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Do candidates serve parties interests? Party polarization as a discipline device

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Abstract

In this paper we model the interaction between parties and candidates to highlight the mechanisms by which parties selecting candidates may discipline legislators. The selection of candidates happens by majority voting of party members. We assume a multidimensional policy space comprising an ideological and a monetary dimension. Candidates are policy motivated on the ideological dimension only. Hence they care more about winning elections the bigger the ideological distance from the candidate of the opponent party. Therefore, parties can use strategically polarization to provide incentives to candidates. Because of this strategic use of candidate polarization, the polarization of the political race does not always reflect the polarization of voters' preferences. In general, the polarization of the political race seems to be a compromise between policy preferences of party members and electoral goal as parties may use polarization strategically to provide incentives.

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1. Introduction

Parties are organizations of citizens acting as political intermediaries between candidates and citizens¹. One of the most important function of parties is the selection of candidates. Parties choosing candidates may have a chance to influence policies. However, since parties can only indirectly influence policies through candidates, the issue of how much control they have over candidates and therefore over policies is an open question. Indeed, candidates and parties may have conflicting interests due to their different objectives and time horizon. Hence the question arises of whether parties can actually control elected officials. This question is linked to the more general problem of whose interests delegated politicians are serving, which is a fundamental issue in the political science literature. This work relates to this literature seeking a formal answer to the question of whether legislators serve the interests of their constituents and how parties, as political intermediaries with their own internal democratic processes, act to bind individual politicians to their constituents. Therefore, the main objective of this paper is to model the *interaction* between *parties* and *candidates* to highlight the mechanisms by which parties selecting candidates may discipline legislators and ultimately affect policy outcomes.

The selection of candidates happens through a democratic process where party members by majority voting decide which candidate to put forward. Hence, party members delegate to candidates the selection of the policy. If candidates are policy motivated, they will choose their most preferred policy. Hence parties choosing candidates determine the policy outcome. However, in a multidimensional policy space it may happen that individuals are policy motivated on some issue, while they are not on others. For example, consider a public policy that has two dimensions, an ideological dimension and a monetary dimension. The ideological dimension can be represented by the rules to follow in the provision of goods often

¹Often parties originate as associations of citizen bound together by ideological concerns. Hence, parties can be interpreted as associations of individuals with precise preferences over some dimensions of the policy space (ideological dimensions) that get together in parties to gain influence on policy choices. If we believe in this definition of parties, we are implicitly assuming that individuals forms these associations not only to gain the power to *choose* policies but more precisely to choose their *most preferred* policy. In other words, we assume that parties do not pursue the goal of winning the elections *per se* - as downsian parties will do - but they care about winning elections in order to choose the policy they prefer. The political science literature on parties goals is vast. For a brief overview on the goals of political parties see J. A. Schlesinger, "The Primary Goals of Parties: a Clarification of Positive Theory", The American Political Science Review, 1975.

provided by the state such as health or education², and the monetary dimension is the total cost of provision, paid by the citizens through taxes. Suppose that individuals are policy motivated on the ideological dimension only. This implies that this dimension is fixed and determined by their ideology. On the other hand, the monetary dimension is not fixed and the politicians' choice will depend on the incentives they will face. For example, if private firms are involved in various ways in the provision of these goods, they could lobby the incumbent legislator to set an high cost of provision and share the benefits. Or, alternatively if a policy can be realized at a low cost, but the legislator can divert money from the public budget, then the legislator may just decide to impose a high level of taxes and capture the monetary surplus from the realization of the low cost policy. Finally, it could happen that a public policy can be realized at low cost if the legislator exerts a high level of effort, while for a high cost project less effort is required. In this case, again the legislator might gain from choosing the high cost policy. In all these examples, the politician faces incentives inducing a policy against the interests of the constituents. However, the choice of a high cost can be punished by the electorate replacing the incumbent legislator with the candidate of an opponent party. When this happens, an incumbent legislator will hurt not only all citizens, imposing a high cost of provision for the public policy, but also his own party loosing elections. Hence, when a candidate becomes legislator, although he has been elected to serve the interests of a particular group of citizens, he could choose policies against the interests of the party members. When a party cannot prevent his own candidate from choosing a policy against the party's interest, clearly a problem of party discipline arises. Hence, it is relevant to ask whether parties have instruments to control candidates and therefore policies.

The outline of the model we propose is as follows. Citizens have preferences over different dimensions of the policy space. In particular, the policy space comprises an ideological dimension and a monetary dimension. Citizens with similar preferences for the ideological dimension belong to the same party. We assume that two parties exists and party members vote to put forward one candidate. Candidates are policy motivated on the ideological dimension of the policy but they are not on the monetary dimension. The monetary dimension of the policy is the cost that is paid by all the citizens through non-distortionary taxation. The cost can be high or low. Any policy can be implemented at a low cost, but

²Examples of these rules can be universal versus restricted access, different procedures to regulate the access to the service , different allocations of total spending to different aspects of the service, etc.

the legislator can arbitrarily decide to set a high cost. The high cost generates a private benefit for the legislator, while the low cost does not. Hence, a legislator might decide to set the high cost policy to obtain the private benefit. A legislator obtaining private benefits from the public policy hurts the interests of the citizens and hence can lose elections. Therefore, the question arises of how citizens - as voters and party members - can find mechanisms to discipline candidates.

