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Performance of Inflation Targeting Based On Constant Interest Rate Projections*

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Abstract

Monetary policy is sometimes formulated in terms of a target level of inflation, a fixed time horizon and a constant interest rate that is anticipated to achieve the target at the specified horizon. These requirements lead to constant interest rate (CIR) instrument rules. Using the standard New Keynesian model, it is shown that some forms of CIR policy lead to both indeterminacy of equilibria and instability under adaptive learning. However, some other forms of CIR policy perform better. We also examine the properties of the different policy rules in the presence of inertial demand and price behaviour.

JEL classification: E52, E61, E32

Key words: Indeterminacy, instability under learning, inflation targeting, inertia in demand, inflation inertia

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1 Introduction

Inflation targeting has become a fairly common objective of monetary policy in the past ten to fifteen years; see e.g. (Svensson 2003a). This general objective can be implemented in a number of different ways. One possibility is to formulate an explicit objective function, i.e. a "general targeting rule" using the terminology suggested by Lars Svensson. A different approach has been the use of a particular target level for the inflation rate. This target is usually specified at some given horizon for the future and we may speak of inflation forecast targeting in this case. Formally, the inflation target is set for some fixed forecast horizon h and policy tries to achieve that target:

$$E_t \pi_{t+h} = \bar{\pi}. \tag{1}$$

Here $E_t \pi_{t+h}$ is the forecast of the inflation for period $t+h$ and in the analysis it will be taken to be the rational expectations (RE) forecast. In other words, the central bank acts as if private agents have RE, which are computed under knowledge of the structural model of the economy.

It can be noted that if the horizon is long, there will typically be many different paths for interest rates up to the target horizon that achieve the specified inflation target. If this is the case, a fixed inflation target at the specified horizon does not yield a unique value or time path for the interest rate, which is the actual instrument of monetary policy. A further specialization for achieving the fixed target is to use inflation forecasts that are derived as constant interest rate (CIR) projections, see e.g. the discussions in (Leitemo 2003) and (Svensson 1999). CIR inflation targeting has been advocated as an easily understandable and hence practical approach to conducting monetary policy; for general discussions of its merits and problems see (Goodhart 2000), (Kohn 2000), (Svensson 2003b) and (Woodford 2003), pp. 620-623.

In practice there appear to be at least two different ways for computing and employing the CIR projections in setting the value for the monetary policy instrument. One approach, which is arguably close to the practice in the UK, has been described by (Goodhart 2000), p.177: "When I was a member of the MPC I thought that I was trying, at each forecast round, to set the level of interest rates so that, without the need for future rate changes, prospective (forecast) inflation would on average equal the target at the policy".[1] Given a model of the macroeconomy, setting the forecast of

[1]Charles Goodhart has commented to us as a qualification that this practice is not to

inflation based on constant interest rates at a given target level of inflation implies a rule for the interest rate.

A second approach to CIR policy-making is in general terms described by the quote "... if the overall picture of inflation prospects (based on an unchanged repo rate) indicates that in twelve to twenty-four months' time inflation will deviate from the target, then the repo rate should normally be adjusted accordingly"; see (Riksbanken 1999). This way of conducting monetary policy seems (at least implicitly) to be the practice in Sweden.[2] In this approach the CIR projection is computed at the interest rate prevailing before any policy decision and the rate of interest is then adjusted depending on the difference between the CIR projection and the inflation target.

We will refer to these two ways of conducting monetary policy in general as CIR inflation targeting and corresponding interest rate rules as CIR rules. In addition, we will refer to the two approaches as CIR_{UK} and CIR_S policies, respectively.

CIR inflation targeting necessarily introduces a further element of forward-looking behavior into the economy in addition to the forward-looking behavior of private agents that is assumed in many current models for monetary policy. If the model has forward-looking elements, issues of determinacy of rational expectations equilibria (REE) and their stability under (adaptive) learning have been raised in the recent literature. (Bullard and Mitra 2002) have derived constraints on the interest rate instrument (or Taylor) rules that achieve stability and determinacy in a standard New Keynesian model of monetary policy. (Evans and Honkapohja 2003a) and (Evans and Honkapohja 2003b) have shown that some standard ways for implementing optimal policy under discretion or commitment can lead to the difficulties of indeterminacy and instability under learning. They also propose expectations-based optimal rules to overcome these problems. (Evans and Honkapohja 2004) survey this literature and provide further references.

Our principal goal in this paper is to analyze CIR policies from the point of view of determinacy and stability under learning. We will study both CIR_{UK} and CIR_S policies in these respects. We will argue that CIR_{UK} policies can very often lead to unpleasant outcomes, i.e. the resulting REE can exhibit both indeterminacy and instability under learning. CIR_S policies

be followed if it leads to drastic policy changes. Our second interpretation of CIR policy has a more gradualist approach to changing interest rates.

[2] Anders Vredin pointed to us that in practice policy appears to respond to other aspects of the economy besides CIR forecasts of inflation.

policies perform better with regard to both determinacy and stability under learning, but they are not always problem-free either. We also examine the (more realistic) policy of flexible inflation targeting and the consequences of inherent inertia in inflation and output for the indeterminacy and instability results.

2 The Framework

2.1 The Basic Model

The model we employ is the standard New Keynesian model of monopolistic competition and (Calvo 1983) price stickiness. This model has been employed in numerous recent studies of monetary policy; see e.g. (Clarida, Gali, and Gertler 1999) for a survey. The log-linearized model is described by two equations

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t, \tag{2}$$

which is the "IS" curve derived from the Euler equation for consumer optimization, and

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{3}$$

which is the price setting rule for the monopolistically competitive firms.[3] We remark that in a later section we will add inertia terms to (2) and (3). The inertia is usually justified by empirical relevance even though the micro foundations of the model are then fairly weak.[4]

Here x_t and π_t denote the output gap and inflation for period t, respectively. i_t is the nominal interest rate, expressed as the deviation from the steady state real interest rate. The determination of i_t will be discussed below. $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote the private sector expectations of the output gap and inflation next period. Since our focus is on learning behavior, these expectations need not be rational (E_t without $*$ denotes RE). The parameters φ and λ are positive and β is the discount factor so that $0 < \beta < 1$.

[3]See e.g. (Woodford 1996) or (Woodford 2003) for further details of the linearization and the original nonlinear model.

[4]See (Christiano, Eichenbaum, and Evans 2001) and (Gali and Gertler 1999) for possible justifications.

The shocks g_t and u_t are assumed to be observable and follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = V \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \tag{4}$$

where

$$V = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

$0 < |\mu| < 1$, $0 < |\rho| < 1$ and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$ are independent white noise. g_t represents shocks to government purchases and or potential output. u_t represents any cost push shocks to marginal costs other than those entering through x_t. For simplicity, we assume throughout the paper that μ and ρ are known (if not, they could be estimated).

For brevity, details of the derivation of equations (2) and (3) are not discussed. The derivation is based on individual Euler equations under (identical) subjective expectations, together with aggregation and definitions of the variables. The Euler equations for the current period give the decisions as functions of the expected state next period. Rules for forecasting the next period's values of the state variables are the other ingredient in the description of individual behavior. Given forecasts, agents are assumed to make decisions according to the Euler equations.[5]

For further analysis we write the model in matrix-vector form

$$\begin{aligned} y_t &= AE_t^* y_{t+1} + Bw_t + Di_t, \\ w_t &= Vw_{t-1} + v_t, \end{aligned} \tag{5}$$

where $y_t = (x_t, \pi_t)'$, $w_t = (g_t, u_t)'$ and $v_t = (\tilde{g}_t, \tilde{u}_t)'$. $E_t^* y_{t+1}$ denotes private expectations of y_{t+1}. The coefficient matrices are

$$A = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}, D = \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix}. \tag{6}$$

In the next two subsections we introduce the formalization of CIR_{UK} and CIR_S policies. In this and the next section, we consider the case when the

[5]This kind of behavior is boundedly rational but in our view reasonable since agents attempt to meet the margin of optimality between the current and the next period. Other models of bounded rationality are possible. Recently, (Preston 2002) has proposed a formulation in which long horizons matter in individual behavior. See also (Honkapohja, Mitra, and Evans 2002) for further discussion.

central bank tries to hit a certain inflation target at a specified horizon. Such a policy is often termed one of *strict inflation targeting*, following (Svensson 1999) and (Svensson 2003a). Even though this policy is not entirely realistic from a practical point of view since most central banks have output concerns (either implicitly or explicitly), it does serve as a useful benchmark. The case when the bank pursues a policy of *flexible inflation targeting* is considered in Section 4.1.

