Discussion Paper Series

2005 – 03

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Aligning Ambition and Incentives*

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March 3, 2005

Abstract

In many economic situations several principals contract with the same agents sequentially. Asymmetric learning about agents' abilities provides the first principal with an informational advantage and has profound implications for the design of incentive contracts. We show that the principal always strategically distorts information revelation to future principals about the ability of her agents. The second main result is that she can limit her search for optimal incentive schemes to the class of relative performance contracts that cannot be replicated by contracts based on individual performance only. This provides a new rationale for the optimality of such compensation schemes.

JEL: D82, J33, L14

Keywords: relative performance contracts, reputation, asymmetric learning

*We are indebted to Jacques Crémer, Christian Ewerhart, Bruno Jullien, Georg Nöldeke, David Perez Castrillo, Patrick Rey, Bernard Salanié and Jean Tirole for stimulating discussions and their insightful comments. The first author thanks the University of Toulouse for its kind hospitality.

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1 Introduction

Many economic situations arise in which several different principals contract with the same agents sequentially. For example, Topel and Ward (1992) report that an average white male in the USA holds ten different full-time jobs during his working life, seven of which during the first ten years of his career. In the labor market context, firms' hiring and remuneration decisions depend to a large extent on the employment history and track record of a worker. Therefore, workers' incentives for effort are not only created directly through explicit incentive contracts but also shaped by their *career concerns*. Good current performance will enhance the labor market's perception of their ability and increase future earnings, providing a strong motivation for effort. However, during any contractual relationship the current employer acquires more information about her workers than is directly available to potential future employers. In this paper we show that such interaction between different principals has profound implications for the design of optimal incentive contracts. We demonstrate how contracts allow a principal to shape agents' reputational incentives and prove that in our model a principal's contracts always distort information revelation to future principals about the performance of her agents. Moreover, we show that this provides a new rationale for the optimality of relative performance contracts.

The idea that career concerns can reduce the need for explicit incentives dates back to Fama (1980) and was first formalized by Holmström (1982/99). Such models are typically cast in terms of *symmetric learning*, where symmetrically informed firms try to infer the ability of an agent from publicly observable measures of his past performance. Agents interfere with the updating process by exerting effort to influence these performance measures. Ex ante, the parties cannot internalize the impact of agents' actions on reputation, either because no formal compensation contracts can be written, or because of limited pre-commitment powers. This prevents the dynamic incentive problem from simply collapsing to a static one.

Under symmetric learning the impact of current actions on future reputation, and thus the strength of reputational incentives, can either increase or decrease with improved information.[1] However, as pointed out above, a lot of economic situations are characterized by sequential contracting where there is *asymmetric learning*. Waldman (1984) is an early example for

[1]Dewatripont, Jewitt, and Tirole (1999) characterize the impact of different information systems on implicit incentives for situations where explicit incentives are not possible. Meyer and Vickers (1997) show that better information can either enhance or weaken incentives in environments where implicit incentives are complemented by explicit incentives. Incentives from the reputation enhancing effect of effort can be outweighed by disincentives arising from the ratchet effect.

the impact of a firm's actions on workers' reputations. In his analysis, a firm learns about its workers' types during the first period of their employment and then decides on job assignments for the second period. Outsiders can observe the job offer that a worker receives. Promotion to a job that requires higher ability sends a favorable signal to the labor market and enhances an agent's reputation. Because outside options improve, the firm has to increase the worker's compensation to retain him. As a result, the firm sets the ability threshold for promotion too high compared to the socially efficient level.[2]

Because of such strategic effects the distinction between asymmetric and symmetric learning environments is particularly important when implicit incentives are complemented by explicit incentives. If a principal acquires superior information about agents' abilities then the explicit compensation scheme provides agents with signals that affect their reputation, and therefore interact with implicit incentives. Thus, the explicit incentive contract now has two functions, which might conflict with each other. First, it is supposed to directly affect effort incentives through monetary transfers. Second, it is supposed to indirectly affect effort by controlling the flow of information to outsiders to create appropriate reputational incentives. The model of Zábojník and Bernhardt (2001) illustrates this dual role of explicit incentives. A firm sets up a tournament in which ex ante identical workers compete in human capital investments and are subject to a permanent human capital shock. The promotion scheme ranks workers by their realized human capital. Reputational incentives arise because the expected human capital shock for a tournament winner is larger than that for the next highest in rank, etc.[3]

This raises the question about the optimal structure of incentive schemes and whether tournaments can indeed be optimal contracts in such a sequential contracting environment where parties can only commit to spot contracts. The first step in answering this question is to design a model that excludes all the non-reputation based reasons for the use of relative performance contracts that the literature has identified. First, correlation between stochastic components in the outputs of different agents can be used to insure risk-averse agents against common performance shocks (e.g., Lazear and Rosen (1981), Holmström (1982), Nalebuff and Stiglitz (1983), Green and Stokey (1983), and Mookherjee (1984)).[4] Second, relative performance con-

[2] Other models with asymmetric learning are Greenwald (1986), Ricart I Costa (1988), Bernhardt (1995), and Waldman (1990). In Lazear (1986) both the incumbent employer and outsiders obtain signals about workers' abilities.

[3] The analysis abstracts from the strategic impact of promotion that arises in Waldman (1984) by assuming that the firm can commit to its promotion rule ex ante. See Waldman (2003) for a discussion of conflicts between ex ante incentives and ex post optimal promotion rules, and the role of commitment.

[4] Meyer and Vickers (1997) show that this static insurance effect can be outweighed by the negative impact on implicit incentives of the ratchet effect in a dynamic model with career concerns.

tracts can help internalize production externalities (e.g., Itoh (1991)). Third, the principal can use relative performance contracts to create proper incentives when agents can monitor each others' efforts (e.g., Ma (1988), Che and Yoo (2001), and Laffont and Rey (2001)). Fourth, if agents with other-regarding preferences interact, the principal may find it optimal to design relative performance schemes that exploit the dependence of an agent's utility on other agents' transfers to enhance incentives (e.g., Itoh (2004)). Finally, special features about the economic environment may restrict the set of feasible contracts in such a way that tournaments become optimal. For example, even if measures of absolute levels of performance are not available, it may still be possible to make ordinal comparisons among workers (e.g., McLaughlin (1988)). Malcomson (1984) argues that even though the principal might observe performance these measures may not be verifiable by outsiders. Then tournaments can be the only credible way of providing incentives (see also Bhattacharya (1983).)

The next step consists in defining the contract space. In some economic applications it is reasonable to assume that the principal can only contract on variables that are publicly observable. In our companion paper (Koch and Peyrache 2003) we take this approach and the principal's decision to reveal performance variables directly impacts her contracting possibilities. We show that it is then optimal for a principal not to disclose performance measures and implement a tournament whenever there is much heterogeneity in experienced agents' productivities, and therefore reputation matters a lot. In this paper we assume that the principal can commit to contracts on output even though it is not publicly observable.[5] We focus on the more realistic case where a principal can only write deterministic contracts, since stochastic contracts impose the strong requirement that a principal can commit to lotteries.

We model the contracting problem between a principal ('she') and two heterogenous agents ('he'). Let us call the agent endowed with a larger ability 'high skilled' and the other 'low skilled'. Both agents work for her during one period. In the second period, the agents contract with other principals who draw inferences about agents' abilities from the publicly observable contracts and hard evidence on transfers that agents received in the first period. In the model, observing an agent's first-period output would reveal his ability and, therefore, would provide a valuable signal for the high-skilled agent. Market inference relies on 'inverting' the transfer scheme to back out the output produced by an agent. As we show, the first principal can profit from using the transfer scheme to pool performance-related signals for low- and high-skilled agents in such a way that reputation increases with output. This provides agents with reputational incentives and lowers the monetary cost of implementing effort.

[5] In other words, output is assumed to be *verifiable* by a third party (e.g., a court).

Our main results obtain under the above assumptions alone. The subsequent analysis will show that the monetary transfers given by the principal influence an agent's reputation. Upon observing such a transfer, the market forms expectations about the agent's type. Moreover, the principal can profit from complementing monetary transfers with cheap-talk messages since this allows her to provide different agents who receive the same monetary transfer with distinct signals to the market.[6] Therefore, we additionally require contracts to be renegotiation proof to guarantee that our findings do not rely on contracting parties to renegotiate or on agents buying cheap-talk signals in equilibrium.

The first main result of the paper is that the principal always distorts the flow of information to future principals. Second, we show that rank order tournaments can be optimal contracts when they implement effort at no monetary cost, otherwise however, they are strictly dominated by individual performance contracts. This complements the result on the optimality of rank order tournaments in our companion paper (Koch and Peyrache 2003). However, in our setup a simple group bonus scheme can strictly outperform any individual performance contract. In fact, we prove that the principal can limit her search for optimal renegotiation proof incentive schemes to the class of relative performance contracts that are non-trivial, in the sense that they cannot be replicated by contracts based on individual performance only. This provides a new rationale for the optimality of relative performance contracts in a setup where the extant reasons for the optimality of such compensation schemes are absent.

The outline of the paper is as follows. Section 2 sets up the model. Section 3 analyzes contracts based on individual performance measures. Section 4 extends the analysis of the contracting problem to the entire set of deterministic contracts. Section 5 concludes. All proofs are in the appendix.

2 The Model

A principal (she) offers two agents (he) contracts to work for her during one period. It is common knowledge that one of the agents is *high-skilled* ($\theta = H$) and that the other one is *low-skilled* ($\theta = L$).[7] Both agents' working lives last for two periods and they have outside options that provide them with a life-time utility normalized to $u = 0$. An agent who enters

[6]For example, the principal can write different types of reference letters or award the agents some symbolic prizes. Technically, the role of such messages is to guarantee existence of an equilibrium.

[7]It is possible to introduce a stage where the principal selects agents from a heterogeneous population to contract with. We refer the interested reader to our companion paper (Koch and Peyrache 2003), where we consider this issue in a related context.

into a contract with the principal for one period then faces new contracting opportunities with other principals in the second period. All parties are risk neutral. Agents are subject to wealth and credit constraints that prevent the principal from imposing negative transfers. Discount rates are normalized to one.

We begin by describing the first-period production technology. An agent of type $\theta \in \{L, H\}$ who works for the principal in the first period can achieve two possible type-dependent output levels, a low one ($q_{\theta l}$) and a high one ($q_{\theta h}$). A high output level can only be reached if the agent exerts effort ($e_\theta = 1$) at a private cost ψ. Formally,

$$Prob\left(\tilde{q} = q_{\theta h} \middle| e_\theta = 1\right) = P_\theta, \tag{1}$$

$$Prob\left(\tilde{q} = q_{\theta l} \middle| e_\theta = 0\right) = 0. \tag{2}$$

Thus, both agents' outputs depend only on their own effort and type.[8] In addition, we assume that the high-skilled agent has a larger productivity of effort than the low-skilled one. That is,

$$P_H > P_L > 0. \tag{3}$$

In sum, stochastic output accruing to the principal from an arbitrary agent can take on four possible realizations $\tilde{q} \in Q \equiv \{q_{Ll}, q_{Lh}, q_{Hl}, q_{Hh}\}$, where $q_{Ll} < q_{Lh} \neq q_{Hl} < q_{Hh}$.

Agents are initially privately informed about their own type. At the beginning of the first period, the principal offers contracts to the two agents, who can accept or reject the contract offered to them. If an agent accepts, he non-cooperatively chooses his effort level, which is not observable by any other party.[9] At the end of the first period, output realizes and agents are paid according to their contracts.

We consider an environment where agents' outputs are contractible but not publicly observable.[10] Contracts map agents' outputs to *transfer/message* (t/m) pairs. These consist of a monetary component $t \in \mathbb{R}_+$ and a message component[11] $m \in M$, which both are *hard evidence*. Allowing for such messages in our analysis provides the principal with the means to make a distinction between agents who receive the same monetary transfers. At the end of the first period, principal and agents can renegotiate about t/m pairs. This entails a cost that can be arbitrarily small. If no renegotiation occurs, or no agreement is reached, the agent receives the t/m pair guaranteed under the contract.

[8] This eliminates production externalities as a rationale for relative performance contracts.

[9] This rules out mutual monitoring as a rationale for relative performance contracts.

[10] This could be the case whenever a third party, such as a court, can verify performance even though it is not publicly observable. Agents can sue the principal in the case of contract breach, and the court then imposes a large penalty on the principal.

[11] Examples for such messages are reference letters, job titles, honorific rewards, and medals.

In the second period, agents leave the first principal and face new contracting opportunities with different principals. In our setup the utility that an agent derives from such a contractual relationship is increasing in his expected type. To fix ideas, think of this as the reduced form of a competitive market for experienced agents. Let $k_\theta > 0$ reflect an experienced agent's productivity, which can differ from that in the first period because of human capital accumulated. Denote by $\Delta k \equiv k_H - k_L > 0$ the difference in productivities between high- and low-skilled experienced agents. Each of the principals in the market for experienced agents meets at most one of the two agents. They form beliefs about the agent's type based on the publicly observable first-period contracts and any piece of hard evidence that this agent chooses to furnish, and then simultaneously offer him contracts. Agents can conceal the t/m pair which they received in the first period or one component of it. In the following, we will simply model these principals as a 'market' that forms homogeneous beliefs about an agent based on observed t/m pairs.

