4	-1.9	**25/1**	47,547	919	-49.7	-5.2
1/10	9	8	-2.2	-0.5	**4/6**	1,001	552	-8.1	-1.7	**7/2**	15,242	3,107	-8.3	-5.2	**28/1**	2,429	57	-31.9	-7.5
1/9	5	5	11.1	-0.4	**8/11**	933	513	-5.0	-2.0	**4/1**	17,407	3,091	-11.2	-4.7	**33/1**	47,590	600	-57.1	-8.7
2/17	1	0	-100	-0.4	**4/5**	1,209	634	-5.6	-0.9	**9/2**	15,819	2,635	-8.4	-4.3	**40/1**	12,521	139	-54.5	-9.9
1/8	24	22	3.1	-0.8	**5/6**	514	262	-6.6	-1.8	**5/1**	19,836	2,901	-12.3	-3.9	**50/1**	29,447	211	-63.5	-12.1
1/7	27	22	-6.9	-0.6	**9/10**	168	73	-17.4	-0.2	**11/2**	13,467	1,856	-10.4	-3.6	**66/1**	19,726	100	-66.0	-8.7
2/13	12	11	5.8	-0.6	**10/11**	967	481	-5.0	-2.3	**6/1**	21,651	2,627	-15.1	-3.4	**80/1**	1,444	4	-77.6	-10.1
1/6	48	40	-2.8	-0.6	**1/1**	1,876	803	-14.4	-2.5	**13/2**	9,990	1,136	-14.7	-3.2	**100/1**	19,085	50	-73.5	-10.0
2/11	40	33	-2.5	-0.8	**11/10**	1,313	588	-6.0	-2.4	**7/1**	27,047	2,753	-18.6	-3.0	**125/1**	403	0	-100	-5.3
1/5	78	68	4.6	-1.2	**6/5**	577	233	-11.2	-1.1	**15/2**	4,417	424	-18.4	-2.8	**140/1**	1	0	-100	-3.3
2/9	82	73	8.8	-1.1	**5/4**	2,024	874	-2.8	-2.7	**8/1**	33,530	2,881	-22.7	-2.7	**150/1**	1,429	1	-89.4	-12.5
1/4	131	100	-4.6	-1.4	**11/8**	1,867	719	-8.5	-2.6	**17/2**	798	73	-13.1	-2.5	**200/1**	1,660	2	-75.8	-10.0
2/7	174	131	-3.2	-0.5	**6/4**	2,834	1,038	-8.4	-2.5	**9/1**	17,790	1,462	-17.8	-4.6	**250/1**	191	0	-100	-8.3
3/10	65	51	2.0	-1.2	**13/8**	1,898	698	-3.5	-2.3	**10/1**	39,316	2,621	-26.7	-4.2	**300/1**	103	0	-100	-12.4
1/3	232	168	-3.5	-1.2	**7/4**	4,067	1,351	-8.6	-2.2	**11/1**	9,330	598	-23.1	-3.9	**400/1**	21	0	-100	-10.2
4/11	268	189	-3.8	-1.4	**15/8**	1,871	613	-5.8	-2.1	**12/1**	44,729	2,399	-30.3	-6.8	**500/1**	110	0	-100	-25.5
2/5	373	265	-0.5	-1.5	**2/1**	5,893	1,756	-10.6	-2.0	**14/1**	45,753	1,892	-38.0	-3.2	**1000/1**	1	0	-100	-50.9

Legend: N_i: ♯ of quoted starting prices i; W_i: ♯ of winners quoted at odds i; $\bar{r}_i^{UK}$: historical average return of bets struck at starting price i; $\bar{r}_i^{sim}$: simulation-based average return of bets struck at odds i (beta(2,5), m=1). [a] Starting prices in UK horse races 2001-2006 (693,554 horses, 60,487 races); see Table 1 also.

Source: Raceform.