2.2 CIR_{UK} Policy

We first consider CIR_{UK} formulation of CIR policy. This has been recently formalized by (Leitemo 2003), which can be consulted for further details. We introduce the (strict) inflation target as in (1), where for simplicity the target $\bar{\pi}$ is assumed to be zero without loss of generality (w.l.o.g.) and h is the targeting horizon.[6]

In the derivation of CIR policies it is assumed that the central bank acts as if expectations of private agents are rational and the forecasts are computed under knowledge of the structural model of the economy. For later purposes it will be useful to express this constraint as

$$0 = K(E_t w'_{t+h}, E_t y'_{t+h})',\ K = (0, 0, 0, 1). \tag{7}$$

To derive the interest rate rule, rewrite (5) as

$$\begin{pmatrix} w_{t+1} \\ E_t y_{t+1} \end{pmatrix} = \Omega \begin{pmatrix} w_t \\ y_t \end{pmatrix} + \Psi i_t + \begin{pmatrix} v_{t+1} \\ 0 \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} \Omega &= \begin{pmatrix} V & 0 \\ -A^{-1}B & A^{-1} \end{pmatrix} \equiv \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ -1 & \varphi\beta^{-1} & 1+\lambda\varphi\beta^{-1} & -\varphi\beta^{-1} \\ 0 & -\beta^{-1} & -\lambda\beta^{-1} & \beta^{-1} \end{pmatrix}, \\ \Psi &= \begin{pmatrix} 0 \\ -A^{-1}D \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & \varphi & 0 \end{pmatrix}'. \end{aligned}$$

[6]This is without loss of generality as the precise values of model constants affect neither determinacy nor stability under learning.

Iterate (8) forward to get

$$\begin{pmatrix} E_t w_{t+h} \\ E_t y_{t+h} \end{pmatrix} = \Omega^h \begin{pmatrix} w_t \\ y_t \end{pmatrix} + \sum_{j=0}^{h-1} \Omega^j \Psi E_t i_{t+h-1-j}, \tag{9}$$

Pre-multiplying (9) by K yields

$$E_t \pi_{t+h} = K\Omega^h \begin{pmatrix} w_t \\ y_t \end{pmatrix} + K \sum_{j=0}^{h-1} \Omega^j \Psi E_t i_{t+h-1-j}. \tag{10}$$

CIR_{UK} targeting policy central bank assumes that

$$E_t i_{t+j} = i_t \text{ for } 0 \leq j \leq h-1, \tag{11}$$

where in (11) it is assumed that expected future interest rates are equal to the contemporaneous interest rate i_t for all horizons $0 \leq j \leq h-1$. In other words, in the formulation of policy, the bank assumes a constant path of interest rates at the current level. Using assumption (11) in (10) leads to

$$E_t \pi_{t+h}(i_t) = K\Omega^h \begin{pmatrix} w_t \\ y_t \end{pmatrix} + K \sum_{j=0}^{h-1} \Omega^j \Psi i_t \tag{12}$$

where $E_t \pi_{t+h}(i_t)$ denotes the *constant-interest-rate forecast of inflation* conditional on the forward-looking variables x_t, π_t and the contemporaneous interest rate i_t. Finally, setting $E_t \pi_{t+h}(i_t)$ in (12) equal to the (target) zero yields the interest rate rule

$$i_t = G \begin{pmatrix} w_t \\ y_t \end{pmatrix}, \; G = -\left(K \sum_{j=0}^{h-1} \Omega^j \Psi \right)^{-1} K\Omega^h. \tag{13}$$

We will refer to (13) as the CIR_{UK} rule I.

The CIR_{UK} rule I, equation (13), has the general form

$$i_t = \chi_g g_t + \chi_u u_t + \chi_x x_t + \chi_\pi \pi_t. \tag{14}$$

(14) is thus an instrument rule like the classic rule studied by (Taylor 1993) and it can be explicitly computed for different values of h. For $h = 2$ we get

$$\chi_g = \varphi^{-1}, \chi_u = -\frac{1 + \beta\rho + \lambda\varphi}{\beta\lambda\varphi}, \chi_x = -\frac{1 + \beta + \lambda\varphi}{\beta\varphi}, \chi_\pi = \frac{1 + \lambda\varphi}{\beta\lambda\varphi}.$$

It is seen that, for $h = 2$, the rule (13) surprisingly has $\chi_x < 0$, i.e. the interest rate should react negatively to the output gap. For higher values of h the expressions $\chi_i, i = g, u, x, \pi$ become cumbersome, but numerical computations indicate that the negative coefficient on the output gap is a robust phenomenon of the CIR_{UK} rule I.

This unexpected result can be given an economic interpretation in the case $h = 2$. Shift the New Phillips curve (3) forward and take RE. Recalling that the inflation target is assumed to be zero, we have

$$E_t\pi_{t+1} = \lambda E_t x_{t+1} + \rho u_t,$$

which pins down the expectations terms in (2) and yields the positive relation between $E_t\pi_{t+1}$ and $E_t x_{t+1}$. By (3) we also have

$$E_t\pi_{t+1} = \beta^{-1}(\pi_t - \lambda x_t - u_t),$$

which indicates that both $E_t\pi_{t+1}$ and $E_t x_{t+1}$ depend negatively on the current output gap under this policy. Finally, rewriting the IS curve (2) as

$$\begin{aligned} \varphi i_t &= -x_t + \varphi E_t\pi_{t+1} + E_t x_{t+1} + g_t \\ &= -(1 + \beta^{-1} + \beta^{-1}\lambda\varphi)x_t + (\beta^{-1}\varphi + \beta^{-1}\lambda^{-1})\pi_t \\ &\quad + g_t - (\beta^{-1}\varphi + \lambda^{-1}(\beta^{-1} + \rho))u_t. \end{aligned}$$

it is seen that i_t and x_t are negatively related, both directly as part of the IS relationship and indirectly through the negative dependence of $E_t\pi_{t+1}$ and $E_t x_{t+1}$ on the current x_t.

(13) should be viewed as an instrument rule as it depends on current endogenous variables[7] and another rule depending only on predetermined variables is often suggested instead. It can be derived as follows. Substituting (13) into (8) we have

$$\begin{pmatrix} w_{t+1} \\ E_t y_{t+1} \end{pmatrix} = (\Omega + \Psi G) \begin{pmatrix} w_t \\ y_t \end{pmatrix} + \begin{pmatrix} v_{t+1} \\ 0 \end{pmatrix},$$

for which it is possible to derive the MSV solution of the form

$$y_t = H w_t \tag{15}$$

[7](13) can be viewed as a behavioral rule in the same sense as demand and supply functions of private agents are behavioral rules, i.e. the central bank "goes to the market" with that schedule and is able to adjust the interest rate within the period. Such rules are sometimes said to be non-operational; see (McCallum 1999) for further discussion.

using standard techniques (we omit the precise form of H). (The MSV solutions are REE that are usually employed in the applied literature.) Introducing the partition $G = \begin{pmatrix} G_w & G_y \end{pmatrix}$, we can rewrite the interest rule (13) as

$$i_t = (G_w + G_y H)w_t, \tag{16}$$

which we will call the CIR_{UK} rule II.

2.3 CIR_S Policy

As mentioned in the Introduction, an alternative interest rule, which we call a CIR_S rule, is based on computing CIR forecasts of inflation at the interest rate before any policy decision and then changing the interest rate if the CIR forecast deviates from the target. Formally, the policy-maker first makes a forecast of inflation, conditioned on a constant interest rate at the level of i_{t-1} from period $t-1$. This forecast is compared with the target rate. If the forecast is above the target, the interest rate is raised.