Let us briefly discuss the simplifying assumption that agents leave the first principal. We have in mind markets where there are high rates of turnover, as for example in the professional service industry where employee turnover can be as high as 20 to 25 percent of the workforce per year (Maister (2003), p.15). Since separation is a joint decision of the principal and the agent one could argue that the informational asymmetry between the first principal and outsiders causes a 'lemons' problem (Greenwald 1986). However, in a more realistic setting, an agent's productivity is determined both by ability and the match between the agent's human capital and the job that a principal can offer (e.g., Antel (1985) and McLaughlin (1991)). The turnover pattern in our model would arise endogenously if the match between principal and skills for experienced agents were always better in a different segment of the labor market, *regardless* of agents' ability levels. For example, productive abilities and resources under control might be complements (Rosen 1982). Then, it is efficient for experienced agents to move to a bigger firm if they all sufficiently enhanced their human capital through learning by doing in the first period (while still differing in the attained productivity levels). Turning to empirical studies, Gibbons and Katz (1991) do find an adverse selection effect for white collar workers, which however is not apparent for workers with less than two years of tenure (p.367) – the group that we have in mind in our model. Gibbs, Ierulli, and Milgrom (2002) even report a positive effect on income ensuing a move to another firm. Moreover, both Baker, Gibbs, and Holmström (1994) and Lazear and Oyer (2004) document substantial turnover at all hierarchy levels of firms. Thus, adverse selection appears not to be a severe problem since otherwise the market for experienced labor would break down (Greenwald 1986).

To summarize, the timing of the model is as follows:

- At date 0, the first principal makes contract offers to the two agents.
- At date 1/3, agents can accept or refuse the offer. If an agent rejects he receives the outside utility level $u = 0$. If he accepts he gets hired and the market observes the contract.
- At date 1/2, agents who accepted the contract non-cooperatively choose their effort levels.
- At date 2/3, output realizes. The principal and agents can renegotiate about transfer/message pairs. The default t/m pair is the one guaranteed under the contract.
- At date 1, agents receive a t/m pair according to their contracts or the outcome of the renegotiation with the principal, and the relation with the first principal ends.
- In the second period, agents who worked for the first principal enter the market for experienced labor, where future employers meet at most one of the agents. Agents can show employers their t/m pair or conceal one or both components of it. They get paid their expected productivity given the hard evidence they provided.

The above model structure is common knowledge. We solve for a Perfect Bayesian Equilibrium and restrict attention to contracts that incite both agents to exert effort.[12]

3 Individual Performance Measure Contracts

We begin our analysis by focusing on contracts based on individual performance measures.

Definition 1 (Individual performance measure contract)
An individual performance measure (IPM) contract $f \in \Phi_{IPM}$ is a function from the set of outputs of one agent to the set of transfer-message pairs: $f : Q \to \mathbb{R}_+ \times M$.

Hence, the range of an IPM contract can encompass at most four distinct t/m pairs, each being associated with one realization of the performance measure. To identify the output states in which a t/m pair is given under an IPM contract, we use subscripts for the components of t/m

[12] A sufficient condition for this is that $q_{Lh} - q_{Ll} \geq \frac{2\psi}{P_L}$. Then, even if there existed a contract that offered sufficiently high reputational incentives to incite the high-skilled agent to exert effort at no monetary cost but not the low-skilled one, it would pay to switch to a contract with no reputational incentives that implements effort by both agents (cf. footnote 15).

IPM Contract

Output ($\mathbf{q}_{\theta\,\mathbf{s}}$)	$\mathbf{q}_{\mathbf{Hh}}$	$\mathbf{q}_{\mathbf{Lh}}$	$\mathbf{q}_{\mathbf{Hl}}$	$\mathbf{q}_{\mathbf{Ll}}$
Label	**a**	**b**	**c**	**d**
$f(q_{\theta\,i})$	(t_a, m_a)	(t_b, m_b)	(t_c, m_c)	(t_d, m_d)

Table 1: IPM contracts

pairs. These correspond to the letters assigned to the particular output states listed in Table 1. For example, t_a denotes the monetary transfer associated with output state a, and t_{ab} is a short-hand for $t_a = t_b$.

Upon meeting an agent who shows t/m pair $(t, m) \in \mathbb{R}_+ \times M$, the market forms beliefs about the probability of facing a low-skilled individual: $\beta : \mathbb{R}_+ \times M \to [0, 1]$. Given beliefs β, the market's expectation about the productivity of an agent with t/m pair (t, m) is:[13]

$$E[k_\theta | t, m] = \beta(t, m)\, k_L + [1 - \beta(t, m)]\; k_H. \tag{4}$$

One polar case are *perfectly revealing* t/m pairs which induce beliefs of facing either a high- or a low-skilled agent with probability one:

Definition 2 (Perfectly revealing transfer-message pairs and contracts)
A transfer-message pair $(t, m) \in \mathbb{R}_+ \times M$ *is perfectly revealing if* $\beta(t, m) \in \{0, 1\}$*. A contract is perfectly revealing if all of its t/m pairs are perfectly revealing.*

Before describing the belief formation process, it is useful to understand how t/m pairs are valued by an agent. Since t/m pairs (which are hard evidence) serve as a signal to the labor market, their value to an agent does not only depend on their monetary component but also on the reputation that they confer. This is captured by the concept of a *perceived transfer*. It reflects the combination of the direct monetary value t and the reputation $E[k_\theta | t, m]$ associated with a t/m pair.

Definition 3 (Perceived transfer)
The perceived transfer for a transfer-message pair $(t, m) \in \mathbb{R}_+ \times M$ *is given by* $T(t, m) = t + E\,[k_\theta |\, t, m]$.

To identify the set of t/m pairs that an agent receives under an IPM contract $f \in \Phi_{IPM}$, we define the correspondence X as follows:

$$X(f) = \{(t_a, m_a), (t_b, m_b), (t_c, m_c), (t_d, m_d)\}\,. \tag{5}$$

[13] To avoid cumbersome notation, we do not index this expectation by the contract f.

As a first step in describing the belief formation process, consider only the t/m pairs that can arise on the equilibrium path with no renegotiation for a given contract f. Given the market's belief that both agents exert effort, the contract $f \in \Phi_{IPM}$ induces a probability distribution over the t/m pairs $(t, m) \in X(f)$ for each agent. Denote the probability that the market observes t/m pair $(t, m) \in X(f)$ for an agent of type θ by $\alpha(t, m|\theta, e_L = 1, e_H = 1)$. The market forms its beliefs about the agent's type according to Bayes' rule. Thus, upon observing a t/m pair $(t, m) \in X(f)$, the probability assigned to the individual being low-skilled is

$$b(t, m) = \frac{\alpha(t, m|L, e_L = 1, e_H = 1)}{\alpha(t, m|L, e_L = 1, e_H = 1) + \alpha(t, m|H, e_L = 1, e_H = 1)}, \quad (t, m) \in X(f). \tag{6}$$

Off the equilibrium path, Bayes' rule no longer applies and restrictions on beliefs have to be imposed. A minimum requirement is that beliefs account for the possibility of agents hiding the t/m pair that they received. One way of achieving this is to assume that the market considers any agent who shows up empty handed to be low-skilled. The only other restriction that we impose on out-of-equilibrium beliefs is that they support an equilibrium where contracts are not renegotiated after output has realized.[14] The following lemma states the conditions on t/m pairs and beliefs required for renegotiation proofness:

Lemma 1 (Renegotiation proofness)

Suppose that both parties incur a positive (but possibly infinitesimally small) cost of renegotiation. Given market beliefs β, a contract is renegotiation proof if $\forall\, (t', m') \in X(f)$ with $t' > 0$ and $\forall\ (t'', m'') \in \mathbb{R}_+ \times M$, for which $(t'', m'') \neq (t', m')$, none of the following conditions is violated:

$$\begin{array}{lccc} (i) & t' > t'' & \Rightarrow & t' + E\left[k_\theta \middle|\, t', m'\right] > t'' + E\left[k_\theta \middle|\, t'', m''\right], \\ (ii) & E\left[k_\theta \middle|\, t', m'\right] < E\left[k_\theta \middle|\, t'', m''\right] & \Rightarrow & t' < t'' \text{ or } t = 0 \quad \forall\, (t, m) \in X(f), \\ (iii) & t' = t'' > 0 & \Rightarrow & E\left[k_\theta \middle|\, t', m'\right] = E\left[k_\theta \middle|\, t'', m''\right]. \end{array}$$

Condition (i) states that perceived transfers have to be strictly increasing in the monetary component. Otherwise, the agent could renegotiate with the principal, offering her to replace the contractually guaranteed t/m pair (t', m') by a pair (t'', m'') involving a lower monetary transfer. The principal never renegotiates to a t/m pair with a higher monetary transfer because agents cannot commit to repay her anything after having used the t/m pair as a signal in the market. Conditions (ii) and (iii) guarantee that it is never profitable to buy a message from the principal if the agent has cash. If $t' = 0$, renegotiation is not possible because the agent lacks the funds to bribe the principal into renegotiating to another t/m pair.

[14] For example, the simplest out-of-equilibrium beliefs that sustain contracts that are renegotiation-proof on the equilibrium path, are: $\beta(t, m) = 1$ if $(t, m) \notin X(f)$.

Note that the above conditions also guarantee that an agent never has an incentive to break a t/m pair apart and conceal one or all of its components: under the conditions in Lemma 1 an agent is never willing to give up (or hide) a monetary transfer $t > 0$ in exchange for a zero monetary transfer. Moreover, the perceived transfer can never decrease as a consequence of revealing to the market a hard evidence message. If the agent receives $t > 0$ then his reputation depends only on the monetary transfer (this follows from (iii)). If he receives $t = 0$ then he will be taken as a low-skilled agent if he shows up without any hard evidence message (this follows from our assumption about beliefs when agents show up empty handed).

Equipped with the necessary concepts, we can now derive the incentive constraints for both types of agents:

$$\begin{aligned} E\left[T(t,m)|\,\theta = H, e_L = 1, e_H = 1\right] - \psi \quad &\geq \quad E\left[T(t,m)|\,\theta = H, e_L = 1, e_H = 0\right], \quad (IC:H) \\ E\left[T(t,m)|\,\theta = L, e_L = 1, e_H = 1\right] - \psi \quad &\geq \quad E\left[T(t,m)|\,\theta = L, e_L = 0, e_H = 1\right]. \quad (IC:L) \end{aligned}$$

To decompose total incentives into monetary and reputational incentives, we rewrite the incentive constraint for the type θ agent with an IPM contract $f \in \Phi_{IPM}$ as follows:[15]

$$\underbrace{t\left(q_{\theta h}\right) - t\left(q_{\theta l}\right)}_{\text{monetary incentives}} \quad \geq \frac{\psi}{P_\theta} - \underbrace{\left[E\left[k_\theta|\, f\left(q_{\theta h}\right)\right] - E\left[k_\theta|\, f\left(q_{\theta l}\right)\right]\right]}_{\text{reputational incentives}}. \tag{7}$$

Our assumption on individuals' productivities as experienced agents leads to a minimum second-period wage of $k_L > 0$. In conjunction with the wealth and credit constraints this guarantees that agents' always receive more than their outside option value $u = 0$. Thus, agents' individual rationality constraints are always satisfied.[16]

Best IPM Contracts

As a first step in our analysis, we restrict attention to IPM contracts only and derive optimal contracts within this class of contracts (best IPM contracts). The following two rather straightforward results already greatly reduce the set of candidate IPM contracts.

Lemma 2

Under any IPM contract, if a transfer/message pair that is given to an agent who has produced low output is perfectly revealing, then the principal always sets the corresponding monetary transfer equal to zero.

[15] From this we obtain the sufficient condition stated in footnote 12. The gain in expected output from making the low-skilled agent exert effort is $P_L\left(q_{Lh} - q_{Ll}\right)$ and the expected cost of providing effort for both agents is bounded above by $2\,\psi$.

[16] The possibility of binding individual rationality constraints is analyzed in a related framework in our companion paper (Koch and Peyrache 2003). We do not treat this case here since it does not add much economic insight but greatly complicates expressions.

Lemma 3

Under any IPM contract that gives agents the same transfer/message pairs in all low-output states, the principal always sets the corresponding monetary transfer equal to zero.

Before proceeding to characterize optimal incentive schemes in the class of IPM contracts that implement effort by both agents, we discuss a few examples to build some intuition about how the principal can use t/m pairs to create reputational incentives.

As a benchmark, consider the solution to the traditional moral hazard contract with limited liability, implemented as a perfectly revealing IPM contract. Beliefs are insensitive to an agent's output since $E[k_\theta|f(q_{\theta h})] = E[k_\theta|f(q_{\theta l)}]$. Hence, the incentive constraint for the type θ agent in (7) imposes the following condition on monetary transfers:

$$t(q_{\theta h}) - t(q_{\theta l}) \geq \frac{\psi}{P_\theta}. \tag{8}$$

Thus, the contract takes the following form:

$$[(t_a, m_a), (t_b, m_b), (t_c, m_c), (t_d, m_d)], \quad \text{where} \quad t_a = \frac{\psi}{P_H}, \quad t_b = \frac{\psi}{P_L}, \quad t_c = t_d = 0, \quad m_c \neq m_d.$$

The principal has to rely exclusively on monetary incentives, yielding an expected implementation cost of $2\,\psi$. The above contract helps illustrate the restrictions on the set of equilibria that renegotiation proofness imposes. If $\frac{\psi}{P_L} + k_L < k_H$ an equilibrium where the benchmark contract above is perfectly revealing cannot occur. By way of contradiction, suppose that market beliefs are such that the contract is perfectly revealing. Then, a low-skilled agent who is entitled to t/m pair (t_b, m_b) would renegotiate with the principal to obtain t/m pair $(0, m_c)$, which yields a higher perceived transfer. Thus, the assumed beliefs are inconsistent.