Table 2: UK horse races 2001-2006 vs model-based simulation

References

Ali, Mukhtar M., 1977, Probability and utility estimates for racetrack bettors, *Journal of Political Economy* 85, 803–15.

Bruce, Alistair C., and Johnnie E. V. Johnson, 2000, Investigating the roots of the favourite-longshot bias: An analysis of decision making by supply- and demand-side agents in parallel betting markets, *Journal of Behavioral Decision Making* 13, 413–430.

Busche, Kelly, 1994, Efficient market results in an Asian setting, in Donald Hausch, Victor Lo, and William T. Ziemba, ed.: *Efficiency of Racetrack Betting Markets*, 183–198 (Academic Press: New York).

Busche, Kelly, and Christopher D. Hall, 1988, An exception to the risk preference anomaly, *Journal of Business* 61, 337–46.

Busche, Kelly, and W. David Walls, 2000, Decision costs and betting market efficiency, *Rationality and Society* 12, 477–492.

Busche, Kelly, 2001, Breakage and betting market efficiency: Evidence from the horse track, *Applied Economics Letters* 8, 601–04.

Cain, Michael, David Law, and David Peel, 2001, The incidence of insider trading in betting markets and the Gabriel and Marsden anomaly, *The Manchester School* 69, 197–207.

Cain, Michael, David Law, and David Peel, 2003, The favourite longshot bias, bookmaker margins and insider trading in a variety of betting markets, *Bulletin of Economic Research* 55, 263–273.

Cason, Timothy N, 2000, The opportunity for conspiracy in asset markets organized with dealer intermediaries, *Review of Financial Studies* 13, 385–416.

Christie, William G., Jeffrey H. Harris, and Paul H. Schultz, 1994, Why did NASDAQ market makers stop avoiding odd-eighth quotes? *Journal of Finance* 49, 1841–60.

Christie, William G., and Paul H. Schultz, 1994, Why do NASDAQ market makers avoid odd-eighth quotes? *Journal of Finance* 49, 1813–40.

Christie, William G., and Paul H. Schultz, 1995, Policy watch: Did NASDAQ market makers implicitly collude? *Journal of Economic Perspectives* 9, 199–208.

Clark, Derek J., and Christian Riis, 1997, Contest success functions: An extension, *Economic Theory* 11, 201–204.

Coleman, Les, 2004, New light on the longshot bias, *Applied Economics* 36, 315–326.

Coleman, Les, and Martin McGrath, 2006, Biases and insider trading in exotic bets on thoroughbreds, in Leighton Vaughan Williams, ed.: *Information Efficiency in Financial and Betting Markets*, 231–246 (Cambridge University Press: Cambridge).

de Jong, Frank, Theo Nijman, and Ailsa Roell, 1995, A comparison of the cost of trading french shares on the paris bourse and on seaq international, *European Economic Review* 39, 1277–1301.

Dowie, Jack, 1976, On the efficiency and equity of betting markets, *Economica* 43, 139–150.

Duembgen, Lutz, 2001, BinoCBs: Matlab routine for computing exact one- and two-sided confidence bounds for the parameter p of the binomial distribution, available at `http://www.imsv.unibe.ch/~duembgen`.

Gabriel, Paul E., and James R. Marsden, 1990, An examination of market efficiency in British racetrack betting, *Journal of Political Economy* 98, 874–85.

Gabriel, Paul E., and James R. Marsden, 1991, An examination of efficiency in British racetrack betting: Errata and corrections, *Journal of Political Economy* 99, 657–59.

Gandar, John M., Richard A. Zuber, and R. Stafford Johnson, 2001, Searching for the favourite-longshot bias down under: An examination of the New Zealand pari-mutuel betting market, *Applied Economics* 33, 1621–1629.

Gandar, John M., Richard A. Zuber, and R. Stafford Johnson, 2004, A reexamination of the efficiency of the betting market on National Hockey League games, *Journal of Sports Economics* 5, 152–168.