One simple rule that reflects this way of thinking is[8]

$$i_t - i_{t-1} = \omega(E_t \pi_{t+h}(i_{t-1}) - \bar{\pi}), \tag{17}$$

where $E_t \pi_{t+h}(i_{t-1})$ denotes the inflation forecast conditioned on the last period interest rate, i.e., the bank assumes

$$E_t i_{t+j} = i_{t-1}, \text{ for all } 0 \leq j \leq h-1$$

instead of (11). We again assume $\bar{\pi} = 0$, w.l.o.g. $\omega > 0$ determines the magnitude of the extent of increase in i_t when the inflation forecast is above target. From (10) we now have

$$E_t \pi_{t+h}(i_{t-1}) = K\Omega^h \begin{pmatrix} w_t \\ y_t \end{pmatrix} + K \sum_{j=0}^{h-1} \Omega^j \Psi i_{t-1} \tag{18}$$

Written explicitly, (18) takes the form

$$E_t \pi_{t+h}(i_{t-1}) = \psi_g g_t + \psi_u u_t + \psi_x x_t + \psi_\pi \pi_t + \psi_i i_{t-1} \tag{19}$$

[8] This is a simplified version of a rule proposed by Anders Vredin in the discussion at the conference.

where the coefficients $\psi_g, \psi_u, \psi_x, \psi_\pi, \psi_i$ can be computed for different values of h. For example, when $h = 2$, these take the values

$$\begin{aligned}\psi_g &= \lambda\beta^{-1}, \psi_u = -\beta^{-2}(1+\beta\rho+\lambda\varphi), \psi_x = -\lambda\beta^{-2}(1+\beta+\lambda\varphi), \\ \psi_\pi &= \beta^{-2}(1+\lambda\varphi), \psi_i = -\lambda\varphi\beta^{-1}.\end{aligned}$$

Using (19) in (17), we obtain CIR_S rule I of the form

$$i_t = \omega(\psi_g g_t + \psi_u u_t + \psi_x x_t + \psi_\pi \pi_t) + (1+\omega\psi_i)i_{t-1} \tag{20}$$

Note that the CIR_S rule I, (20), captures a form of interest smoothing frequently observed in the data.

As before, we can also define an interest rule which depends solely on pre-determined variables using the MSV solution of the model when CIR_S rule I is employed. We now consider the formulation of the CIR_S rule II associated with (17). Since the CIR_S rule I introduces the lagged interest rate as a predetermined endogenous variable, the MSV solution of the model (5) with CIR_S rule I, (20), takes the form

$$x_t = b_x i_{t-1} + b_x^g g_t + b_x^u u_t, \tag{21}$$

$$\pi_t = b_\pi i_{t-1} + b_\pi^g g_t + b_\pi^u u_t, \tag{22}$$

$$i_t = b_i i_{t-1} + b_i^g g_t + b_i^u u_t, \tag{23}$$

where the coefficients $b_x, ...$ need to be determined. Furthermore, for a determinate MSV solution, we have $|b_i| < 1$, and using this solution in CIR$_S$ rule I, (20), we may obtain uniquely CIR_S rule II below[9]

$$i_t = b_i' i_{t-1} + \tilde{\psi}_g g_t + \tilde{\psi}_u u_t. \tag{24}$$

Here $b_i' = \omega\psi_x b_x + \omega\psi_\pi b_\pi + (1+\omega\psi_i)$ and $\tilde{\psi}_g, \tilde{\psi}_u$ describe the dependence on the shocks (their precise form is not needed in the computations).

2.4 Calibration Scenarios

In several places we will need to revert to numerical results in the study the properties of CIR policies introduced above. For our numerical analysis, we will frequently adopt three calibration scenarios proposed in the literature.[10]

[9] In cases when the model has indeterminacy and hence potentially multiple stationary MSV solutions with CIR_S rule I, there is no unique way to define CIR_S rule II.

[10] Both the (Clarida, Gali, and Gertler 2000) and (Woodford 1999) calibrations are for quarterly data. However, (Woodford 1999) uses quarterly interest rates and measures

Calibration W: $\beta = 0.99$, $\varphi = (0.157)^{-1}$, and $\lambda = 0.024$.

Calibration CGG: $\beta = 0.99$, $\varphi = 4$, and $\lambda = 0.075$.

Calibration MN: $\beta = 0.99$, $\varphi = 0.164$, and $\lambda = 0.3$.

These are taken, respectively, from (Woodford 1999), (Clarida, Gali, and Gertler 2000), and (McCallum and Nelson 1999). We remark that these calibrations are based on U.S. data and thus the numerical results are not necessarily relevant for the British and Swedish cases. For an analysis of E-stability, we sometimes need the values of ρ and μ and we set these at $\rho = 0.9$ and $\mu = 0.35$ in accordance with the literature.

3 Determinacy and Learning Stability

3.1 Results for CIR_{UK} Policies, Types I and II

We consider whether CIR_{UK} interest rate rules, either in the form (13) or (16), yield determinacy and stability under learning of the MSV REE. We will assess stability under learning using the concept of E-stability, which is known to be the relevant condition for convergence of adaptive learning formulated using least squares and closely related learning rules. Formal analysis of determinacy is standard; see e.g. (Blanchard and Kahn 1980) or Chapter 10 of (Evans and Honkapohja 2001). For an analysis of E-stability in models like this, we refer the reader to (Bullard and Mitra 2002) for an overview and to (Evans and Honkapohja 2001) for a detailed discussion. The analysis is conducted using the forward-looking model (2) and (3), together with either (13) or (16).

It should be emphasized that the analysis of determinacy and learning is based on the assumption that the private sector expectations are based on expectation functions with the actually implemented interest rate rule and not on expectations used as part of the computation of the hypothetical CIR policy. This highlights an aspect of the time-inconsistency of CIR policy: computation of the policy presumes a constant interest rate through the

inflation as quarterly changes in the log price level, while (Clarida, Gali, and Gertler 2000) use annualized rates for both variables. We adopt the Woodford measurement convention, and therefore our CGG calibration divides by 4 the λ value and multiplies by 4 the φ value reported by (Clarida, Gali, and Gertler 2000).

horizon, whereas the actual interest rate is known to be adjusted in response to variations in inflation and output gap as well as in the shocks.[11]

It is convenient to start with CIR_{UK} rule II (16). Plugging this rule into the model (5) leads to the reduced form

$$y_t = AE_t^* y_{t+1} + \{B + D(G_w + G_y H)\} w_t. \tag{25}$$

In the basic model (16) has unpleasant properties on both counts:[12]

Proposition 1 *CIR$_{UK}$ rule II, i.e. equation (16), leads to both indeterminacy and instability under adaptive learning.*

The indeterminacy result means that there other stationary REE to the model under the CIR_{UK} rule II besides the MSV solution used above. These equilibria include various sunspot solutions and it is possible to examine whether the non-MSV solutions are stable under learning. Using results of (Honkapohja and Mitra 2004), it can be shown that the non-MSV REE are also E-unstable. Thus there are no E-stable REE under CIR_{UK} rule II.

The difficulties spelled out by Proposition 1 naturally raise the question whether the instrument rule form of CIR monetary policy, i.e. CIR_{UK} rule I given by equation (13) has better determinacy or learnability properties. Unfortunately, this is not the case:

Proposition 2 *In model (2)-(3) CIR_{UK} rule I leads to indeterminacy (when $h \geq 2$) and to E-instability (when $h \geq 3$).*

In Appendix A we prove the result for values $h \leq 4$. For higher values of h we have computed the relevant conditions numerically using the three baseline calibrations. The results clearly indicate that the CIR_{UK} rule I delivers neither determinacy nor stability under learning.

3.2 Results for CIR_S Policies, Types I and II

We now turn to an analysis of the performance of CIR$_S$ type policy. To analyze determinacy, we plug the rule (20) into the basic model (5) and

[11] See (Leitemo 2003) for a further discussion of time-inconsistency issues in CIR policies.