From the preceding discussion two questions arise. First, can the principal benefit from not revealing agents' types through the t/m pairs she uses? Second, what role do the messages play in doing so? We will explore these issues in turn.

Setting $m_{cd} = m_c = m_d$, i.e., not perfectly revealing the agents' types in the low-output states has two countervailing effects on incentives. Upon receiving t/m pair $(0, m_{cd})$ the expected productivity for the agent is $k_L < E[k_\theta|0, m_{cd}] < k_H$. On the one hand, this creates reputational incentives for the high-skilled agent because moving from the low-output state to the (perfectly revealing) high-output state increases his reputation from $E[k_\theta|0, m_{cd}]$ to k_H. On the other hand, this leads to reputational disincentives because a low-skilled agent loses in terms of reputation by moving from the low-output state to the high-output state. This decreases his reputation from $E[k_\theta|0, m_{cd}]$ to k_L. It can easily be shown that on balance the principal benefits from setting $m_c = m_d$, i.e., not revealing the agents' types in the low-output states.[17] Thus, there is no real role for messages in this contract. However, the following example illustrates when messages actually are useful as tools for distinguishing agents with the same monetary transfers.

Consider altering the contract structure in the benchmark contract by pooling the t/m pairs for a high-skilled agent in a low-output state and a low-skilled agent in a high-output state:

$$[(t_a, m_a), (t_{bc}, m_{bc}), (t_d, m_d)], \quad \text{where} \quad t_d = 0.$$

This is an example of a contract with multiple performance standards that is not perfectly revealing. Once the output of an agent surpasses a given performance standard, he receives a different t/m pair. The contract is designed to group agents of different types in some tiers of performances. Thereby, the principal affects the probabilities of meeting the thresholds for the different types of agents, and controls how much information about agents' types is transmitted. In the above contract the performance standards create the following reputational incentives: for the high-skilled agent,

$$E[k_\theta | t_a, m_a] - E[k_\theta | t_{bc}, m_{bc}] = k_H - \frac{(1-P_H)\, k_H + P_L\, k_L}{1 - P_H + P_L} = \frac{P_L}{1 - P_H + P_L}\,\Delta\, k, \tag{9}$$

and for the low-skilled agent,

$$E[k_\theta | t_{bc}, m_{bc}] - E[k_\theta | t_d, m_d] = \frac{(1-P_H)\, k_H + P_L\, k_L}{1 - P_H + P_L} - k_L = \frac{1 - P_H}{1 - P_H + P_L}\,\Delta\, k. \tag{10}$$

On the one hand, these reputational incentives permit reducing some monetary transfers because of the effect on incentives captured by equation (7). On the other hand, pooling of t/m pairs in states b and c forces the principal to pay the monetary transfer t_{bc} also to a high-skilled agent who is in a low-output state instead of nothing under a perfectly revealing contract. A contracts' *total implementation cost* is given by the sum of perceived transfers that are required to implement effort by both agents. By pooling t/m pairs across states, the principal increases the total implementation cost relative to the fully revealing benchmark contract with expected cost $2\,\psi$. In fact, this latter contract minimizes the total implementation cost.[18] However, what matters to the principal is the *monetary* cost of implementing effort. In perfectly revealing contracts the total implementation cost is equal to the monetary implementation cost. Thus, if the reputational incentives that a non-revealing contract generates

[17] The first part of the proof of Proposition 1 in Appendix B shows this: the perfectly revealing IPM_1 is dominated by IPM_5 which sets $m_c = m_d$.

[18] The benchmark contract is the solution to the static moral hazard problem with limited liability. It minimizes the expected monetary implementation cost in the static problem, which is equal to the total implementation cost in our setting.

are larger than the increase in total implementation cost relative to the perfectly revealing benchmark contract, the principal's expected cost decreases below $2\,\psi$.

Consider the situation where the reputational incentives are sufficiently large so that the principal can incite agents to exert effort at no cost. This is the case here for $\frac{\Delta k}{\psi} > \max\left\{\frac{1+P_L-P_H}{P_L(1-P_H)}, \frac{1+P_L-P_H}{P_L\,P_H}\right\}$ (see IPM$_4$ in Appendix B). Then the principal sets $t_a = t_{bc} = t_d = 0$. Now messages are useful because they allow the principal to distinguish the three different output states. Using three distinct messages m_a, m_{bc}, and m_d she can maintain the desired reputational incentives even though the transfer scheme is totally flat.[19].

The previous examples illustrated the method for finding the best renegotiation proof IPM contract. Checking the renegotiation proofness criteria of Lemma 1 and comparing the resulting profits of the candidate IPM contracts that implement effort by both agents yields the following result:

Proposition 1

In the class of renegotiation proof individual performance measure (IPM) contracts where both agents exert effort, the profit maximizing contracts (best IPM contracts) are non perfectly revealing contracts with (multiple) performance standards.

Specifically, we show in the proof that the best IPM contract takes one of the following forms: $[(t_a, m_a), (t_{bc}, m_{bc}), (t_d, m_d)]$ (denoted IPM$_4$), $[(t_a, m_a), (t_b, m_b), (t_{cd}, m_{cd})]$ (denoted IPM$_5$), or $[(t_{ab}, m_{ab}), (t_{cd}, m_{cd})]$ (denoted IPM$_6$). The above finding implies that the principal always benefits from offering some form of non-revealing IPM contract that creates ambiguity about agents' types. This is reminiscent of Calzolari and Pavan (2002)'s model where optimal information transmission is always imperfect. In their sequential contracting model with pure asymmetric information, either the first principal never discloses information (in the absence of complementarities between the contractual relationships) or she partially discloses information. Full disclosure would eliminate all information rents in the second contractual relationship. In our model, full disclosure would eliminate all first-period reputational incentives. Strategic information revelation permits the principal to shift part of the moral hazard cost to future principals. Thus reputational incentives can be interpreted as an information rent accruing to the first principal.

[19]Technically, the messages guarantees the existence of an equilibrium by serving as a means of distinguishing two identical monetary transfers in terms of the reputation that they confer. Without messages one would need to introduce a grid for such transfers to achieve existence of an equilibrium in a situation where monetary transfers differ just to create distinct reputations.

4 Individual versus Relative Performance Measure Contracts

This section extends the analysis to the complete set of deterministic contracts Φ. This set can be partitioned into two subclasses, *individual performance measure* (IPM) contracts (the focus of Section 3) and *relative performance measure* (RPM) contracts, which condition t/m pairs on both agents' outputs in a non-trivial way, i.e., they cannot be replicated by two IPM contracts.

Definition 4 (Relative performance measure contract)
A relative performance measure (RPM) contract $F \in \Phi_{RPM}$ is a mapping from the set of outputs of both agents to the set of tuples of transfer-message pairs that cannot be replicated using IPM contracts:[20]

$$
\begin{gathered}
F : Q \times Q \to (\mathbb{R}_+ \times M) \times (\mathbb{R}_+ \times M), \\
\textit{for which } \exists\, q_{\theta' s} \in Q \textit{ such that } F(q_{\theta' s}, q_{\theta'' l}) \neq F(q_{\theta' s}, q_{\theta'' h}), \\
\textit{where} \quad s \in \{l, h\}, \quad \theta', \theta'' \in \{L, H\}, \quad \textit{and} \quad \theta' \neq \theta''.
\end{gathered}
$$

Note that we have tailored the definition to our case where one agent is high-skilled and the other is low-skilled. Thus, an RPM can be represented by a matrix that contains at most 16 distinct t/m pairs,[21] corresponding to the possible combinations of output levels (see Table 2). A symmetric RPM contract ignores the identities of the agents and thus contains at most eight distinct t/m pairs.

Let $F_i(q_i, q_j)$ denote the t/m pair received by agent $i \in \{1, 2\}$ under contract F. We extend our previous definition of the correspondence X so that it identifies the set of t/m pairs that any agent can receive under an IPM or an RPM contract:

$$
X(\phi) = \begin{cases} \{(t_a, m_a), (t_b, m_b), (t_c, m_c), (t_d, m_d)\} & \phi \in \Phi_{IPM}, \\ \left\{ (t, m) \in \mathbb{R}_+ \times M : \begin{array}{c} \exists (q_1, q_2) \in Q \times Q \text{ and } \exists\, i \in \{1, 2\} \\ \text{such that } (t, m) = F_i(q_1, q_2) \end{array} \right\} & \phi \in \Phi_{RPM}. \end{cases}
$$

Similar to the case of IPM contracts, a given RPM contract $F \in \Phi_{RPM}$ induces a probability distribution over the t/m pairs $(t, m) \in X(F)$. This distribution determines the market's equilibrium beliefs about an agent who presents a t/m pair $(t, m) \in X(F)$. Hence, the incentive constraint for the high-skilled agent, say this is agent 1, under an RPM contract $F \in \Phi_{IPM}$ is

[20] In the contract proposal game, the principal needs to specify what happens if only one agent accepts the contract. Since in equilibrium both agents accept the contract, to keep things simple, we do not include this contingency in the definition of the contract. For example, the contract could stipulate to then apply the perfectly revealing benchmark IPM contract from Section 3.

[21] Since in deterministic contracts different messages only can serve to distinguish between these cells.

RPM Contract

Output of agent 1	**Output of agent 2**			
	$\mathbf{q_{Ll}}$	$\mathbf{q_{Lh}}$	$\mathbf{q_{Hl}}$	$\mathbf{q_{Hh}}$
$\mathbf{q_{Hl}}$	$(t_1, m_1), (t_2, m_2)$	$(t_3, m_3), (t_4, m_4)$		
$\mathbf{q_{Hh}}$	$(t_5, m_5), (t_6, m_6)$	$(t_7, m_7), (t_8, m_8)$		
$\mathbf{q_{Ll}}$			$(t_1', m_1'), (t_2', m_2')$	$(t_5', m_5'), (t_6', m_6')$
$\mathbf{q_{Lh}}$			$(t_3', m_3'), (t_4', m_4')$	$(t_7', m_7'), (t_8', m_8')$

Symmetric RPM Contract

Output combinations	$\mathbf{q_{Hl}}$	$\mathbf{q_{Hh}}$
$\mathbf{q_{Ll}}$	$(t_1, m_1), (t_2, m_2)$	$(t_3, m_3), (t_4, m_4)$
$\mathbf{q_{Lh}}$	$(t_5, m_5), (t_6, m_6)$	$(t_7, m_7), (t_8, m_8)$

Table 2: RPM contracts

given by

$$\underbrace{\left[P_L\, t\left(q_{Hh}, q_{Lh}\right) + \left(1 - P_L\right)\, t\left(q_{Hh}, q_{Ll}\right)\right] - \left[P_L\, t\left(q_{Hl}, q_{Lh}\right) + \left(1 - P_L\right)\, t\left(q_{Hl}, q_{Ll}\right)\right]}_{\text{monetary incentives}}$$
$$\geq \frac{\psi}{P_H} - \underbrace{\begin{array}{l} \Big\{ \Big[P_L\, E\left[k_\theta \middle| F_1\left(q_{Hh}, q_{Lh}\right)\right] + \left(1 - P_L\right)\, E\left[k_\theta \middle| F_1\left(q_{Hh}, q_{Ll}\right)\right]\Big] \\ - \Big[P_L\, E\left[k_\theta \middle| F_1\left(q_{Hl}, q_{Lh}\right)\right] + \left(1 - P_L\right)\, E\left[k_\theta \middle| F_1\left(q_{Hl}, q_{Ll}\right)\right]\Big] \Big\} \end{array}}_{\text{reputational incentives}}. \tag{11}$$

The low-skilled agent's incentive constraint can be decomposed in a similar way. The same conditions for renegotiation proofness as for IPM contracts apply to RPM contracts (see Lemma 1). As before, the wealth and credit constraints guarantee that agents' individual rationality constraints are always satisfied.

First, consider the polar case of perfectly revealing RPM contracts. These generate no reputational incentives in equilibrium so that all that matters to an agent is the expected monetary reward received in each of the possible output states. Because the two agents are risk neutral and their outputs are independent random variables, conditioning contracts on the other agent's output cannot decrease implementation cost. This is immediately apparent from a comparison of the incentive constraints under perfectly revealing IPM contracts and RPM contracts (cf. equations (7) and (11)). Hence, we obtain the following result:

Lemma 4

Perfectly revealing relative performance measure contracts never have a (strictly) lower monetary implementation cost than the least costly perfectly revealing individual performance measure contract.

From the literature we know that the principal can gain from relative performance measure contracts if agents' performances are correlated and agents are risk averse or if there is mutual monitoring. Our model excluded these elements, and Lemma 4 confirms that RPM contracts that do not create any reputational incentives cannot strictly dominate IPM contracts. As a direct implication of Proposition 1 and Lemma 4 we obtain the following important result.

Proposition 2

Fully revealing contracts are not optimal.