Gandar, John M., Richard A. Zuber, and R. Stafford Johnson, and W. Dare, 2002, Re-examining the betting market on Major League baseball games: Is there a reverse favourite-longshot bias? *Applied Economics* 34, 1309–1317.

Gramm, Marshall, and Douglas H. Owens, 2005, Determinants of betting market efficiency, *Applied Economics Letters* 12, 181–185.

Grossman, Sanford J., Merton H. Miller, Kenneth R. Cone, and Daniel R. Fischel, 1997, Clustering and competition in asset markets, *Journal of Law and Economics* 40, 23–60.

Gwilym, Owain Ap, Andrew Clare, and Stephen Thomas, 1998, Extreme price clustering in the London equity index futures and options markets, *Journal of Banking and Finance* 22, 1193–1206.

Harris, Lawrence, 1991, Stock price clustering and discreteness, *Review of Financial Studies* 4, 389–415.

Hartley, Roger, and Lisa Farrell, 2002, Can expected utility theory explain gambling? *American Economic Review* 92, 613–624.

Henery, Robert J., 1985, On the average probability of losing bets on horses with given starting price odds, *Journal of the Royal Statistical Society. Series A (General)* 148, 342–349.

Huang, Roger D., and Hans R. Stoll, 1996, Dealer versus auction markets: A paired comparison of execution costs on nasdaq and the nyse, *Journal of Financial Economics* 41, 313–357.

Huang, Roger D., and Hans R. Stoll, 2001, Tick size, bid-ask spreads, and market structure, *Journal of Financial and Quantitative Analysis* 36, 503–522.

Hurley, William, and Lawrence McDonough, 1995, A note on the Hayek hypothesis and the favorite-longshot bias in parimutuel betting, *American Economic Review* 85, 949–955.

Independent Gambling Authority, 2000, South Australia bookmakers licensing rules 2000, Version No. 000, as at 1 October 2000 available at `http://www.iga.sa.gov.au`.

Johnson, Johnnie E. V., Raymond O'Brien, and Hyun Song Shin, 1999, A violation of dominance and the consumption value of gambling, *Journal of Behavioral Decision Making* 12, 19–36.

Jullien, Bruno, and Bernard Salanié, 2000, Estimating preferences under risk: The case of racetrack bettors, *Journal of Political Economy* 108, 503–530.

Jullien, Bruno, and Bernard Salanié, 2008, Empirical evidence on the preferences of racetrack bettors, in D.B. Hausch, and W.T. Ziemba, ed.: *Efficiency of Sport and Lottery Markets, Handbooks In Finance Series* (Elsevier).

Levitt, Steven D., 2004, Why are gambling markets organised so differently from financial markets? *Economic Journal* 114, 223–246.

Madhavan, Ananth, 1992, Trading mechanisms in securities markets, *Journal of Finance* 47, 607–41.

NJPC (National Joint Pitch Council), 2006, Annual report and accounts 2005, available at http://www.njpc-ltd.co.uk/data/documents/60-NJPC_05_6.pdf.

Ottaviani, Marco, and Peter Norman Sørensen, 2005, Parimutuel versus fixed-odds markets, mimeo available at www.econ.ku.dk/sorensen/Papers/pvfom.pdf.

Ottaviani, Marco, and Peter Norman Sørensen, 2006, Noise, information, and the favorite-longshot bias, FRU Working Papers 2006/04 University of Copenhagen. Finance Research Unit available at http://ideas.repec.org/p/kud/kuiefr/200604.html.

Pagano, Marco, and Ailsa Roell, 1992, Auction and dealership markets : What is the difference? *European Economic Review* 36, 613–623.

Sauer, Raymond D., 1998, The economics of wagering markets, *Journal of Economic Literature* 36, 2021–2064.

Schnytzer, Adi, and Yuval Shilony, 2006, Insider trading and bias in a market for state-contingent claims, in Leighton Vaughan Williams, ed.: *Information Efficiency in Financial and Betting Markets*, 287–312 (Cambridge University Press: Cambridge).