[12] The result follows directly from Proposition 2 in (Evans and Honkapohja 2003a) stating that, in the New Keynesian model, any interest rate rule that depends only on the exogenous shocks lead to both indeterminacy and instability under learning.

obtain the following system after defining $z_t = (x_t, \pi_t, i_{t-1})'$

$$B_1 E_t z_{t+1} = B_2 z_t + shocks, \tag{26}$$

where

$$B_1 = \begin{bmatrix} 1 & \varphi & 0 \\ \lambda & \beta + \lambda\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 + \varphi\omega\psi_x & \varphi\omega\psi_\pi & \varphi(1 + \omega\psi_i) \\ \lambda\varphi\omega\psi_x & 1 + \lambda\varphi\omega\psi_\pi & \lambda\varphi(1 + \omega\psi_i) \\ \omega\psi_x & \omega\psi_\pi & 1 + \omega\psi_i \end{bmatrix}$$

The matrix for computing determinacy is then given by $B_3 \equiv B_1^{-1} B_2$, which explicitly is

$$B_3 = \begin{bmatrix} 1 + \beta^{-1}\lambda\varphi + \varphi\omega\psi_x & \beta^{-1}\varphi(\beta\omega\psi_\pi - 1) & \varphi(1 + \omega\psi_i) \\ -\beta^{-1}\lambda & \beta^{-1} & 0 \\ \omega\psi_x & \omega\psi_\pi & 1 + \omega\psi_i \end{bmatrix}$$

Since x_t, π_t are free while i_{t-1} is pre-determined, REE is determinate if and only if exactly one eigenvalue of B_3 is inside the unit circle.

When $h = 2$, we can obtain a partial result on determinacy. The following proposition is proved in Appendix B:

Proposition 3 *Let $h = 2$. CIR_S rule I yields determinacy of REE for all sufficiently small $\omega > 0$.*

In general, determinacy depends on the structural parameters. It is easily checked that in the case $h = 2$ determinacy obtains for all $0 < \omega < 1$ under the three baseline calibrations. For higher horizons we will examine determinacy numerically below.

To analyze E-stability, we need to put the system in the following form

$$\begin{aligned} \varsigma_t &= \digamma \hat{E}_t \varsigma_{t+1} + \delta \varsigma_{t-1} + \varkappa w_t, \qquad (27) \\ \digamma &= \varrho \begin{bmatrix} 1 & \varphi(1 - \omega\beta\psi_\pi) & 0 \\ \lambda & \beta + \lambda\varphi + \beta\varphi\omega\psi_\pi & 0 \\ \omega(\lambda\psi_\pi + \psi_x) & \omega\{(\beta + \lambda\varphi)\psi_\pi + \varphi\psi_x\} & 0 \end{bmatrix}, \\ \delta &= \varrho \begin{bmatrix} 0 & 0 & -\varphi(1 + \omega\psi_i) \\ 0 & 0 & -\lambda\varphi(1 + \omega\psi_i) \\ 0 & 0 & 1 + \omega\psi_i \end{bmatrix}, \varrho^{-1} = 1 + \varphi\omega(\lambda\psi_\pi + \psi_x). \end{aligned}$$

where $\varsigma_t = (x_t, \pi_t, i_t)'$ and ψ_x, ψ_π, ψ_i are the coefficients in (19).

The MSV solution of the model (27) takes the form

$$\varsigma_t = \bar{a} + \bar{b}\varsigma_{t-1} + \bar{c}w_t \tag{28}$$

with $\bar{a} = 0$ and $\bar{b}$ to be determined from $\bar{b} = (I - \digamma\bar{b})^{-1}\delta$, provided the relevant inverse exists. In the determinate case, there is only one solution of $\bar{b}$ with eigenvalues inside the unit circle; in the indeterminate case there may exist more than one.

For the analysis of learning, agents have a PLM of the form

$$\varsigma_t = a + b\varsigma_{t-1} + cw_t,$$

from which one can compute expectations[13] as $\hat{E}_t\varsigma_{t+1} = (I + b)a + b^2\varsigma_{t-1} + (bc + cV)w_t$ and inserting $\hat{E}_t\varsigma_{t+1}$ into the model gives the ALM

$$\varsigma_t = (\digamma + \digamma b)a + (\digamma b^2 + b)\varsigma_{t-1} + (\digamma bc + \digamma cV + \varkappa)w_t$$

When the time t information set is $(1, \varsigma'_{t-1}, w_t)'$, the E-stability conditions for an MSV solution require us to have the eigenvalues of the matrices $\bar{b}' \otimes \digamma + I \otimes \digamma\bar{b} - I$, $V \otimes \digamma + I \otimes \digamma\bar{b} - I$, $\digamma + \digamma\bar{b} - I$, to have negative real parts; see Chapter 10 of (Evans and Honkapohja 2001) for details. Otherwise, the solution is not E-stable.

We now look numerically at E-stability of the determinate MSV solution for different horizons h, ranging from 2 to 8. Table 1 below reports a pair of critical values of ω, $(\breve{\omega}, \hat{\omega})$ such that for all $0 < \omega \leq \breve{\omega}$, one has determinacy with the determinate MSV solution being E-stable. For values of ω such that $\breve{\omega} < \omega \leq \hat{\omega}$, one has determinacy but the determinate MSV solution is E-unstable. Values of $\omega > \hat{\omega}$ lead to indeterminacy.[14] For $h = 2$, we find that $\breve{\omega} = \hat{\omega}$ so that only one number is reported for the $h = 2$ column.

Table 1. Regions of Determinacy and E-stability for different horizons

h	2	3	4	5	6	7	8
W	3.39	(.84, .84)	(.3, .34)	(.14, .17)	(.07, .1)	(.04, .06)	(.03, .03)
CGG	1.87	(.44, .46)	(.14, .18)	(.06, .09)	(.03, .04)	(.01, .02)	(.01, .01)
MN	10.39	(2.4, 2.4)	(.94, .94)	(.45, .47)	(.25, .28)	(.15, .18)	(.10, .12)

[13] We assume that the time t information set does not include ς_t.

[14] We have not conducted an analysis of E-stability in the indeterminate region. We did a grid search of 0.01 for ω up to an upper bound for ω of 100.

The determinate solution usually turns out to be E-stable (there are only some exceptions). However, the range for which determinacy and E-stability hold shrinks quite rapidly as the horizon increases. When $h = 8$, only very small values of ω yield determinacy and E-stability.

Note that, in the interest rule (20), the responses to x_t, π_t, and i_{t-1} are, respectively, $\omega\psi_x$, $\omega\psi_\pi$, and $1+\omega\psi_i$ and these resemble the form of a Taylor type rule with interest rate smoothing. With this rule, the Taylor principle (TP) corresponds to the requirement

$$TP \equiv 1 + \omega\psi_i + \omega\psi_\pi + \lambda^{-1}(1-\beta)\omega\psi_x > 1; \tag{29}$$

see Chapter 4 of (Woodford 2003) for a discussion of the Taylor principle. It can be verified analytically that for all horizons $h = 2, 3, .., 8$, $TP = 1 + \omega$ so that the rule does indeed satisfy the Taylor principle for all $\omega > 0$. In fact, Proposition 4.4, p. 255, of (Woodford 2003), shows that the necessary and sufficient condition for determinacy, for a rule of the form (20), in the model (5) is that the Taylor principle be satisfied, i.e. that $TP > 1$ *provided* $\psi_x > 0, \psi_\pi > 0$, and $1 + \omega\psi_i > 0$. The latter means that the individual responses to x_t, π_t, and i_{t-1} are all positive (as is perhaps true in realistic versions of the Taylor type rule). However, for the rule (20), $\psi_x < 0$ for all calibrations and often $1+\omega\psi_i < 0$, (even though $\psi_\pi > 0$ for all calibrations).