This generalizes our earlier finding in Proposition 1 that the principal wants to design the transfer scheme so that it creates ambiguity about agents' types ex post. A corollary of Proposition 1 is that the principal can never benefit from any randomization scheme that is independent of agents' types, i.e., that does not affect reputational incentives. This rules out asymmetric RPM contracts:

Lemma 5

Asymmetric relative performance contracts, where agents are identified by randomly assigned indices, never have a strictly lower monetary implementation cost than symmetric relative performance contracts.

The previous results have narrowed the set of candidate contracts considerably: we are left with the IPM contracts characterized in Proposition 1 and the class of non-revealing symmetric RPM contracts.

Let us first consider rank order tournaments, which are a prominent example of RPM contracts. Such a tournament selects the agent with the highest output as the winner, who then receives the t/m pair (B^e, "winner"), consisting of an explicit bonus $B^e \geq 0$ and a message announcing the agent as the winner. The loser receives t/m pair (0, "loser"). In a setting where $q_{Hl} > q_{Lh}$ the high-skilled agent wins the tournament with certainty. That is, rank order tournaments are then perfectly revealing contracts, and by Proposition 2 cannot strictly dominate IPM contracts. In contrast, if $q_{Hl} < q_{Lh}$ rank order tournaments create the following reputational incentives:

$$E\left[k_\theta\right|(B^e, \text{“winner”})] - E\left[k_\theta\right|(0, \text{“loser”})] \quad = \quad [1 - 2\, P_L\, (1 - P_H)]\, \Delta\, k. \qquad (12)$$

The combination of explicit bonus and reputational incentives generates a perceived bonus $B = B^e + [1 - 2\, P_L\, (1 - P_H)]\, \Delta\, k$. Satisfying both agents' incentive constraints[22] requires that $B \geq \max\left\{\frac{\psi}{P_L P_H}, \frac{\psi}{P_L (1-P_H)}\right\}$. Since wealth constraints prevent the principal from imposing negative transfers, the explicit bonus is

$$B^e = \max\left\{\max\left\{\frac{\psi}{P_L P_H}, \frac{\psi}{P_L\,(1 - P_H)}\right\} - R\,(B^e)\,, 0\right\}. \tag{13}$$

Clearly, rank order tournaments can be optimal contracts whenever the heterogeneity in experienced agents' productivities, $\Delta\, k$, is sufficiently large. Then the principal obtains the maximum possible expected profit since reputational incentives are sufficient to implement effort and thus $B^e = 0$. However, a rank order tournament can never strictly dominate IPM contracts, as the next result states:

Proposition 3

Rank order tournaments cannot be strictly more profitable than individual performance measure (IPM) contracts. IPM contracts strictly dominate rank order tournaments whenever the latter require a strictly positive explicit bonus, B^e, to provide incentives.

The proof consists in showing that at least one of the best IPM contracts identified in the proof of Proposition 1 strictly dominates a rank order tournament whenever $B^e > 0$. One of the objectives of this paper was to answer the question whether tournaments can be optimal contracts in a sequential contracting environment with asymmetric learning. Proposition 3 states that rank order tournaments can only be optimal contracts when they implement effort at no monetary cost.[23]

Proposition 3 raises the question whether other RPM contracts can dominate IPM contracts. Intuition would suggest that RPM contracts can at least achieve the same expected profit as IPM contracts do since they provide the principal with more flexibility in designing t/m pairs. Therefore, one could expect that by appropriately adjusting the monetary transfers the principal can guarantee herself at least the same profit while leaving each agent with the same expected payoff as under the best IPM contract. However, such reasoning applies only if the best IPM contract involves some perfectly revealing t/m pairs (i.e., under IPM_4 or IPM_5). Then it is indeed straightforward to construct an RPM that yields the same expected profit as the IPM contract. Suppose IPM_4 is the best IPM contract, i.e. the high-skilled agent receives

[22] Recall that the individual rationality constraint is always satisfied. A detailed derivation of the following results is in Appendix D.

[23] In contrast, in our companion paper (Koch and Peyrache 2003) we show that rank order tournaments can be uniquely optimal contracts if the principal can only contract on publicly observable information.

a perfectly revealing t/m pair with monetary transfer t_a in the high-output state. Let us construct an RPM by setting $t(q_{Hh}, q_{Lh}) = t_a + \eta$ and $t(q_{Hh}, q_{Ll}) = t_a - \alpha\,\eta$, while leaving all other transfers and messages as under the IPM. Obviously, the new transfers under the RPM are perfectly revealing, just as t_a. Therefore, the expected perceived transfer that the agent receives in the high-output state is equal under both contracts if

$$P_L\,\eta - (1 - P_L)\,\alpha\,\eta = 0 \quad \Rightarrow \quad \alpha = \frac{P_L}{1 - P_L}.$$

For such an η, the RPM 'perturbs' the perfectly revealing monetary transfer of the IPM in such a way that the expected perceived transfer for the agent and the expected profit of the principal are equal under the RPM and the IPM.[24] Therefore, whenever at least one of the transfers of the optimal IPM contract is fully revealing, there always exists a symmetric RPM contract that yields at least the same profit.

The above reasoning of 'perturbing' transfers does not work if none of the transfers are perfectly revealing (if IPM$_6$ is the best IPM). In this case, making the transfer of one agent contingent on the output of the other agent leads to a discrete change in the reputation attached to a t/m pair. Total implementation cost and reputational incentives change in complex ways once the composition of t/m pairs is altered, and the wealth constraints as well as the renegotiation proofness conditions are considered. Therefore, it is far from obvious that a renegotiation-proof RPM exists that can dominate this IPM contract. However, it turns out that such an RPM does exist. Interestingly, a simple 'group bonus scheme' which rewards both agents in the same way when both produce high output and provides an individualized bonus to the high achiever if only one agent produces high output dominates IPM$_6$. This is shown in the proof of Proposition 4. This directly implies the general result that the class of IPM contracts is dominated by the class of non-revealing symmetric RPM contracts.

Proposition 4

Among the class of deterministic contracts which implement effort by all agents, non-revealing symmetric relative performance measure (RPM) contracts generically dominate individual performance measure (IPM) contracts. RPM contracts are strictly more profitable than IPM contracts for a non-degenerate range of parameter values.

The result tells us that the complex contracting problem can be reduced to a search on the subclass of non-revealing symmetric RPM contracts. This provides a new rationale for the use of relative performance contracts since the assumptions in our model were chosen so that

[24] Renegotiation proofness of such an RPM is guaranteed generically as the proof of Proposition 4 shows.

the known reasons for the use of such contracts are absent. In our setup the production process of one agent is independent of that of the other. Nevertheless, since output contains information on the agent's ability it is optimal for the principal to tie the incentives of the two agents together by pooling t/m pairs across different output states (Proposition 2). This permits her to create ambiguity about the agent's type since inverting the incentive scheme does not allow the market to back out perfectly the output that the agent realized. Under such a non-revealing contract the agent faces a lottery over future reputation that depends on the output he produces. Specifically, the principal chooses a contract that partitions the joint distribution of agents' t/m pairs in such a way that, for at least one type of agent, the reputation derived from using t/m pairs as a signal in the labor market is increasing in the output that he produces. This gives rise to reputational incentives that permit the principal to reduce monetary transfers whenever this does not create scope for renegotiation. Ideally, the principal would want to write a stochastic contract to fine tune these lotteries over perceived transfers. However, the principal might only be able to credibly commit to deterministic incentive schemes (as in our setup) since in contrast to stochastic contracts these are easy to verify by third parties such as courts. Then the principal can benefit from RPM contracts because they provide her with more flexibility in creating lotteries over perceived transfers. Even though agents' wealth constraints and renegotiation proofness constrain these choices, non-revealing symmetric RPM contracts can be shown to dominate IPM contracts (Proposition 4).

5 Conclusion

The paper characterizes the class of optimal contracts in a sequential agency setting with moral hazard. The first principal acquires information by observing agents' outputs that is not directly available to future principals. The latter form expectations about agents' types based on the first-period contract that they observe for an agent and any hard evidence from the first agency relation that the agent is able or willing to show. The first-period contract provides agents with such pieces of hard evidence through monetary transfers and cheap-talk messages. We show that the first principal always distorts the flow of agent-related information to future principals in order to sharpen incentives. Moreover, the first principal can limit her search for optimal renegotiation proof incentive schemes to the class of symmetric relative performance contracts that make each agent's transfers a function of all agents' performances in a non-trivial way, in the sense that they cannot be replicated using individual performance contracts only. This provides a new rationale for the use of relative performance contracts in an environment where the extant reasons for the optimality of such incentive schemes are absent.

A Proof of Lemma 1

Consider an agent who is entitled to a t/m pair (t', m'). Due to his wealth and credit constraints, if $t' = 0$ he cannot directly compensate the principal for the cost that she incurs by renegotiating to another t/m pair. Moreover, he cannot commit to pay the principal at a later stage. Hence, all t/m pairs $(0, m)$, $m \in M$, are renegotiation proof. If $t' > t''$, the principal always renegotiates to t'' since this decreases her cost. The agent accepts this offer if the perceived transfer for the t/m pair (t'', m'') is larger than the one for (t', m'), guaranteed by the contract. This yields condition (i). Suppose now that for some (t'', m'') we have $E[k_\theta | t'', m''] > E[k_\theta | t', m']$. Then the agent may want to renegotiate to (t'', m''). The principal agrees to this only if $t' > t''$, which is excluded by (i). If $t' < t''$, the agent cannot commit to repay the principal part of a transfer after having used it as a signal in the market. Hence, the principal refuses to renegotiate to any t/m pairs with higher monetary transfer components. Finally, if the agent never receives any cash under the contract, i.e., $t = 0 \quad \forall (t, m) \in X(\phi)$, he can never bribe the principal. This yields condition (ii). Suppose now that there exists another t/m pair with an identical monetary transfer. If the perceived transfer from this alternative t/m pair is larger than that from (t', m'), and the market beliefs are continuous in the monetary transfers in this point, then the agent can use part of t' to profitably bribe the principal into replacing m' with m''. This yields condition (iii).

B Individual Performance Measure Contracts

Proof of Lemma 2

Consider a positive monetary transfer for an agent of type θ who is in the low-output state. Reducing this monetary transfer relaxes the agent's incentive constraint and, therefore, the principal can also reduce the expected monetary transfer to the agent in the high-output state by the same amount. Since the t/m pair in the low-output state is perfectly revealing, this modification in monetary transfers does not change the reputation effect of moving from low to high output.

Proof of Lemma 3

Decreasing the monetary transfer in low-output states has no impact on the reputation, but relaxes the agents' incentive constraints. This makes it possible to reduce the monetary transfers in the high-output states.

Proof of Proposition 1

We only consider contracts under which both types of agents exert effort. Thus, all contracts ensure an expected output of:

$$\hat{q} \equiv q_{Hl} + P_H\,(q_{Hh} - q_{Hl}) + q_{Ll} + P_L\,(q_{Lh} - q_{Ll}). \tag{14}$$

Using Lemmas 2 and 3 we can characterize all types of IPM contracts, and their respective profits are given in Table 3 (the derivation of this table is in Appendix C). The proof below consists in showing that one of the candidate contracts always dominates the other IPM contracts.

1. **$\mathbf{IPM_1}$ is strictly dominated by $\mathbf{IPM_5}$.**

   If IPM_5 is not renegotiation proof, then IPM_1 is not either. This stems from the fact that $\frac{1}{P_L} < \frac{2-P_H-P_L}{P_L(1-P_L)} \equiv C_4$ since $\frac{2-P_H-P_L}{1-P_L} - 1 = \frac{1-P_H}{1-P_L} > 0$. Moreover, if

   - $\frac{\Delta\,k}{\psi} < \min\left\{C_3, \frac{1}{P_L}\right\}$, then $\Pi_5 - \Pi_1 = \frac{P_H-P_L}{2-P_H-P_L}\Delta\,k > 0$.
   - $C_3 \leq \frac{\Delta\,k}{\psi} < \frac{1}{P_L}$, then $\Pi_5 - \Pi_1 = \psi - \frac{P_L(1-P_H)}{2-P_H-P_L}\Delta\,k > 0$ since $C_4 \equiv \frac{2-P_H-P_L}{P_L(1-P_L)} > \frac{1}{P_L} > \frac{\Delta\,k}{\psi}$.