Party members as *ordinary citizens* care about the ideological as well as the monetary dimension of the policy. However, as *party members* they are particularly concerned by the ideological dimension that they can directly control through the selection of candidates. Hence, compared to ordinary citizens, party members have a different interest in disciplining legislators. Accountable legislators are rewarded by reelection. Therefore for a party member an incumbent legislator of his own party winning elections insures that the ideological dimension of the policy will not change. Therefore parties pursuing their primary long term objective (control of the ideological dimension of the policy), can provide incentives to legislators to forgo private benefit and seek for re-election. In particular, since legislators are policy motivated on the ideological dimension of the policy, they care more about winning elections the bigger the ideological distance from the candidate of the opponent party. Therefore parties - shaping the political race - can control the behavior of the incumbent legislator.

The main insight from this analysis is that parties determine the polarization of the political race and they can use strategic polarization to provide incentives to candidates. Because of this strategic use of candidate polarization we obtain that the polarization of voters' *preferences* and the polarization of the *political race* do not necessarily go in the same direction. For a medium degree of polarization, the political race tends to reflect the polarization of preferences but for more extreme distributions we can obtain different results. Hence for example, when the *polarization* of preferences is *low*, parties may choose to increase the polarization of the political race in order to provide incentives to candidates. As a consequence the candidate of the incumbent party may be more extreme than the median party member. On the other hand, if the *polarization* of preferences is *high*, parties may choose less extreme candidates to get closer to the median voter. Hence, again the candidate will not represent the preferences of the median party member. In general, the polarization of the political race seems to be a compromise between policy preferences of party members and electoral goal and parties may use polarization strategically to provide incentives.

The plan of the paper is as follows. In section 2 we present a brief overview of

the related literature. In section 3 we outline the model. In section 4 we discuss the main assumptions and we state the main propositions. In section 5 we discuss the results. In section 6 we summarize and conclude.

2. Related literature

The questions addressed by this paper are common to different streams of literature. The problem of the incentives provided by voters to policy makers relates this work to the *agency* models of political competition (Banks and Sundaram (1997), Persson, Roland and Tabellini (1997), Coate and Morris (1995)). Even though the problem of the incentives provided by voters is important in our work, the focus of this paper is different from the agency model in many respects.

First, the problem of the incentives provided by voters is analyzed in a game that also includes *parties*. The introduction of parties with their own internal democratic process and the formalization of the relationship between candidates and parties are the main contributions of this paper that identifies precisely in the *candidate choice* the major mechanism by which parties as long-lived institutions provide incentives to short-lived candidates. The role of parties as long-lived institutions providing incentives to short-lived candidates is not new to the literature. In particular, Alesina and Spear (1988) show that parties making transfers to candidates can prevent them from choosing their most preferred policies when these policies will hurt parties' interests. In this respect, our argument for the role of parties as long lived institutions proving incentives is very closed to theirs. However, while in Alesina and Spear the role of parties is to make transfers, in our model the role of a party as an institution is to insure the existence of a mechanism that selects *always* a precise *type* of candidate. As long as an incumbent legislator cares about the *type* of policy and knows that remaining *loyal* to the party (i.e. acting in a way that enhances the electoral prospects of future party candidates) a certain *type* of policy will be chosen, then *citizens* - as *party members* and *voters* - can discipline legislators.

A second departure of our model from the agency literature is our assumption of perfect information. Although the problem of information in political games is relevant, agency models may face the criticism of extremely simplifying political competition due to the absence of heterogeneity in policy preferences. The heterogeneity of preferences plays a crucial role in our model and indeed parties are the institutional device translating heterogeneous policy preferences into candidate locations on the policy space. Therefore, with this model we can address

the fundamental question of *how citizens preferences are reflected in party locations and finally in policy outcomes*. The Downsian model of party competition (Downs, 1957) predicts that parties will only represent the interests of the median voter since parties care only about winning elections. On the other hand, the citizen-candidate model (Besley and Coate, 1997) of political competition, assuming that candidates are policy motivated, shows that the median voter outcome is only one of the possible equilibria arising in the political race. Alesina (1988) shows that in a two party system, where parties care about policies, partial or total converge to the median voter are possible. In our model we consider jointly *candidates* and *parties*. We assume that candidates are policy motivated (at least on one dimension) but we introduce parties as a mechanism to select candidates. Although in our model the electoral goal represents a legitimate objective of a party, we assume that parties aim to win elections in order to implement policies and they do so, choosing candidates that are policy motivated. Therefore, parties represent the interests of their members taking into account the preferences of the median voter and the agency problem arising from the limited control they have on candidates. Hence, our model shows that the use of polarization as a discipline device can be an alternative explanation for the non-convergence to the median voter.