We conjecture that the key to the failure of determinacy and E-stability is the fact that the rule (20) fails to be super-inertial since $1+\omega\psi_i < 1$. With strict inflation targeting, it can be verified analytically that $\psi_i < 0$ for all horizons h so that the rule can merely be inertial.[15] An explanation for this conjecture will be provided later.

For CIR_S rule II we have both indeterminacy and E-instability (the proof is in Appendix B):

Proposition 4 *CIR_S-rule II associated with (19), i.e. equation (24), leads to both indeterminacy and instability under adaptive learning of the MSV solution for all horizons h.*

[15] In fact, as ω increases, the response $1 + \omega\psi_i$ becomes an increasingly large negative number.

4 Extensions

4.1 Flexible Inflation Targeting

4.1.1 CIR_{UK} Rules

We examine some extensions to our basic model. The analysis of Section 2 has assumed that the central bank pursues a policy of strict inflation targeting, which serves as a useful benchmark. We now turn to the arguably more realistic case where the bank also has concerns for output in its loss function.

With flexible inflation targeting and assuming that the target levels for output gap and inflation are $\bar{x}$ and $\bar{\pi}$, the central bank's optimality condition (under discretion) can be shown to be $\alpha x_t + \lambda \pi_t - (\lambda\bar{\pi} + \alpha\bar{x}) = 0$, see (Evans and Honkapohja 2003a) for the details.[16] Thus, the bank seeks to achieve

$$\alpha\lambda^{-1}E_t x_{t+h} + E_t\pi_{t+h} - (\bar{\pi} + \alpha\lambda^{-1}\bar{x}) = 0. \tag{30}$$

We again rewrite this constraint as

$$\bar{\pi} + \alpha\lambda^{-1}\bar{x} = K(E_t w_{t+h}, E_t y_{t+h})',\ K = (0, 0, \alpha\lambda^{-1}, 1). \tag{31}$$

Except for the change in K, the rest of the analysis formally proceeds as before. The form of CIR_{UK} rule I continues to be given by (13) with K as defined in (31). As before, we assume $\bar{x} = \bar{\pi} = 0$ (w.l.o.g.).

Obviously, some assumptions about α must be made. In the numerical analysis we assumed that α ranges from 0.1 (low concern for output) to 0.9 (high concern for output) at intervals of 0.1. We continue to have:[17]

Result: Under CIR_{UK} rule I the REE is indeterminate and the MSV solution is E-unstable.

This result was obtained for all the examined α and across all three calibrations. As for CIR_{UK} rule II, Proposition 1 continues to be applicable for the case of flexible inflation targeting since the rule still depends only on the exogenous shocks.

[16] (Evans and Honkapohja 2003a) consider optimal discretionary policy by minimizing a quadratic objective function, $\alpha(x_t - \bar{x})^2 + (\pi_t - \bar{\pi})^2$, subject to (3), which approach leads to the discretionary optimality condition.

[17] We report the main numerical findings as "Results" and not as Propositions.

4.1.2 CIR_S Rules

Since the bank's targeting rule is now given by (31), a rule analogous to (17) in this case can be written as

$$i_t - i_{t-1} = \omega[\alpha\lambda^{-1}E_t x_{t+h}(i_{t-1}) + E_t\pi_{t+h}(i_{t-1}) - (\bar{\pi} + \alpha\lambda^{-1}\bar{x})] \tag{32}$$

If the bank expects the expression within parentheses in (32) to be positive, then interest rates should be raised to reduce inflationary pressures in the economy. According to (32) the nominal interest rate is raised if the optimal combination of forecasted output gap and inflation exceeds the corresponding combination evaluated at the target values.[18]

For the formal analysis we remark that $\alpha\lambda^{-1}E_t x_{t+h}(i_{t-1}) + E_t\pi_{t+h}(i_{t-1})$ will be of the same form as the right hand side of (18) except for the change in K, namely $K = (0, 0, \alpha\lambda^{-1}, 1)$. Again, we simplify by assuming $\bar{x} = \bar{\pi} = 0$ and continue to obtain from (32), an interest rule of the form (20) but with different coefficients. For example, with $h = 2$, the coefficients are

$$\begin{aligned}
\psi_x &= \lambda^{-1}\beta^{-2}[\alpha\{\beta^2 + (1+2\beta)\lambda\varphi + \lambda^2\varphi^2\} - \lambda^2(1+\beta+\lambda\varphi)], \\
\psi_\pi &= \lambda^{-1}\beta^{-2}[\lambda(1+\lambda\varphi) - \alpha\varphi(1+\beta+\lambda\varphi)], \\
\psi_i &= \varphi\lambda^{-1}\beta^{-1}[\alpha(2\beta+\lambda\varphi) - \lambda^2].
\end{aligned}$$

Note that for large enough α, $\psi_i > 0$ (and $\psi_x > 0, \psi_\pi < 0$). One can verify analytically for higher horizons that the same features are true and, consequently, the interest rules are super-inertial for α large enough.

It can also be verified analytically that for all horizons $h = 2, 3, .., 8$ the interest rule (20), continues to satisfy the Taylor principle since

$$TP = 1 + \omega + \lambda^{-2}\alpha\omega(1-\beta) > 1$$

for all α, $\omega > 0$.

Since only K changes, the rest of the formal analysis proceeds as in Section 2.3. We examine determinacy and E-stability for CIR_S rule I for values of $0 < \omega \leq 15$ and for values of α between 0.1 and 0.9, both at intervals of 0.1. Remarkably, numerical results suggest the following general conclusion:

[18] Responding to deviations from the optimality condition is similar in spirit to the approximate targeting rule proposed by (McCallum and Nelson 2000). However, in the McCallum-Nelson rule a deviation from optimality leads to an increase in the real interest rate.

Result: Most values of α lead to determinacy and E-stability of the determinate solution for horizons $h = 2, 3, .., 8$.

The conclusion shows that if the bank has sufficient concerns for output in its loss function and adopts a rule of the form (32), CIR_S rule I policy performs well in terms of determinacy and E-stability.

The detailed findings are as follows. For the W and CGG calibrations, we find that for all h, all values of α, ω examined lead to determinate REE which are also E-stable. The corresponding rule (20) has its coefficients satisfying $\psi_x > 0$, $\psi_\pi < 0$, and $1 + \omega\psi_i > 1$. So even though the response to π_t is of the "wrong" sign, the rule is nevertheless super-inertial.

For the MN calibration, the general theme is unchanged. We find that values of $\alpha \geq 0.3$ lead to determinacy and E-stability for all h. In other words, a sufficient concern for output eliminates problems of determinacy and E-stability. In addition, the associated rules tend to be super-inertial since $1 + \omega\psi_i > 1$, i.e., $\psi_i > 0$ (they also satisfy $\psi_x > 0$ and $\psi_\pi < 0$). Determinacy sometimes fails for small values of α like $\alpha = .1, .2$ when the horizon h is large (say, $h \geq 4$). These failures of determinacy typically coincide with interest rules which satisfy $1 + \omega\psi_i < 1$ (along with $\psi_x < 0$, $\psi_\pi > 0$), i.e. when policy rules that are not super-inertial.

We note that the MSV solution has $b_x < 0$, $b_\pi < 0$, and $0 < b_i < 1$ for all calibrations. These results together with those of the previous section suggest that an important reason for determinacy and E-stability is the super-inertial nature of the associated interest rule, which seems to be true even when $\psi_\pi < 0$, or $\psi_x < 0$.[19] (Bullard and Mitra 2001) examined super-inertial interest rules (with $\psi_\pi > 0$, $\psi_x > 0$ and dependence on lagged data) and found these to be conducive to E-stability of the MSV solution for the basic model (5). They also found that superinertial rules that depend on contemporaneous data on inflation and output and the lagged interest rate, as in rule (20), were conducive to determinacy and E-stability.

Finally, we remark that Proposition 4 continues to be applicable for CIR_S rule II since it was applicable for *any* determinate MSV solution under all parameter values.

[19] For some (intermediate) values of α, the individual responses in the interest rule to output, inflation and lagged interest rates are all positive (as with the MN calibration) and since the rule also always satisfies the Taylor principle, the determinacy result in Proposition 4.4 of (Woodford 2003) is applicable.