2. **$\mathbf{IPM_2}$ is strictly dominated by $\mathbf{IPM_5}$ or $\mathbf{IPM_6}$.**

   (a) Suppose that IPM_5 is renegotiation proof, i.e., $\frac{\Delta\,k}{\psi} < C_4$. Given that $C_4 - \frac{P_H-P_L}{P_H\,P_L} = \frac{P_H\,(1-P_H)+P_L\,(1-P_L)}{P_H P_L (1-P_L)} > 0$, the number of cases to consider are reduced. Then, if

   - $\frac{\Delta\,k}{\psi} < min\left\{C_3, \frac{P_H-P_L}{P_H P_L}\right\}$, then $\Pi_5 - \Pi_2 = \frac{P_H-P_L}{P_L}\,\psi - P_H\,\Delta\,k + \frac{P_H-P_L}{2-P_H-P_L}\Delta\,k > \frac{P_H-P_L}{P_L}\,\psi - P_H\,\Delta\,k > 0$, since $\frac{\Delta\,k}{\psi} < \frac{P_H-P_L}{P_H P_L}$.
   - $C_3 \leq \frac{\Delta\,k}{\psi} < \frac{P_H-P_L}{P_H P_L} < C_4$, then $\Pi_5 - \Pi_2 = \frac{P_H}{P_L}\,\psi - \frac{2\,P_H - 2\,P_H P_L + P_L - P_H^2}{2-P_H-P_L}\Delta\,k$. Since $\psi > \frac{P_H P_L}{P_H-P_L}\Delta\,k$ this is greater than $\left[\frac{P_H^2}{P_H-P_L} - \frac{2\,P_H - 2\,P_H P_L + P_L - P_H^2}{2-P_H-P_L}\right]\Delta\,k = \frac{P_L[P_H(1-P_L)+P_L(1-P_H)]}{(P_H-P_L)(2-P_H-P_L)}\,\Delta\,k > 0$.
   - $\frac{P_H-P_L}{P_H P_L} \leq \frac{\Delta\,k}{\psi} < C_3$, then $\Pi_5 - \Pi_2 = -\frac{P_H-P_L}{P_H}\,\psi + P_L\Delta\,k + \frac{P_H-P_L}{2-P_H-P_L}\Delta\,k > -\frac{P_H-P_L}{P_H}\,\psi + P_L\Delta\,k \geq 0$, since $\frac{P_H-P_L}{P_H P_L} \leq \frac{\Delta\,k}{\psi}$.

	Transfer/message pairs	Profit
Perfectly revealing IPM contracts		
1.	$[(t_a, m_a), (t_b, m_b), (t_c, m_c), (t_d, m_d)]$	$\Pi_{IPM_1} = \begin{cases} \hat{q} - 2\,\psi & \text{if } \frac{\Delta\, k}{\psi} \leq \frac{1}{P_L}, \\ \text{not renegotiation proof} & \text{otherwise.} \end{cases}$
IPM contracts with three distinct t/m pairs		
2.	$[(t_{ab}, m_{ab}), (t_c, m_c), (t_d, m_d)]$	$\Pi_{IPM_2} = \begin{cases} \hat{q} - \frac{P_H + P_L}{P_H}\,\psi - P_L\,\Delta\, k & \text{if } \frac{\Delta\, k}{\psi} > \frac{P_H - P_L}{P_H P_L}, \\ \hat{q} - \frac{P_H + P_L}{P_L}\,\psi + P_H\,\Delta\, k & \text{otherwise.} \end{cases}$
3.	$[(t_{ad}, m_{ad}), (t_b, m_b), (t_c, m_c)]$	$\Pi_{IPM_3} = \hat{q} - \frac{1 + 2\,P_H}{P_H}\,\psi - \Delta\, k.$
4.	$[(t_a, m_a), (t_{bc}, m_{bc}), (t_d, m_d)]$	$\Pi_{IPM_4} = \begin{cases} \hat{q} - \frac{1+2P_L}{P_L}\,\psi + \Delta\, k & \text{if } P_H \leq \frac{1}{2} \text{ and } \frac{\Delta\, k}{\psi} < C_1, \\ & \text{or } P_H > \frac{1}{2} \text{ and } \frac{\Delta\, k}{\psi} \leq C_2, \\ \hat{q} - \psi + \frac{P_H P_L}{1 + P_L - P_H}\,\Delta\, k & \text{if } P_H \leq \frac{1}{2} \text{ and } C_1 \leq \frac{\Delta\, k}{\psi} < C_2, \\ \text{not renegotiation proof} & \text{if } P_H > \frac{1}{2} \text{ and } C_2 < \frac{\Delta\, k}{\psi} < C_1, \\ \hat{q} & \text{if } \frac{\Delta\, k}{\psi} \geq \max\{C_1, C_2\}, \end{cases}$ $C_1 \equiv \frac{1+P_L-P_H}{P_L(1-P_H)}$, and $C_2 \equiv \frac{1+P_L-P_H}{P_L\,P_H}$.
5.	$[(t_a, m_a), (t_b, m_b), (t_{cd}, m_{cd})]$	$\Pi_{IPM_5} = \begin{cases} \hat{q} - 2\psi + \frac{P_H - P_L}{2 - P_H - P_L}\,\Delta\, k & \text{if } \frac{\Delta\, k}{\psi} < C_3, \\ \hat{q} - \psi - \frac{P_L(1-P_H)}{2 - P_H - P_L}\,\Delta\, k & \text{if } C_3 \leq \frac{\Delta\, k}{\psi} < C_4, \\ \text{not renegotiation proof} & \text{otherwise,} \end{cases}$ $C_3 \equiv \frac{2 - P_H - P_L}{P_H(1-P_L)}$ and $C_4 \equiv \frac{2 - P_H - P_L}{P_L(1-P_L)}$.
IPM contracts with two distinct t/m pairs		
6.	$[(t_{ab}, m_{ab}), (t_{cd}, m_{cd})]$	$\Pi_{IPM_6} = \begin{cases} \hat{q} - \frac{P_L + P_H}{P_L}\,\psi + \frac{P_H - P_L}{2 - P_H - P_L}\,\Delta\, k & \text{if } \frac{\Delta\, k}{\psi} < C_5, \\ \hat{q} & \text{otherwise,} \end{cases}$ $C_5 \equiv \frac{(P_H + P_L)(2 - P_H - P_L)}{P_L(P_H - P_L)}$.

Table 3: IPM contracts for which both agents exert effort

- $\max\left\{C_3, \frac{P_H-P_L}{P_H P_L}\right\} \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_5 - \Pi_2 = \frac{P_L}{P_H}\psi + P_L \Delta k - \frac{P_L(1-P_H)}{2-P_H-P_L}\Delta k = \frac{P_L}{P_H}\psi + \frac{P_L(1-P_L)}{2-P_H-P_L}\Delta k > 0$.

(b) Suppose now that IPM_5 is not renegotiation proof, i.e., $\frac{\Delta k}{\psi} \geq C_4$, then if

- $C_4 \leq \frac{\Delta k}{\psi} < C_5$, then $\Pi_6 - \Pi_2 = -\frac{(P_H-P_L)(P_L+P_H)}{P_L P_H}\psi + \frac{P_H-P_L}{2-P_H-P_L}\Delta k + P_L \Delta k$. Since $\psi \leq \frac{1}{C_4}\Delta k = \frac{P_L(1-P_L)}{2-P_H-P_L}\Delta k$ this expression is greater or equal to $\left[-\frac{(1-P_L)(P_H-P_L)(P_L+P_H)}{P_H(2-P_H-P_L)} + \frac{P_H-P_L}{2-P_H-P_L} + P_L\right]\Delta k = \frac{P_L(1-P_L)(P_L+P_H)}{P_H(2-P_H-P_L)} > 0$.
- $C_5 \leq \frac{\Delta k}{\psi}$, then $\Pi_6 - \Pi_2 = \frac{P_H+P_L}{P_H}\psi + P_L \Delta k > 0$.

3. **$\mathbf{IPM_3}$ is strictly dominated by $\mathbf{IPM_5}$ or by $\mathbf{IPM_6}$.**

(a) Suppose IPM_5 is renegotiation proof, i.e., $\frac{\Delta k}{\psi} < C_4$, then if

- $\frac{\Delta k}{\psi} < C_3$, then $\Pi_5 - \Pi_3 = \frac{\psi}{P_H} + \Delta k + \frac{P_H-P_L}{2-P_H-PL}\Delta k > 0$.
- $C_3 \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_5 - \Pi_3 = \frac{1+P_H}{P_H}\psi + \frac{(2-PH)(1-PL)}{2-PH-PL}\Delta k > 0$.

(b) Suppose now that IPM_5 is not renegotiation proof, i.e., $\frac{\Delta k}{\psi} \geq C_4$:

- $C_4 \leq \frac{\Delta k}{\psi} < C_5$. Then $\Pi_6 - \Pi_3 = \frac{P_L + P_H P_L - P_H^2}{P_H P_L}\psi + \frac{2(1-P_L)}{2-P_H-PL}\Delta k > -\frac{P_H}{P_L}\psi + \frac{2(1-P_L)}{2-P_H-PL}\Delta k$. Since $\psi \leq \frac{\Delta k}{C_4} = \frac{P_L(1-P_L)}{2-P_H-P_L}\Delta k$ this expression is greater or equal to $\frac{(2-P_H)(1-P_L)}{2-P_H-P_L}\Delta k > 0$.
- $C_5 \leq \frac{\Delta k}{\psi}$. Then $\Pi_6 - \Pi_3 = \frac{1+2P_H}{P_H}\psi + \Delta k > 0$.

Hence, only IPM_5, IPM_6, or IPM_4 can be optimal in the class of individual performance measure contracts. Each of these contracts can be shown not to be always dominated by one of the other two candidate contracts:

1. Looking at the profits of IPM contracts in Table 3, it is obvious that IPM_4 is optimal whenever $\frac{\Delta k}{\psi} \geq \max\{C_1, C_2\}$ and that IPM_6 is optimal whenever $\frac{\Delta k}{\psi} \geq C_5$. For example, fix $P_L = 0.4$, then $\max\{C_1, C_2\} < C_5$ for $P_H < 0.76$ and IPM_4 is uniquely optimal among IPM contracts for values of $\frac{\Delta k}{\psi}$ between these thresholds.

2. Similarly, IPM_6 can be uniquely optimal since the above inequality is reversed for larger values of P_H.

3. Finally, IPM_5 is the best IPM contract for $P_H > \frac{1}{2}$ and $C_2 < \frac{\Delta k}{\psi} < \min\{C_1, C_4\}$:[25]

   - IPM_5 strictly dominates IPM_6 for $\frac{\Delta k}{\psi} < C_4$. First, recall that $C_4 > C_3$ and note that $C_5 - C_4 = \frac{(2-P_H-P_L)^2}{(P_H-P_L)(1-P_L)} > 0$. Hence, there are only two cases to consider:

[25] This interval is non-degenerate. One can easily see that for $P_H > \frac{1}{2}$ we have $C_1 > C_2$ and $C_4 - C_2 = [(2P_H - 1)(1-P_L) - (P_H - P_L)(P_H + P_L - 1)]/[P_H P_L (1-P_L)] > (2P_H - 1)(1-P_H)/[P_H P_L (1-P_L)] > 0$.

- $\frac{\Delta k}{\psi} < C_4$, then $\Pi_{IPM_5} - \Pi_{IPM_6} = \frac{P_H - P_L}{P_L}\,\psi > 0$.
- $C_3 \le \frac{\Delta k}{\psi} < C_4$, then $\Pi_{IPM_5} - \Pi_{IPM_6} = \frac{P_H}{P_L}\,\psi - \frac{P_H(1-P_L)}{2-P_H-P_L}\,\Delta k > 0$, since $\frac{\Delta k}{\psi} < C_4 \equiv \frac{2-P_H-P_L}{P_L(1-P_L)}$.

- IPM$_4$ is not renegotiation proof for $P_H > \frac{1}{2}$ and $C_2 < \frac{\Delta k}{\psi} < C_1$.

C Individual performance measure contracts

We characterize here the generic types of IPM contracts that can arise and derive their profits, which are summarized in Table 3.

Perfectly revealing IPM contracts

$\mathbf{IPM_1}$: $[(\mathbf{t_a}, \mathbf{m_a}), (\mathbf{t_b}, \mathbf{m_b}), (\mathbf{t_c}, \mathbf{m_c}), (\mathbf{t_d}, \mathbf{m_d})]$

IPM contracts with four distinct t/m pairs are perfectly revealing. Applying Lemma 2, the monetary transfers in the low-output states are zero. This corresponds to the traditional moral hazard contract, implemented as a perfectly revealing contract, discussed in Section 3. It has expected profit

$$\Pi_1 = \begin{cases} \hat{q} - 2\,\psi & \text{if } \frac{\Delta k}{\psi} \le \frac{1}{P_L}, \\ \text{not renegotiation proof} & \text{otherwise.} \end{cases} \tag{15}$$

IPM contracts with three distinct t/m pairs

$\mathbf{IPM_2}$: $[(\mathbf{t_{ab}}, \mathbf{m_{ab}}), (\mathbf{t_c}, \mathbf{m_c}), (\mathbf{t_d}, \mathbf{m_d})]$

The t/m pairs in states c and d are perfectly revealing and, therefore, it follows from Lemma 2 that $t_c = t_d = 0$. Moreover, $E\left[k_\theta \middle|\, t_{ab}, m_{ab}\right] = \frac{P_H\,k_H + P_L\,k_L}{P_H + P_L}$. The respective incentive constraints are

$$P_H\left[t_{ab} + E\left[k_\theta \middle|\, t_{ab}, m_{ab}\right]\right] + (1-P_H)\ k_H - \psi \ge k_H, \tag{$IC:H$}$$

$$P_L\left[t_{ab} + E\left[k_\theta \middle|\, t_{ab}, m_{ab}\right]\right] + (1-P_L)\ k_L - \psi \ge k_L. \tag{$IC:L$}$$

Depending on which constraint binds, $t_{ab} = \max\left\{\frac{\psi}{P_H} + \frac{P_L}{P_H+P_L}\,\Delta k, \frac{\psi}{P_L} - \frac{P_H}{P_H+P_L}\,\Delta k\right\}$. If $\frac{\Delta k}{\psi} > \frac{P_H - P_L}{P_H P_L}$ the binding constraint is $(IC:H)$. Applying Lemma 1, the contract is renegotiation proof since $t_{ab} + E\left[k_\theta \middle|\, t_{ab}, m_{ab}\right] = \frac{\psi}{P_H} + k_H > k_H$. Otherwise, $(IC:L)$ is the binding constraint. Renegotiation proofness requires that $t_{ab} + E\left[k_\theta \middle|\, t_{ab}, m_{ab}\right] = \frac{\psi}{P_L} + k_L > k_H$, which is equivalent to $\frac{\Delta k}{\psi} < \frac{1}{P_L}$. Since $\frac{\Delta k}{\psi} \le \frac{P_H - P_L}{P_H P_L} < \frac{1}{P_L}$, the contract is renegotiation proof. IPM_2 yields expected profit

$$\Pi_2 = \begin{cases} \hat{q} - \frac{P_H+P_L}{P_H}\,\psi - P_L\,\Delta k & \text{if } \frac{\Delta k}{\psi} > \frac{P_H-P_L}{P_H P_L}, \\ \hat{q} - \frac{P_H+P_L}{P_L}\,\psi + P_H\,\Delta k & \text{otherwise.} \end{cases} \tag{16}$$

IPM : $[(\mathbf{t_{ac}}, \mathbf{m_{ac}}), (\mathbf{t_b}, \mathbf{m_b}), (\mathbf{t_d}, \mathbf{m_d})]$

Under such a contract only the low-skilled agent exerts effort.