Securities and Exchange Commission, 1996, Report persuant to section 21(a) of the Securities Exchange Act of 1934 regarding the NASD and the NASDAQ market, available at http://www.sec.gov/litigation/investreport/nasdaq21a.htm.

Shin, Hyun Song, 1991, Optimal betting odds against insider traders, *Economic Journal* 101, 1179–85.

Shin, Hyun Song, 1992, Prices of state contingent claims with insider traders, and the favourite-longshot bias, *Economic Journal* 102, 426–35.

Shin, Hyun Song, 1993, Measuring the incidence of insider trading in a market for state-contingent claims, *Economic Journal* 103, 1141–53.

Skaperdas, Stergios, 1996, Contest success functions, *Economic Theory* 7, 283–290.

Smith, Michael A., David Paton, and Leighton Vaughan Williams, 2006, Market efficiency in person-to-person betting, *Economica* 73, 673–689.

Snowberg, Erik, and Justin Wolfers, 2005, Explaining the favorite-longshot bias: Is it risk-love, or misperceptions? mimeo Wharton School, University of Pennsylvania.

Snyder, Wayne W., 1978, Horse racing: Testing the efficient markets model, *Journal of Finance* 33, 1109–18.

Sobel, Russell S., and S. Travis Raines, 2003, An examination of the empirical derivatives of the favourite-longshot bias in racetrack betting, *Applied Economics* 35, 371–385.

Swidler, Steve, and Ron Shaw, 1995, Racetrack wagering and the "uninformed" bettor: A study of market efficiency, *Quarterly Review of Economics and Finance* 35, 305–314.

Szymanski, Stefan, 2003, The economic design of sporting contests, *Journal of Economic Literature* 41, 1137–1187.

Terrell, Dek, and Amy Farmer, 1996, Optimal betting and efficiency in parimutuel betting markets with information costs, *Economic Journal* 106, 846–68.

Thaler, Richard H., and William T. Ziemba, 1988, Anomalies: Parimutuel betting markets: Racetracks and lotteries, *Journal of Economic Perspectives* 2, 161–174.

Tullock, Gordon, 1980, Efficient rent-seeking, in James M. Buchanan, Robert D. Tollison, and Gordon Tullock, ed.: *Toward a Theory of the Rent-Seeking Society*, 97–112 (Texas A. & M. University Press: College Station).

Vaughan Williams, Leighton, 1999, Information efficiency in betting markets: A survey, *Bulletin of Economic Research* 51, 1–39.

Vaughan Williams, Leighton, and David Paton, 1997, Why is there a favourite-longshot bias in British racetrack betting markets? *Economic Journal* 107, 150–58.

Vaughan Williams, Leighton, and David Paton, 1998, Why are some favourite-longshot biases positive and others negative? *Applied Economics* 30, 1505–10.

Walls, W. David, and Kelly Busche, 2003a, Breakage, turnover, and betting market efficiency: New evidence from Japanese horse tracks, in Leighton Vaughan Williams, ed.: *The Economics of Gambling*, 2–13 (Routledge: London).

Walls, W. David, and Kelly Busche, 2003b, Broken odds and the favourite-longshot bias in parimutuel betting: a direct test, *Applied Economics Letters* 10, 311–314.

Woodland, Bill M., and Linda M. Woodland, 1994, Market efficiency and the favorite-longshot bias: The baseball betting market, *Journal of Finance* 49, 269 – 279.

Woodland, Bill M., and Linda M. Woodland, 2001, Market efficiency and profitable wagering in the National Hockey League: Can bettors score on longshots? *Southern Economic Journal* 67, 983–995.

Woodland, Linda M., and Bill M. Woodland, 2003, The reverse favourite-longshot bias and market efficiency in Major League baseball: An update, *Bulletin of Economic Research* 55, 113–123.