4.2 Inflation and Output Inertia

The model given by (2) and (3) is entirely forward-looking and as a result has difficulty capturing the inertia in output and inflation evident in the data; see (Fuhrer and Moore 1995b), (Fuhrer and Moore 1995a) and (Rudebusch and Svensson 1999) for empirical results. We now look at an extension of this model considered in (Clarida, Gali, and Gertler 1999), Section 6, with important backward-looking elements. This model consists of the structural equations

$$\begin{aligned} x_t &= -\varphi\left(i_t - E_t^*\pi_{t+1}\right) + \theta E_t^* x_{t+1} + (1-\theta)x_{t-1} + g_t & (33)\\ \pi_t &= \lambda x_t + \beta\gamma E_t^*\pi_{t+1} + (1-\gamma)\pi_{t-1} + u_t & (34)\end{aligned}$$

The parameters θ and γ capture the inertia in output and inflation inherent in the model and are assumed to be between 0 and 1. The shocks g_t and u_t continue to follow the process (4).

We outline the formal analytical procedures in Appendix C.

4.2.1 CIR_{UK} Rules

The CIR_{UK} rule I, equation (42), under strict inflation targeting has the general form

$$i_t = \vartheta_g g_t + \vartheta_u u_t + \vartheta_{xL} x_{t-1} + \vartheta_{\pi L}\pi_{t-1} + \vartheta_x x_t + \vartheta_\pi \pi_t \qquad (35)$$

and it is possible to compute this rule explicitly for different values of h. For instance, with $h = 2$, the rule is

$$\begin{aligned} \vartheta_g &= \varphi^{-1}, \vartheta_u = -\frac{\theta + \beta\gamma\theta\mu + \lambda\varphi}{\beta\gamma\lambda\varphi}, \vartheta_{xL} = \frac{1-\theta}{\varphi}, \vartheta_{\pi L} = -\frac{(1-\gamma)(\theta+\lambda\varphi)}{\beta\gamma\lambda\varphi},\\ \vartheta_x &= -\frac{\theta+\beta\gamma+\lambda\varphi}{\beta\gamma\varphi}, \vartheta_\pi = \frac{\theta\{1-\beta\gamma(1-\gamma)\}+\lambda\varphi}{\beta\gamma\lambda\varphi}. & (36)\end{aligned}$$

Note that the response of the interest rule to the contemporaneous output gap is negative (as in the non-inertial model) and, in addition, the response to lagged inflation is now negative. Similar qualitative responses follow for other horizons.

We examine determinacy and E-stability in the model with inertia (33), (34) for the CIR_{UK} rule I (35) when the central bank pursues strict inflation targeting. When $h = 2$, we are able to obtain analytical results and we analyze this case first.[20] When $h = 2$, there is indeterminacy for all values of output and inflation inertia. The MSV solution turns out to be unique but it is not E-stable:

Proposition 5 *CIR_{UK} rule I leads to indeterminacy in the model (39) when $h = 2$. There exists a unique MSV solution, which is E-unstable.*

The result is proved in Appendix C. For $h > 2$, we need to resort to numerical analysis and we let γ and θ take values from 0.1 to 0.9 at intervals of 0.1. Table 2 below reports the results when $h = 4$ for the W calibration. In this table, the third column shows determinacy (D) or indeterminacy (I). The fourth column shows the number of stationary MSV solutions. Obviously, in the determinate case, there is only one stationary solution whereas there may be more than one in the indeterminate region. The final column examines E-stability of the stationary MSV solutions whether in the determinate or indeterminate region.

Table 2. CIR$_{UK}$ Rule I: E-stability of MSV solution when h=4

[20] For E-stability, we continue to assume that agents's expectations are based on information of endogenous variables at time $t-1$, which we believe is more realistic since contemporaneous data on output and inflation are not usually available for making forecasts.

γ	θ	Det/Indet	# of stat solns	E-stability
.1	{.1,..,.5}	D	1	No
.1	{.6,..,.9}	I	2	No in both cases
.2	.1	D	1	Yes
.2	{.2,..,.5}	D	1	No
.2	{.6,..,.9}	I	2	No in both cases
.3	.1	D	1	Yes
.3	.2,.3,.4	D	1	No
.3	{.5,..,.9}	I	2	No in both cases
.4	.1	D	1	Yes
.4	.2,.3	D	1	No
.4	{.4,..,.9}	I	2	No in both cases
.5	.1	D	1	Yes
.5	.2	D	1	No
.5	{.3,..,.9}	I	2	No in both cases
{.6,.7}	{.1,..,.9}	I	2	No in both cases
.8	{.1,..,.6},.9	I	2	No in both cases
.8	.7,.8	I	3	No in all cases
.9	{.1,..,.5},.8	I	2	No in both cases
.9	.6,.7,.9	I	3	No in all cases

The table shows that most values of inflation and output inertia lead to indeterminacy when $h = 4$. Even if determinacy obtains, the (locally) unique solution is usually E-unstable. In the indeterminate region, all MSV solutions always turn out to be E-unstable.

Similar results follow for higher horizons. These results indicate that policy using CIR_{UK} rule I continues to have undesirable properties in the presence of inertia.

For brevity, we do not report the performance of CIR_{UK} rule II here. We have checked numerically that the qualitative features of this rule are basically unchanged from those of CIR_{UK} rule I. Most parameter values continue to lead to indeterminacy and all MSV solutions are E-unstable.

4.2.2 CIR_S Rules

We consider the performance of CIR_S rule I in the presence of flexible inflation targeting. For simplicity, we assume $\mu = \rho = \bar{x} = \bar{\pi} = 0$. In the presence of inflation inertia, the targeting rule (31) takes the form

$$0 = K(E_t y_{1,t+h}, E_t y_{2,t+h})',\ K = (0,0,0,0,\alpha(1-\beta\tilde{a}_\pi)\lambda^{-1},1), \tag{37}$$

compare (6.4) in (Clarida, Gali, and Gertler 1999). $0 \leq \tilde{a}_\pi < 1$ is the solution of the lagged inflation term in equation (6.5) of (Clarida, Gali, and Gertler 1999). When $\gamma = 1$, $\tilde{a}_\pi = 0$. We first summarize the general nature of the results in this case.

Results: The CIR_S rule I continues to perform well in the presence of low levels of output and inflation inertia. The presence of high levels of inertia hampers the performance of this rule.

Consider first determinacy under the rule. Table 3 depicts the region of determinacy when $h = 8$ for the W and CGG calibrations. For each value of γ in the first row, the table reports the critical value $\bar{\theta}$ such that one has determinacy for all $\theta \geq \bar{\theta}$ for all values of α and ω examined.[21] There typically exists no stationary REE for values of $\theta < \bar{\theta}$.

Table 3. Region of Determinacy when h = 8

γ	.1	.2	.3	.4	.5	$\geq .6$
W	.6	.6	.5	.4	.3	.1
CGG	.6	.5	.4	.3	.2	.1

It seems, therefore, that determinacy can fail in the presence of high levels of inertia in the model. In addition, even when determinacy holds, the (determinate) solution can fail to be E-stable.[22] Thus, sufficient inertia worsens significantly the performance of CIR_S rule I.

5 Concluding Remarks

The results in this paper suggest that the conduct of inflation targeting by using CIR policy is subject to two fundamental difficulties. First, there may be multiple stationary RE solutions under such a policy. Second, the suggested

[21] We examined determinacy for values of γ, θ, α between .1 and .9 and for values of ω between .1 and 2, all at intervals of length .1.

[22] For simplicity, we considered E-stability in the presence of inflation inertia only, i.e., the case $\theta = 1$.

interest rates rules in this approach can lead to instability of equilibria under learning. We remark that optimal inflation targeting policies, discussed e.g. in (Svensson 2003a), are an alternative to CIR policies and there are ways to implement them to achieve determinacy and learnability; see (Evans and Honkapohja 2004).