$\mathbf{IPM_3}$: $[(\mathbf{t_{ad}}, \mathbf{m_{ad}}), (\mathbf{t_b}, \mathbf{m_b}), (\mathbf{t_c}, \mathbf{m_c})]$

Lemma 2 implies that $t_c = 0$; $E[k_\theta | t_{ad}, m_{ad}] = \frac{P_H k_H + (1-P_L) k_L}{1+P_H-P_L}$. The respective incentive constraints are:

$$P_H [t_{ad} + E[k_\theta | t_{ad}, m_{ad}]] + (1-P_H) k_H - \psi \geq k_H, \qquad (IC:H)$$

$$P_L [t_b + k_L] + (1-P_L) [t_{ad} + E[k_\theta | t_{ad}, m_{ad}]] - \psi \geq t_{ad} + E[k_\theta | t_{ad}, m_{ad}]. \qquad (IC:L)$$

Hence, transfers are $t_{ad} = \frac{\psi}{P_H} + \frac{1-P_L}{1+P_H-P_L} \Delta k$ and $t_b = t_{ad} + \frac{\psi}{P_L} + \frac{P_H}{1+P_H-P_L} \Delta k$. Note, $t_{ad} + E[k_\theta | T_{ad}] = \frac{\psi}{P_H} + k_H > k_H$ and $t_b + k_L = \frac{P_L+P_H}{P_L P_H} \psi + k_H > k_H$. Moreover, $t_b > t_{ad}$ and $t_b + k_L > t_{ad} + E[k_\theta | t_{ad}, m_{ad}]$. Hence, the renegotiation proofness conditions of Lemma 1 are satisfied. The expected profit is:

$$\Pi_3 = \hat{q} - \frac{1 + 2 P_H}{P_H} \psi - \Delta k. \qquad (17)$$

$\mathbf{IPM_4}$: $[(\mathbf{t_a}, \mathbf{m_a}), (\mathbf{t_{bc}}, \mathbf{m_{bc}}), (\mathbf{t_d}, \mathbf{m_d})]$

Lemma 2 implies that $t_d = 0$; $E[k_\theta | t_{bc}, m_{bc}] = \frac{(1-P_H) k_H + P_L k_L}{1+P_L-P_H}$. The respective incentive constraints are:

$$P_H [t_a + k_H] + (1-P_H) [t_{bc} + E[k_\theta | t_{bc}, m_{bc}]] - \psi \geq t_{bc} + E[k_\theta | t_{bc}, m_{bc}], \qquad (IC:H)$$

$$P_L [t_{bc} + E[k_\theta | t_{bc}, m_{bc}]] + (1-P_L)\, k_L - \psi \geq k_L. \qquad (IC:L)$$

Since monetary transfers have to be nonnegative, the conditions on the transfers are:

$$t_{bc} \geq \max\left\{ \frac{\psi}{P_L} - \frac{(1-P_H)}{1+P_L-P_H} \Delta k,\, 0 \right\}, \qquad (18)$$

$$t_a \geq \max\left\{ t_{bc} + \frac{\psi}{P_H} - \frac{P_L}{1+P_L-P_H} \Delta k,\, 0 \right\}. \qquad (19)$$

Thus,

$$t_{bc} = \begin{cases} \frac{\psi}{P_L} - \frac{(1-P_H)}{1+P_L-P_H} \Delta k & \text{if} \quad \frac{\Delta k}{\psi} < \frac{1+P_L-P_H}{P_L(1-P_H)}, \\ 0 & \text{otherwise.} \end{cases} \qquad (20)$$

In addition to being nonnegative, t_a has to satisfy condition (19).

Suppose first, that $P_H \leq \frac{1}{2}$, then

$$t_a = \begin{cases} \frac{\psi}{P_H} - \frac{P_L}{1+P_L-P_H} \Delta k & \text{if} \quad \frac{1+P_L-P_H}{P_L(1-P_H)} < \frac{\Delta k}{\psi} < \frac{1+P_L-P_H}{P_H P_L}, \\ \frac{P_H+P_L}{P_L P_H} \psi - \Delta k & \text{if} \quad \frac{\Delta k}{\psi} \leq \frac{1+P_L-P_H}{P_L(1-P_H)}, \\ 0 & \text{otherwise.} \end{cases} \qquad (21)$$

The first line uses the fact that for $P_H < \frac{1}{2}$ we have (threshold for $t_{bc} > 0$) $\frac{1+P_L-P_H}{P_L(1-P_H)} < \frac{(P_H+P_L)}{P_L P_H}$ (threshold for $t_a > 0$). For $P_H \leq \frac{1}{2}$ the contract satisfies the renegotiation proofness criteria of Lemma 1: if $t_{bc} > 0$ then $t_a > t_{bc}$. Also, for $t_a > 0$ we then obviously have $t_a + k_H > t_{bc} + E\left[k_\theta | t_{bc}, m_{bc}\right]$.

Suppose now, that $P_H > \frac{1}{2}$, then

$$t_a \quad = \quad \begin{cases} \frac{P_H+P_L}{P_L P_H}\,\psi - \Delta\,k & \text{if} \quad \frac{\Delta\,k}{\psi} < \frac{P_H+P_L}{P_H P_L}, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

For $\frac{\Delta\,k}{\psi} \leq \frac{1+P_L-P_H}{P_L P_H}$ we have $t_a \geq t_{bc}$ so that $t_a + k_H > t_{bc} + E\left[k_\theta | t_{bc}, m_{bc}\right]$. Now, if $\frac{1+P_L-P_H}{P_H P_L} < \frac{\Delta\,k}{\psi} < \frac{1+P_L-P_H}{P_L(1-P_H)}$, we have $t_{bc} > t_a$. For this range of parameter values IPM$_4$ is not renegotiation proof: renegotiation proofness would require that $t_{bc} + E\left[k_\theta | t_{bc}, m_{bc}\right] > t_a + k_H \Leftrightarrow t_{bc} - t_a > \frac{P_L\,\Delta\,k}{1+P_H-P_L}$. For the range $\frac{P_H+P_L}{P_L\,P_H} > \frac{\Delta\,k}{\psi} > \frac{1+P_H-P_L}{P_L\,P_H}$ we have $t_{bc} - t_a = \frac{P_L\,\Delta\,k}{1+P_H-P_L} - \frac{\psi}{P_H}$. For the range $\frac{1+P_H-P_L}{P_L(1-P_H)} > \frac{\Delta\,k}{\psi} \geq \frac{P_H+P_L}{P_L\,P_H}$ we have $t_{bc} - t_a = t_{bc} = \frac{\psi}{P_L} - \frac{(1-P_H)}{1+P_L-P_H}\,\Delta\,k$. But $t_{bc} - \frac{P_L\,\Delta\,k}{1+P_H-P_L} = \frac{\psi}{P_L} - \Delta\,k < 0$, since $\frac{\Delta\,k}{\psi} \geq \frac{P_H+P_L}{P_L\,P_H} > \frac{1}{P_L}$.

Hence, the expected profit is:

$$\Pi_4 \quad = \quad \begin{cases} \hat{q} - \frac{1+2P_L}{P_L}\,\psi + \Delta\,k & \begin{array}{l}\text{if} \quad P_H \leq \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} < C_1, \\ \text{or } P_H > \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} \leq C_2,\end{array} \\ \hat{q} - \psi + \frac{P_H P_L}{1+P_L-P_H}\,\Delta\,k & \text{if} \quad P_H \leq \frac{1}{2} \text{ and } C_1 \leq \frac{\Delta\,k}{\psi} < C_2, \\ \text{not renegotiation proof} & \text{if} \quad P_H > \frac{1}{2} \text{ and } C_2 < \frac{\Delta\,k}{\psi} < C_1, \\ \hat{q} & \text{if} \quad \frac{\Delta\,k}{\psi} \geq \max\left\{C_1, C_2\right\}, \end{cases} \tag{23}$$

where $C_1 \equiv \frac{1+P_L-P_H}{P_L(1-P_H)}$, and $C_2 \equiv \frac{1+P_L-P_H}{P_L\,P_H}$.

IPM : $\quad [(\mathbf{t_a}, \mathbf{m_a}), (\mathbf{t_{bd}}, \mathbf{m_{bd}}), (\mathbf{t_c}, \mathbf{m_c})]$

Under such a contract only the high-skilled agent exerts effort.

IPM$_5$: $\quad [(\mathbf{t_a}, \mathbf{m_a}), (\mathbf{t_b}, \mathbf{m_b}), (\mathbf{t_{cd}}, \mathbf{m_{cd}})]$

Lemma 3 implies that $t_{cd} = 0$. Moreover, $E\left[k_\theta | \, t_{cd}, m_{cd}\right] = \frac{(1-P_H)\,k_H + (1-P_L)\,k_L}{2-P_H-P_L}$. The respective incentive constraints are:

$$P_H\left[t_a + k_H\right] + (1 - P_H)\; E\left[k_\theta | \, t_{cd}, m_{cd}\right] - \psi \geq E\left[k_\theta | \, t_{cd}, m_{cd}\right], \tag{$IC:H$}$$

$$P_L\left[t_b + k_L\right] + (1 - P_L)\; E\left[k_\theta | \, t_{cd}, m_{cd}\right] - \psi \geq E\left[k_\theta | \, t_{cd}, m_{cd}\right]. \tag{$IC:L$}$$

Since monetary transfers have to be nonnegative, the transfers are:

$$t_a \quad = \quad \begin{cases} \frac{\psi}{P_H} - \frac{1-P_L}{2-P_H-P_L}\,\Delta\,k & \text{if} \quad \frac{\Delta\,k}{\psi} < \frac{2-P_H-P_L}{P_H(1-P_L)}, \\ 0 & \text{otherwise,} \end{cases} \tag{24}$$

$$t_b \quad = \quad \frac{\psi}{P_L} + \frac{1 - P_H}{2 - P_H - P_L}\,\Delta\,k. \tag{25}$$

If $t_a > 0$ we always have $t_b + k_L > t_a + k_H$. In contrast, if $t_a = 0$, we obtain $t_b + k_L > k_H$ only if $\frac{\Delta\,k}{\psi} < \frac{2-P_H-P_L}{P_L(1-P_L)}$ (which is greater than the threshold for $t_a = 0$). Thus, the conditions of Lemma 1 are satisfied if and only if $\frac{\Delta k}{\psi} < \frac{2-P_H-P_L}{P_L(1-P_L)}$. Hence, the expected profit is:

$$\Pi_5 \quad = \quad \begin{cases} \hat{q} - 2\psi + \frac{P_H-P_L}{2-P_H-P_L}\,\Delta k & \text{if} \quad \frac{\Delta k}{\psi} < C_3, \\ \hat{q} - \psi - \frac{P_L(1-P_H)}{2-P_H-P_L}\,\Delta k & \text{if} \quad C_3 \le \frac{\Delta k}{\psi} < C_4, \\ \text{not renegotiation proof} & \text{otherwise,} \end{cases} \tag{26}$$

where $C_3 \equiv \frac{2-P_H-P_L}{P_H(1-P_L)}$ and $C_4 \equiv \frac{2-P_H-P_L}{P_L(1-P_L)}$.

IPM contracts with two distinct t/m pairs

IPM$_\mathbf{6}$: $[(\mathbf{t_{ab}}, \mathbf{m_{ab}}), (\mathbf{t_{cd}}, \mathbf{m_{cd}})]$

Lemma 3 implies $t_{cd} = 0$. Moreover, $E\left[k_\theta | \, t_{ab}, m_{ab}\right] = \frac{P_H\,k_H + P_L\,k_L}{P_H+P_L}$ and $E\left[k_\theta | \, t_{cd}, m_{cd}\right] = \frac{(1-P_H)\,k_H + (1-P_L)\,k_L}{2-P_H-P_L}$. The respective incentive constraints are:

$$P_H\left[t_{ab} + E\left[k_\theta | \, t_{ab}, m_{ab}\right]\right] + (1-P_H)\; E\left[k_\theta | \, t_{cd}, m_{cd}\right] - \psi \ge E\left[k_\theta | \, t_{cd}, m_{cd}\right], \tag{$IC:H$}$$

$$P_L\left[t_{ab} + E\left[k_\theta | \, t_{ab}, m_{ab}\right]\right] + (1-P_L)\; E\left[k_\theta | \, t_{cd}, m_{cd}\right] - \psi \ge E\left[k_\theta | \, t_{cd}, m_{cd}\right]. \tag{$IC:L$}$$

To satisfy the wealth and incentive constraints,

$$t_{ab} = \begin{cases} \frac{\psi}{P_L} - \frac{P_H-P_L}{(P_H+P_L)(2-P_H-P_L)}\,\Delta k & \text{if} \quad \frac{\Delta k}{\psi} < \frac{(P_H+P_L)(2-P_H-P_L)}{P_L(P_H-P_L)}, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

Note that the renegotiation proofness conditions from Lemma 1 are satisfied since $t_{ab} + E\left[k_\theta | \, t_{ab}, m_{ab}\right] - E\left[k_\theta | \, t_{cd}, m_{cd}\right] = \frac{\psi}{P_L} > 0$. Hence, the expected profit is:

$$\Pi_6 \quad = \quad \begin{cases} \hat{q} - \frac{P_L+P_H}{P_L}\,\psi + \frac{P_H-P_L}{2-P_H-P_L}\,\Delta k & \text{if} \quad \frac{\Delta k}{\psi} < C_5, \\ \hat{q} & \text{otherwise,} \end{cases} \tag{28}$$

where $C_5 \equiv \frac{(P_H+P_L)(2-P_H-P_L)}{P_L(P_H-P_L)}$.

IPM : $[(\mathbf{t_{abc}}, \mathbf{m_{abc}}), (\mathbf{t_d}, \mathbf{m_d})]$ **and** $[(\mathbf{t_{acd}}, \mathbf{m_{acd}}), (\mathbf{t_b}, \mathbf{m_b})]$

Under such a contract only the low-skilled agent exerts effort.

IPM : $[(\mathbf{t_{abd}}, \mathbf{m_{abd}}), (\mathbf{t_c}, \mathbf{m_c})]$ **and** $[(\mathbf{t_a}, \mathbf{m_a}), (\mathbf{t_{bcd}}, \mathbf{m_{bcd}})]$

Under such a contract only the high-skilled agent exerts effort.

IPM : $[(\mathbf{t_{ad}}, \mathbf{m_{ad}}), (\mathbf{t_{bc}}, \mathbf{m_{bc}})]$

Under such a contract only one of the two types of agents exerts effort, depending on which of the two incentive constraints binds.

IPM : $[(\mathbf{t_{ac}}, \mathbf{m_{ac}}), (\mathbf{t_{bd}}, \mathbf{m_{bd}})]$

Under such a contract none of the agents has an incentive to work.

IPM contracts with a unique t/m pair

Under such a contract none of the agents has an incentive to work.

D Rank Order Tournaments

This section provides the details for the analysis of rank order tournaments in Section 4. If $q_{Hl} < q_{Lh}$ rank order tournaments create reputational incentives, as will be demonstrated.

Reputation Effects

The market observes the explicit bonus B^e that the principal sets as well as the tournament outcome. Suppose that the market's belief is that the explicit bonus B^e implements effort by all agents. Then the winner of the tournament is more likely to be high-skilled and the loser is more likely to be low-skilled, and the market's expectations about productivities are:

$$
\begin{aligned}
E\left[k_\theta \middle| B^e_{11}, winner\right] &= \left[P_H + (1-P_L)(1-P_H)\right] k_H + P_L (1-P_H) k_L, && (29)\\
E\left[k_\theta \middle| B^e, loser\right] &= \left[P_H + (1-P_L)(1-P_H)\right] k_L + P_L (1-P_H) k_H. && (30)
\end{aligned}
$$

The winner receives a higher wage in the second period than the loser. Hence, the reputation gain of winning is:

$$
\begin{aligned}
R\left(B^e\right) &\equiv E\left[k_\theta \middle| winner, B^e\right] - E\left[k_\theta \middle| loser, B^e\right]\\
&= \left[1 - 2 P_L (1-P_H)\right] \Delta k. && (31)
\end{aligned}
$$

The winner's *perceived bonus* B is the sum of the explicit bonus paid by the principal and the reputation effect of winning the tournament: $B = B^e + R\left(B^e\right)$.

Incentive constraints

When both agents exert effort, the high-skilled agent has the highest performance and wins the tournament if he is in the high-output state ($q_{Hh} > q_{Lh} > q_{Ll}$) or if both agents are in the low-output state ($q_{Hl} > q_{Ll}$). Otherwise the low-skilled agent wins the tournament. Thus, to incite both agents to exert effort requires meeting the following incentive constraints for the respective agents:

$$
\begin{aligned}
\left[P_H + (1-P_H)(1-P_L)\right] B - \psi &\geq (1-P_L)\, B\\
\Leftrightarrow \quad B &\geq \tfrac{\psi}{P_L P_H}, && (IC:TH)\\
P_L\, (1-P_H)\, B - \psi &\geq 0\\
\Leftrightarrow \quad B &\geq \tfrac{\psi}{P_L (1-P_H)}. && (IC:TL)
\end{aligned}
$$

To satisfy both agents' incentive constraints[26] requires that the perceived bonus has to satisfy $B \geq \max\left\{\frac{\psi}{P_L P_H}, \frac{\psi}{P_L(1-P_H)}\right\}$.

[26]Recall that the individual rationality constraints are always satisfied.

Equilibrium

Combining the above results now allows us to pin down the equilibrium wage and explicit bonus such that all agents exert effort and the market has consistent beliefs. Since wealth constraints prevent the principal from imposing negative transfers, she sets

$$B^e = \max\left\{\max\left\{\frac{\psi}{P_L P_H}, \frac{\psi}{P_L\,(1-P_H)}\right\} - R\,(B^e)\,, 0\right\}. \tag{32}$$

Denote by $\hat{q}$ the expected output when both agents exert effort (see (**??**)). Under a rank order tournament that implements effort by all worker types, the principal has expected profit

$$\Pi_T = \begin{cases} \hat{q} - \frac{\psi}{P_L P_H} + [1 - 2\,P_L\,(1-P_H)]\,\Delta\,k & \text{if} \quad P_H \leq \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} < TC_1, \\ \hat{q} - \frac{\psi}{P_L(1-P_H)} + [1 - 2\,P_L\,(1-P_H)]\,\Delta\,k & \text{if} \quad P_H > \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} < TC_2, \\ \hat{q} & \text{if} \quad \frac{\Delta\,k}{\psi} \geq \max\{TC_1, TC_2\}\,, \end{cases} \tag{33}$$

where $TC_1 \equiv \frac{1}{P_L P_H\,[1-2\,P_L(1-P_H)]}$, and $TC_2 \equiv \frac{1}{P_L(1-P_H)\,[1-2\,P_L(1-P_H)]}$.

E Proof of Proposition 3

The expected profit under a rank order tournament (T) is given by

$$\Pi_T = \begin{cases} \hat{q} - \frac{\psi}{P_L P_H} + [1 - 2\,P_L\,(1-P_H)]\,\Delta\,k & \text{if} \quad P_H \leq \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} < TC_1, \\ \hat{q} - \frac{\psi}{P_L(1-P_H)} + [1 - 2\,P_L\,(1-P_H)]\,\Delta\,k & \text{if} \quad P_H > \frac{1}{2} \text{ and } \frac{\Delta\,k}{\psi} < TC_2, \\ \hat{q} & \text{if} \quad \frac{\Delta\,k}{\psi} \geq \max\{TC_1, TC_2\}\,, \end{cases} \tag{34}$$

where $TC_1 \equiv \frac{1}{P_L P_H\,[1-2\,P_L(1-P_H)]}$, and $TC_2 \equiv \frac{1}{P_L(1-P_H)\,[1-2\,P_L(1-P_H)]}$.

1. IMP$_4$ dominates T whenever it is renegotiation proof

First, consider $P_H \leq \frac{1}{2}$. Note that then $TC_1 = \max\{TC_1, TC_2\}$ and $C_2 = \max\{C_1, C_2\}$. Since

$$TC_1 - C_2 = \frac{P_H - P_L + 2\,P_L\,(1-P_H)\,(1+P_L-P_H)}{P_L\,P_H\,[1-2\,P_L\,(1-P_H)]} > 0 \tag{35}$$

we have that $\Pi_T = \hat{q} \Rightarrow \Pi_{IPM_4} = \hat{q}$. Thus, what remains to be considered are the cases:

- $C_1 \leq \frac{\Delta\,k}{\psi} < C_2$. Then $\Pi_{IPM_4} - \Pi_T = \frac{1-P_H\,(1+2\,P_L)}{P_L\,P_H}\,\psi + 2\,P_L\,(1-P_H)\,\Delta\,k > 0$ since $1 - P_H\,(1+2\,P_L) > 1 - P_H\,(1+2\,P_H) \geq 0$ and $P_H \leq \frac{1}{2}$.

- $C_1 \leq \frac{\Delta\,k}{\psi} < C_2$ $(< TC_1)$. Then $\Pi_{IPM_4} - \Pi_T > 0$ if

$$\frac{\Delta\,k}{\psi} < C_2\,\underbrace{\frac{1-P_L\,P_H}{(1-P_H)\,\{1+P_L\,[1-2\,(1+P_L-P_H)]\}}}_{>1}\,. \tag{36}$$

This condition is satisfied since the right-hand side (RHS) is larger than C_2.

Now consider $P_H > \frac{1}{2}$. Then, $TC_2 - C_1 > 0$ and we again have that $\Pi_T = \hat{q} \Rightarrow \Pi_{IPM_4} = \hat{q}$. Thus, what remains to be considered are the cases:

- $\frac{\Delta k}{\psi} \leq C_2$ $(< C_1 < TC_2)$. Then $\Pi_{IPM_4} - \Pi_T > 0$ if

$$\frac{\Delta k}{\psi} < C_1 \frac{1 - P_L(1-P_H)}{1 - P_L(1-P_H) - P_L[1 + 2(1-P_H)(P_H - P_L)]}. \tag{37}$$

  The denominator on the RHS is positive since

$$\begin{aligned} P_L(1-P_H) - P_L[1 + 2(1-P_H)(P_H - P_L)] &= 1 - P_L(2 - P_H) + 2(1-P_H)(P_H - P_L) \\ &> 1 - P_L(2-P_H) > (1-P_H)^2 > 0, \end{aligned}$$

  and it clearly is smaller than the numerator. Hence, the expression on the RHS of (37) is larger than C_1 and the condition is satisfied.

- $C_2 \leq \frac{\Delta k}{\psi} < C_1$.

  IPM_4 is not renegotiation proof.

2. $\mathbf{IMP_6}$ dominates T whenever IPM_4 is not renegotiation proof

Two case need to be considered:

- $C_2 \leq C_5 \leq \frac{\Delta k}{\psi} < C_1$ $(< TC_2)$. Then we have $\Pi_{IPM_6} = \hat{q} > \Pi_T$.

- $C_2 \leq \frac{\Delta k}{\psi} < C_1 < C_5$. Then, $\Pi_{IPM_6} - \Pi_T > 0$ if

$$\frac{\Delta k}{\psi} < \frac{(2 - P_H - P_L)[1 - (1-P_H)(P_L + P_H)]}{2(1-P_H)[1 - P_L(2 - P_H - P_L)]} \frac{1}{P_L(1-P_H)}. \tag{38}$$

  The expression on the RHS being greater than C_1, the condition is satisfied. Indeed, we have that $\frac{1}{P_L(1-P_H)} > C_1$ and the remaining fraction is larger than one since

$$\begin{aligned} &(2 - P_H - P_L)[1 - (1-P_H)(P_L + P_H)] - 2(1-P_H)[1 - P_L(2 - P_H - P_L)] \\ &= (P_H - P_L)[2P_H - 1 + P_L(1-P_H) + P_H(1-P_H)] > 0 \quad (\text{since } P_H > \tfrac{1}{2}). \end{aligned}$$

F Proof of Proposition 4

1. $\mathbf{RPM_1}$ dominates $\mathbf{IPM_6}$.