We have examined two versions of CIR inflation targeting, which we called CIR_{UK} and CIR_S. It was found that CIR_{UK} policies are particularly vulnerable to the twin problems of indeterminacy and E-instability in all versions of the models examined. CIR_S rule I, on the other hand, has more appealing features in terms of determinacy and E-stability in the forward-looking model, especially when flexible inflation targeting is employed. However, its performance can be problematic in the presence of high inflation or output inertia. One reason for the poor performance may be the relative simplicity of the rule itself. In inertial models one may have to look at other rules to deliver a robust performance. We leave a detailed investigation of these issues to the future.

It is perhaps remarkable that the performance of CIR_S instrument rule in terms of learnability and determinacy is superior to CIR_{UK} policy, which is an optimal policy under RE. The response coefficient ω makes CIR_S more flexible than CIR_{UK} policy. In fact, CIR_{UK} rule I is a special case of CIR_S rule I when $\omega = -\psi_i^{-1}$, which is easily verified comparing equation (13) with (17)-(19).

The basic analysis can be extended in various ways. First, we made the strong assumption that the central bank knows the structural parameters of the economy when it computes the CIR interest rate rule. If structural parameters are not known, they can be estimated from data as in (Evans and Honkapohja 2003a) and (Evans and Honkapohja 2004). A result of (Evans and Honkapohja 2003a) shows that an interest rate rule that leads to instability under learning when the policy-maker knows the structural parameters does not fare better when structural parameters are estimated. We conjecture that an analogous result will hold for CIR interest rate rules.

Second, we have limited attention to the computation of CIR policy suggested by (Leitemo 2003). While Leitemo's approach is very natural, the appendix of (Svensson 1998), which is the unpublished version of (Svensson 1999), suggests a different formulation of what is meant by inflation targeting with a fixed target at fixed horizon. Svensson's approach is quite general as he constructs consistent internal forecasts relative to any fixed interest rate rule beyond a specified horizon. However, formulations of Svensson's approach in the basic forward-looking model make the interest rate dependent only on exogenous shocks, and a result analogous to Proposition 1 is then applicable. Moreover, as pointed out by (Leitemo 2003), further consistency restrictions naturally arise. For example, Leitemo's rules of the form (13) and (16) do not meet consistency beyond and within the targeting horizon.

Appendices

A Proof of Proposition 2

To prove Proposition 2 we first note that it is unnecessary to consider the exogenous shocks for these results. They play no role in indeterminacy and also, in this setting, E-instability also follows from considering the model without the shocks. We first consider determinacy for $4 \geq h \geq 2$ and E-stability for $4 \geq h > 2$.

Substituting (14) into (5) and omitting the shocks, we have the system

$$\begin{aligned} My_t &= NE_t^* y_{t+1}, \text{ where} \\ M &= \begin{pmatrix} 1+\varphi\chi_x & \varphi\chi_\pi \\ -\lambda & 1 \end{pmatrix}, N = \begin{pmatrix} 1 & \varphi \\ 0 & \beta \end{pmatrix}. \end{aligned}$$

Since both variables are free, we need both eigenvalues of $M^{-1}N$ to be inside the unit circle for determinacy whereas for E-stability we need the real parts of the eigenvalues of $M^{-1}N$ to be less than 1. Alternatively, conditions for determinacy and E-stability may be given in terms of the trace and determinant of $M^{-1}N$ as the system is two-dimensional.

The necessary and sufficient condition for determinacy turns out to be

$$\begin{aligned} 0 &> Abs[Det(M^{-1}N)] - 1 \\ 0 &> Abs[Tr(M^{-1}N)] - 1 - Det(M^{-1}N), \end{aligned}$$

where Abs refers to the absolute value of the bracketed expression. The necessary and sufficient condition for E-stability turns out to be

$$\begin{aligned} Tr(M^{-1}N - I) &< 0, \\ Det(M^{-1}N - I) &> 0. \end{aligned}$$

We now examine determinacy and E-stability for various horizons h using the above conditions. In the case $h = 2$ matrix M is singular, but we can assess determinacy by computing

$$N^{-1}M = \begin{pmatrix} -\beta^{-1} & (\beta\lambda)^{-1} \\ -\lambda\beta^{-1} & \beta^{-1} \end{pmatrix}.$$

Both eigenvalues of $N^{-1}M$ are zero, so that we have indeterminacy.[23]

In the case $h = 3$ we get

$$M^{-1}N = \begin{pmatrix} \frac{1+2\beta+\lambda\varphi}{\beta} & \frac{-1+(\beta-1)\lambda\varphi}{\beta\lambda} \\ \frac{\lambda(1+2\beta+\lambda\varphi)}{\beta} & \frac{(\beta-1)(1+\beta+\lambda\varphi)}{\beta} \end{pmatrix}.$$

It is easy to check that $Det(M^{-1}N)-1 = 2\beta+\lambda\varphi > 0$, so that indeterminacy prevails. In addition, $Tr(M^{-1}N - I) = \beta + \lambda\varphi > 0$ implying E-instability.

In the case $h = 4$ we have

$$\begin{aligned} Det(M^{-1}N) - 1 &= \frac{3\beta^2 + 3\beta\lambda\varphi + \lambda\varphi(1+\lambda\varphi)}{1+2\beta+\lambda\varphi} > 0, \\ Tr(M^{-1}N - I) &= \frac{2\beta^2 + 3\beta\lambda\varphi + \lambda\varphi(1+\lambda\varphi)}{1+2\beta+\lambda\varphi} > 0 \end{aligned}$$

so that both indeterminacy and E-instability prevail.

B Results for CIR_S Rules

Proof of Proposition 3: When $h = 2$, the characteristic polynomial of the determinacy matrix B_3 is

$$\begin{aligned} p(\tau) &\equiv \tau^3 + C_2\tau^2 + C_1\tau + C_0, \\ C_2 &= -\beta^{-2}(1+2\beta+\lambda\varphi)(\beta-\lambda\varphi\omega), \\ C_1 &= \beta^{-2}(2+\beta+\lambda\varphi)(\beta-\lambda\varphi\omega), \\ C_0 &= -\beta^{-2}(\beta-\lambda\varphi\omega). \end{aligned}$$

Then computing $p(1) = 1 + C_2 + C_1 + C_0$ and $p(-1) = -1 + C_2 - C_1 + C_0$ one obtains

$$\begin{aligned} p(1) &= \beta^{-1}\lambda\varphi\omega > 0, \\ p(-1) &= -\beta^{-2}[4\beta^2 + \beta(4+\lambda\varphi(2-3\omega)) - 2\lambda\varphi\omega(2+\lambda\varphi)]. \end{aligned}$$

[23] We remark that the analysis of E-stability for $h = 2$ is not considered as the singularity of M implies that the ALM is not unique.

Woodford has given necessary and sufficient conditions for exactly one eigenvalue of B_3 to be inside the unit circle (which is the condition for determinacy) in terms of the characteristic polynomial, see Proposition C.2, p. 672 of (Woodford 2003). Since $p(1) > 0$, we are in Woodford's Cases II or III. When $\omega > 0$ is sufficiently small we have $p(-1) < 0$, which leads to his Case III. For $\omega > 0$ is sufficiently small it is easily verified that $|C_2| > 3$ and so we get determinacy.

Proof of Proposition 4: Using (24) in the basic model (5), we obtain the following system, where $\varsigma_t = (x_t, \pi_t, i_t)'$,

$$\begin{aligned} \varsigma_t &= \hat{F}\hat{E}_t\varsigma_{t+1} + \hat{\delta}\varsigma_{t-1} + \hat{\varkappa}w_t, \qquad (38) \\ \hat{F} &= \begin{bmatrix} 1 & \varphi & 0 \\ \lambda & \beta+\lambda\varphi & 0 \\ 0 & 0 & 0 \end{bmatrix}, \hat{\delta} = \begin{bmatrix} 0 & 0 & -\varphi b_i' \\ 0 & 0 & -\lambda\varphi b_i' \\ 0 & 0 & b_i' \end{bmatrix}. \end{aligned}$$

We note that the form of the MSV solution with CIR$_S$ rule II takes the same form as (21)-(23), except that b_x, b_π, b_i (and also other coefficients) take different values. For future reference, we denote these values respectively by b_x', b_π', b_i'.