$\mathbf{RPM_1}$		$\theta = \mathbf{H}$	
		q_{Hh}	q_{Hl}
$\theta = \mathbf{L}$	q_{Lh}	$[(t_1, m_1), (t_1, m_1)]$	$[(t_1, m_1), (0, m_2)]$
	q_{Ll}	$[(0, m_2), (t_1, m_1)]$	$[(0, m_3), (0, m_3)]$

Given that both agents exert effort under RPM_1, beliefs about the agents' types are:

$$E\left[k_\theta|\, t_1, m_1\right] = \frac{P_H\, k_H + P_L\, k_L}{P_H + P_L}, \tag{39}$$
$$E\left[k_\theta|\, 0, m_2\right] = \frac{P_L\left(1 - P_H\right)\, k_H + P_H\left(1 - P_L\right)\, k_L}{P_H + P_L - 2P_H P_L}, \tag{40}$$
$$E\left[k_\theta|\, 0, m_3\right] = \frac{k_H + k_L}{2}. \tag{41}$$

The high-skilled agent's incentive constraint requires that

$$t_1 \geq \bar{t} \equiv \frac{\psi}{P_H} - \frac{\left(P_H - P_L\right)\left(P_H + P_L + P_L^2 - P_H P_L\right)}{2\left(P_H + P_L\right)\left(P_H + P_L - 2P_H P_L\right)}\,\Delta\, k. \tag{$IC:H$}$$

Similarly, the low-skilled agent's incentive constraint requires that

$$t_1 \geq \underline{t} \equiv \frac{\psi}{P_L} - \frac{\left(P_H - P_L\right)\left(P_H + P_L + P_H^2 - P_H P_L\right)}{2\left(P_H + P_L\right)\left(P_H + P_L - 2P_H P_L\right)}\,\Delta\, k. \tag{$IC:L$}$$

Note that $\bar{t} > \underline{t}$ if and only if $\frac{\Delta k}{\psi} > \frac{2\left(P_H+P_L-2P_HP_L\right)}{P_HP_L\left(P_H-P_L\right)} \equiv C_6$.

Together, the incentive and wealth constraints imply that $t_1 \geq \max\left\{\underline{t}, \bar{t}, 0\right\}$. We have that $\underline{t} > 0$ if and only if $\frac{\Delta k}{\psi} < \frac{2\left(P_H+P_L\right)\left(P_H+P_L-2P_HP_L\right)}{P_L\left(P_H-P_L\right)\left(P_H+P_L+P_H^2-P_HP_L\right)} \equiv C_7$ and $\bar{t} > 0$ if and only if $\frac{\Delta k}{\psi} < \frac{2\left(P_H+P_L\right)\left(P_H+P_L-2P_HP_L\right)}{P_H\left(P_H-P_L\right)\left(P_H+P_L+P_L^2-P_HP_L\right)} \equiv C_8$. Conveniently, $C_6 > C_7 > C_8$, since $C_6 - C_7 = \frac{2\left(P_H+P_L-2\,P_H\,P_L\right)^2}{P_L\,P_H\,\left(P_H-P_L\right)\left(P_H+P_L+P_H^2-P_HP_L\right)} > 0$ and $C_7 - C_8 = \frac{2\left(P_H+P_L\right)\left(P_H+P_L-2P_HP_L\right)^2}{P_HP_L\,\left(P_H+P_L+P_L^2-P_HP_L\right)\left(P_H+P_L+P_H^2-P_HP_L\right)} > 0$. Hence,

$$t_1 = \begin{cases} \frac{\psi}{P_L} - \frac{\left(P_H-P_L\right)\left(P_H+P_L+P_H^2-P_HP_L\right)}{2\left(P_H+P_L\right)\left(P_H+P_L-2P_HP_L\right)}\,\Delta\, k & \text{if } \frac{\Delta k}{\psi} < C_7, \\ 0 & \text{otherwise.} \end{cases} \tag{42}$$

The contract is renegotiation proof if and only if for $\frac{\Delta k}{\psi} < C_7$ we have that $t_1 + E\left[k_\theta|\, t_1, m_1\right] > \max\left\{E\left[k_\theta|\, 0, m_2\right], E\left[k_\theta|\, 0, m_3\right]\right\}$. Note that $E\left[k_\theta|\, 0, m_3\right] - E\left[k_\theta|\, 0, m_2\right] = \frac{P_H-P_L}{2\left(P_H-P_L-2\,P_L\,P_H\right)}\,\Delta\, k > 0$. Moreover, $t_1 + E\left[k_\theta|\, t_1, m_1\right] - E\left[k_\theta|\, 0, m_3\right] = \frac{\psi}{P_L} - \frac{P_H\,\left(P_H-P_L\right)}{2\left(P_H+P_L-2P_HP_L\right)}\,\Delta\, k$. This expression is positive if $\frac{\Delta k}{\psi} < \frac{2\left(P_H+P_L-2P_HP_L\right)}{P_HP_L\left(P_H-P_L\right)} \equiv C_9$. Hence, renegotiation proofness follows from $C_9 - C_7 = \frac{2\left(P_H+P_L-2\,P_L\,P_H\right)^2}{P_H\,P_L\,\left(P_H-P_L\right)\left(P_H+P_L+P_H^2-P_L\,P_H\right)} > 0$.

Using the above results, the expected profit is given by:

$$\Pi_{RPM_3} = \begin{cases} \hat{q} - \frac{P_L+P_H}{P_L}\,\psi + \frac{\left(P_H-P_L\right)\left(P_H+P_L+P_H^2-P_HP_L\right)}{2\left(P_H+P_L-2P_HP_L\right)}\,\Delta\, k & \text{if } \frac{\Delta k}{\psi} < C_7, \\ \hat{q} & \text{otherwise.} \end{cases} \tag{43}$$

Comparing this profit with that of IPM_6 a useful result is that

$$C_5 - C_7 = \frac{\left(P_H + P_L\right)^2\left(1 - P_H\right)}{P_L\left(P_H + P_L + P_H^2 - P_H P_L\right)} \geq 0. \tag{44}$$

This leaves the following cases to be considered:

- $\frac{\Delta k}{\psi} < C_7$, then $\Pi_{RPM_1} - \Pi_{IPM_6} = \frac{(P_H - P_L)^2 (P_H + P_L)(1-P_H)}{2\,(P_H + P_L - 2P_H P_L)(2-P_H-P_L)}\,\Delta k > 0$.
- $C_7 \leq \frac{\Delta k}{\psi} < C_5$, then $\Pi_{RPM_1} - \Pi_{IPM_6} = \frac{P_L + P_H}{P_L}\,\psi - \frac{P_H - P_L}{2-P_H-P_L}\,\Delta k > 0$, since $\frac{\Delta k}{\psi} < C_5$.
- $C_5 \leq \frac{\Delta k}{\psi}$, then $\Pi_{RPM_1} - \Pi_{IPM_6} = 0$.

It now remains to show that RPM_1 strictly dominates IPM_6 whenever the latter is the best IPM contract. First notice that IPM$_6$ has zero implementation cost and therefore is an optimal contract (i.e., cannot be beaten strictly by *any* other contract) if $\frac{\Delta k}{\psi} \geq C_5$. Similarly, RPM$_1$ is an optimal contract if $\frac{\Delta k}{\psi} \geq C_7$. Hence, we can potentially beat all IPM contracts in the range $[C_7, C_5)$. Now, let us remind the reader that

1. IPM$_5$ is not renegotiation proof if $\frac{\Delta k}{\psi} \geq C_4$.
2. IPM$_4$ is not renegotiation proof if $C_2 < \frac{\Delta k}{\psi} < C_1$ (implying that $P_H > 1/2$).

Hence, IPM$_6$ is the best renegotiation-proof IPM but not (necessarily) an optimal contract if

$$\max\{C_2, C_4\} < \frac{\Delta k}{\psi} < \min\{C_1, C_5\}. \tag{45}$$

We can loosen this condition taking into account that $C_1 > C_2$ implies that $P_H > \frac{1}{2}$ and then $C_4 \geq C_2$ (see footnote 25). Therefore, RPM$_1$ is an optimal contract while IPM$_6$, which is the best IPM contract, does not attain the same profit if

$$\max\{C_4, C_7\} < \frac{\Delta k}{\psi} < \min\{C_1, C_5\}. \tag{46}$$

The interval in (46) is non-empty for a non-degenerate range of parameter values. It can be shown that there exists a non-degenerate range of parameter values for which $C_1 > C_5$ (e.g., if $P_L \in [0, P_H)$ and $P_H \in [P_L, 0.75)$). Moreover,

- $C_5 > C_7$ (see equation (44) above).
- $C_5 - C_4 = \frac{P_L\,(1-P_H) + P_L\,(1-P_L)}{(P_H - P_L)\,(1-P_L)} > 0$.

Thus, if $C_5 = \min\{C_1, C_5\}$ it follows that the interval in condition (46) is non-empty.

2. RPM$_2$ yields the same profit as IPM$_5$.

RPM$_2$		$\theta = \mathbf{H}$: q_{Hh}	$\theta = \mathbf{H}$: q_{Hl}
$\theta = \mathbf{L}$	q_{Lh}	$[(t_2, m_2), (t_1, m_1)]$	$[(t_4, m_4), (0, m_3)]$
	q_{Ll}	$[(0, m_3), (t_1, m_1)]$	$[(0, m_3), (0, m_3)]$

We start off with the transfers from the IPM$_5$ contract, which has the following structure: $[(t_a, m_a), (t_b, m_b), (t_{cd}, m_{cd})]$. It can be shown that[27]

$$t_a = \begin{cases} \frac{\psi}{P_H} - \frac{(1-P_L)}{2-P_H-P_L}\,\Delta k & \text{if} \quad \frac{\Delta k}{\psi} < \frac{2-P_H-P_L}{P_H(1-P_L)} \equiv C_3, \\ 0 & \text{otherwise,} \end{cases} \tag{47}$$

$$t_b = \frac{\psi}{P_L} + \frac{(1-P_H)}{2-P_H-P_L}\,\Delta k, \tag{48}$$

$$t_{cd} = 0. \quad \text{(using lemma 3)} \tag{49}$$

The conditions of Lemma 1 are satisfied if and only if $\frac{\Delta k}{\psi} < \frac{2-P_H-P_L}{P_L(1-P_L)} \equiv C_4$.

Using the transfers under IPM$_5$ above, set the transfers for the candidate RPM$_2$ contract as follows: $t_1 = t_a$, $t_2 = t_b + \eta$ and $t_4 = t_b - \frac{P_H}{1-P_H}\,\eta$. Moreover, $m_1 \neq m_2 \neq m_3 \neq m_4$. This leaves the reputation effects and the principal's expected profit under RPM$_2$ as under IPM$_5$. Obviously, since $C_4 > C_3$ an $\eta \in \mathbb{R}$ exists for all parameter values so that RPM$_2$ is renegotiation proof whenever IPM$_5$ is renegotiation proof.

3. RPM$_3$ (generically) yields the same profit as IPM$_4$.

RPM$_3$		$\theta = \mathbf{H}$: q_{Hh}	q_{Hl}
$\theta = \mathbf{L}$	q_{Lh}	$[(t_1, m_1), (t_2, m_2)]$	$[(t_1, m_1), (t_1, m_1)]$
	q_{Ll}	$[(0, m_4), (t_3, m_3)]$	$[(0, m_4), (t_1, m_1)]$

We start off with the transfers from the IPM$_4$ contract, which has the following structure: $[(t_a, m_a), (t_{bc}, m_{bc}), (t_d, m_d)]$. It can be shown that[28]

$$t_{bc} = \begin{cases} \frac{\psi}{P_L} - \frac{(1-P_H)}{1+P_L-P_H}\,\Delta k & \text{if} \quad \frac{\Delta k}{\psi} < C_1, \\ \text{not renegotiation proof} & \text{if} \quad P_H > \frac{1}{2} \text{ and } C_2 < \frac{\Delta k}{\psi} < C_1, \\ 0 & \text{if} \quad \frac{\Delta k}{\psi} \geq \max\{C_1, C_2\}, \end{cases} \tag{50}$$

and

$$t_a = \begin{cases} \frac{P_H+P_L}{P_L P_H}\,\psi - \Delta k & \text{if} \quad P_H \leq \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} < C_1, \\ & \text{or } P_H > \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} \leq C_2, \\ \frac{\psi}{P_H} - \frac{P_L}{1+P_L-P_H}\,\Delta k & \text{if} \quad P_H \leq \frac{1}{2} \text{ and } C_1 \leq \frac{\Delta k}{\psi} < C_2, \\ \text{not renegotiation proof} & \text{if} \quad P_H > \frac{1}{2} \text{ and } C_2 < \frac{\Delta k}{\psi} < C_1, \\ 0 & \text{if} \quad \frac{\Delta k}{\psi} \geq \max\{C_1, C_2\}. \end{cases} \tag{51}$$

[27]This is implicit in the profits given in Table 3. The detailed derivations underlying the table are available from the authors.

[28]This is implicit in the profits given in Table 3. The detailed derivations underlying the table are available from the authors.

Using the transfers under IPM_4 defined above, set the transfers for the candidate RPM_3 contract as follows: $t_1 = t_{bc}$, $t_2 = t_3 = 0$ if $t_a = 0$. Otherwise set $t_2 = t_a + \eta$ and $t_3 = t_a - \frac{P_L}{1-P_L}\,\eta$. Moreover, $m_1 \neq m_2 \neq m_3 \neq m_4$. This leaves the reputation effects and the principal's expected profit under RPM_3 as under IPM_4. Such a $\eta \in \mathbb{R}_+$ exists for all parameter values and leads to a renegotiation proof RPM contract whenever IPM_4 is renegotiation proof, except for the combination of parameter values $P_H = \frac{1}{2}$ and $\frac{\Delta k}{\psi} = C_1 \equiv \frac{1+P_L-P_H}{P_L(1-P_H)} = C_2 \equiv \frac{1+P_L-P_H}{P_H P_L}$. In this case, both transfers used in IPM_4 are equal to zero, i.e., $t_a = t_{bc} = 0$, and therefore IPM_4 is renegotiation proof. However, since transfers cannot be negative, RPM_3 is not feasible in this non-generic point of the parameter space.

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