Defining $z_t = (x_t, \pi_t, i_{t-1})'$, for determinacy we need to look at the system

$$B_1 E_t z_{t+1} = \hat{B}_2 z_t;\ \hat{B}_2 = \begin{bmatrix} 1 & 0 & \varphi b_i' \\ 0 & 1 & \lambda\varphi b_i' \\ 0 & 0 & b_i' \end{bmatrix}$$

and B_1 defined in (26). REE is determinate iff the matrix

$$B_1^{-1}\hat{B}_2 = \begin{bmatrix} 1+\lambda\varphi\beta^{-1} & -\varphi\beta^{-1} & \varphi b_i' \\ -\lambda\beta^{-1} & \beta^{-1} & 0 \\ 0 & 0 & b_i' \end{bmatrix}$$

has exactly one eigenvalue inside the unit circle. It can be verified that one eigenvalue equals b_i' and the remaining two eigenvalues are given by those of the characteristic polynomial

$$p(\mu) \equiv \mu^2 - \mu(1+\beta^{-1}+\lambda\varphi\beta^{-1}) + \beta^{-1}.$$

It is easy to check that $p(0) > 0$ and $p(1) < 0$ so that one eigenvalue of $p(\mu)$ is between 0 and 1 and the other one exceeds 1. Note that for a determinate

MSV solution, we must have $|b_i'| < 1$ so that exactly two eigenvalues of $B_1^{-1}\hat{B}_2$ are inside the unit circle. The arguments show that REE is indeterminate with CIR_S rule II for all horizons and structural parameters.

We now turn to an analysis of E-stability of the system (38), which is formally the same as in the previous section. One of the necessary conditions for E-stability is that the matrix $\hat{F} + \hat{F}\bar{b}$, where

$$\hat{F} + \hat{F}\bar{b} = \begin{bmatrix} 1 & \varphi & b_x' + \varphi b_\pi' \\ \lambda & \beta + \lambda\varphi & \lambda b_x' + (\beta + \lambda\varphi) b_\pi' \\ 0 & 0 & 0 \end{bmatrix},$$

has eigenvalues with real parts less than one. It is easy to see that one eigenvalue of $\hat{F} + \hat{F}\bar{b}$ is zero and the remaining two are given by those of the matrix A in (6). A has an eigenvalue more than 1 so that all MSV solutions of the model (38) are necessarily E-unstable. This result is again independent of the horizon used by the bank and structural parameters.

C Details for the Inertial Model

As in Section 2.2, we can write the model with inertia in matrix form as

$$\begin{aligned} y_t &= A_1 E_t^* y_{t+1} + L_1 y_{t-1} + B w_t + D i_t, \\ w_t &= V w_{t-1} + v_t, \end{aligned} \tag{39}$$

where $y_t = (x_t, \pi_t)'$, $w_t = (g_t, u_t)'$, $v_t = (\tilde{g}_t, \tilde{u}_t)'$ and the matrices are

$$A_1 = \begin{pmatrix} \theta & \varphi \\ \lambda\theta & \beta\gamma + \lambda\varphi \end{pmatrix}, L_1 = \begin{pmatrix} 1-\theta & 0 \\ \lambda(1-\theta) & 1-\gamma \end{pmatrix}. \tag{40}$$

with B and D as defined before in (6). Strict inflation targeting is defined as before by equations (1) and (7) leading to a form corresponding to (8):

$$\begin{pmatrix} y_{1,t+1} \\ E_t y_{2,t+1} \end{pmatrix} = \Omega_1 \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} + \Psi_1 i_t + \begin{pmatrix} v_{t+1} \\ 0 \end{pmatrix}, \tag{41}$$

where $y_{2,t} = (x_t, \pi_t)'$, $y_{1,t} = (g_t, u_t, x_{lt}, \pi_{lt})'$, $v_t = (\tilde{g}_t, \tilde{u}_t)'$, and $x_{lt} \equiv x_{t-1}, \pi_{lt} \equiv \pi_{t-1}$. Also

$$\Omega_1 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{\theta} & \frac{\varphi}{\theta\gamma\beta} & -\frac{1-\theta}{\theta} & \frac{\varphi(1-\gamma)}{\theta\gamma\beta} & \frac{1+\varphi\lambda\gamma^{-1}\beta^{-1}}{\theta} & -\frac{\varphi}{\theta\gamma\beta} \\ 0 & -\frac{1}{\gamma\beta} & 0 & -\frac{(1-\gamma)}{\gamma\beta} & -\frac{\lambda}{\gamma\beta} & \frac{1}{\gamma\beta} \end{pmatrix}, \Psi_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\varphi}{\theta} \\ 0 \end{pmatrix}.$$

It is possible to compute the interest rule based on constant interest rate projections in the same way as before. The CIR_{UK} rule I corresponding to (13) is now

$$i_t = -\left(K \sum_{j=0}^{h-1} \Omega_1^j \Psi_1\right)^{-1} K\Omega_1^h \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix}, \tag{42}$$

with $K = (0, 0, 0, 0, 0, 1)$.

Proof of Proposition 5: The matrix for checking determinacy, namely,

$$\begin{pmatrix} -\frac{1}{\beta\gamma} & \frac{1+\beta\gamma(\gamma-1)}{\beta\gamma\lambda} & 0 & \frac{\gamma-1}{\beta\gamma\lambda} \\ -\frac{\lambda}{\beta\gamma} & \frac{1}{\beta\gamma} & 0 & \frac{\gamma-1}{\beta\gamma} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

has all eigenvalues equal to zero. However, since there are two free and two pre-determined variables, determinacy requires exactly two eigenvalues inside the unit circle. We now show that even though indeterminacy prevails, there exists a unique MSV solution when $h = 2$.

Plugging the interest rule, (35), with the coefficients (36), into the system (39), we get the reduced form system

$$\begin{aligned} y_t &= A_f E_t^* y_{t+1} + A_l y_{t-1} + A_w w_t, \qquad (43) \\ A_f &= \begin{pmatrix} -(1-\gamma)^{-1} & \frac{1-\beta\gamma(1-\gamma)}{\lambda(1-\gamma)} \\ -\lambda(1-\gamma)^{-1} & (1-\gamma)^{-1} \end{pmatrix}, A_l = \begin{pmatrix} 0 & -\lambda^{-1}(1-\gamma) \\ 0 & 0 \end{pmatrix} \\ A_w &= \begin{pmatrix} 0 & [\lambda(\gamma-1)]^{-1}(1-\gamma+\rho) \\ 0 & -\rho(1-\gamma)^{-1} \end{pmatrix}. \end{aligned}$$

Note that the lagged output gap and the g_t shock do not appear in the reduced form system (43); the interest rule has offset both these terms. The MSV solution of (43), consequently, takes the form

$$x_t = a_x + b_x \pi_{t-1} + c_x u_t, \tag{44}$$
$$\pi_t = a_\pi + b_\pi \pi_{t-1} + c_\pi u_t. \tag{45}$$

It is easy to verify that there exists a unique MSV solution of this form and it involves $a_x = a_\pi = 0$, and

$$b_x = -\lambda^{-1}(1-\gamma), b_\pi = 0. \tag{46}$$

We now check E-stability of this unique MSV solution. Assuming agents have a PLM of the form (44)-(45), they compute their forecasts $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ and these forecasts used in (43) lead to an ALM of the same form. If agents use $t-1$ data to compute their forecasts, the E-stability conditions for such a system are given by Proposition 10.1 in (Evans and Honkapohja 2001). For the constant term, the eigenvalues corresponding to the following characteristic polynomial $p(\tau)$ need to have negative real parts for E-stability.

$$p(\tau) = \tau^2 + \tau + \frac{\beta\gamma}{\gamma - 1}$$

However, $p(0) < 0$, $p(\infty) > 0$ which implies that there exists a positive eigenvalue.

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