3. The model

3.1. Economic environment

The economy comprises N individuals. We denote by k the generic individual of the community. We assume that there are two classes of individuals: citizens denoted $k = i$ and legislators denoted $k = j$. Individuals of this economy live t periods where t belongs to the infinite set $T = \{0, 1, 2, 3, \dots\}$. In each period a legislator chooses a public policy and an elected legislator cannot stay in power for more than 2 periods³. The policy choice is multidimensional and it consists

³This assumption reflects the idea that that elected officials know the date of their last mandate because of term limits or retirement age. Note however that as elections and policy choices go far beyond the last mandate of elected politicians, citizens and party members face uncertainty about the last period where they will participate to an election or the policy will be delivered. Or put it differently, their time horizon is infinite. We will discuss later in the paper the relevance of this assumption.

of an ideological dimension that we call *type* and a monetary dimension that we call *cost*.

The *cost* of the policy is paid by the citizens. We denote $C \in \{C^L, C^H\}$, with $C^H > C^L$, the per capita cost paid by the citizens. The policy can always be implemented at a low cost, C^L . However, in each period the legislator has complete discretion on whether to set the high cost, C^H , or the low cost, C^L . The *type* of the policy is a characteristic on which individuals have different tastes. In particular, we assume that policy makers are policy motivated on the ideological dimension. We denote a_j the policy type delivered by the legislator j . Given the policy maker j and the generic individual k , we define a_{kj} the utility enjoyed by the individual k when the legislator j is choosing the policy and we assume that $a_{jj} = \max_k a_{kj}$.

Therefore, the *policy choice* can be represented as a vector (P, C, a_j) , with $C \in \{C^L, C^H\}$ and $P \in \{0, 1\}$, where 0 means that no policy is selected and 1 means that a policy of type a_j and cost C is implemented in period t .

Besides the utility a_{kj} , the policy generates a private non-negative benefit for the legislator denoted by $B(P, C)$ that is increasing in both arguments. Since the cost can be only low or high, for simplicity, we assume that $B(P, C^L) = 0$ and $B(P, C^H) = B > 0$.

In formal terms, denoting $v_k(P, C, a_j)$ the per period utility of the generic individual k when a policy maker j chooses the policy, we can write the per period utility of citizen i and policy maker j as follows:

$$v_i(P, C, a_j) = a_{ij}P - C.P \quad (3.1)$$

$$v_j(P, C, a_j, B(\cdot)) = a_{jj}P + B(P, C) \quad (3.2)$$

3.2. Political game

Citizens get together in an association called a "*party*" because of common ideological concerns that in our model are represented by preferences for policy *types*. The population is distributed according to their most preferred policy *type* and we denote M the median voter of this distribution. Hence, the median voter M splits the population in two groups of equal size according to their ideological preferences and these two groups constitute the two parties labelled respectively L and R . Given each party, we denote m_R and m_L the median party members of

respectively party R and party L and we assume that the distribution is symmetric with respect to the median voter.

The players of the political game are parties, citizens and a legislator. Parties select candidates for the electoral race. Citizens vote for candidates. Elected politicians choose policies.

The timing of the game is as follows. In period $t = 0$, the two parties announce simultaneously their candidates and then one candidate is exogenously selected to become a legislator. The appointed legislator chooses a policy. At the end of the political mandate elections are called where the incumbent legislator faces the challenger named by the opponent party. The selection of candidates only happens during $t = 0$, before the first election. Hence, once two types of legislators are selected, these two "types" participate to all future elections⁴. We also assume that the incumbent cannot re-run for office in future electoral rounds, if he ever lost elections in the past. Hence a politician that leaves office, either because his mandate cannot be renewed or because he encountered an electoral defeat, just becomes an ordinary citizen⁵.

In period $t = 1$ the first election is called and citizens vote. The winner of the electoral competition becomes the new legislator who will choose the policy in period $t = 1$. Then again at the beginning of period $t = 2$ a new election will take place and the elected legislator choose the policy in period $t = 2$ and so on. More formally, in each period $t \geq 1$ we have an election followed by a policy choice. This is our *stage game* that is repeated for t periods.

To summarize, in period $t = 0$, parties choose candidates and one candidate is exogenously selected to become the first legislator choosing the policy in period $t = 0$. Then, from period $t = 1$ onwards an election followed by a policy choice takes place. In this game all actions are observable and players have perfect recall of all actions undertaken in previous periods.

The strategies of parties, citizens and legislators are as follows.

Parties select candidates that participate to the electoral race, hence the par-

⁴Remember that an incumbent legislator can be in office only for two periods. Hence, the fact that the candidate choice only happens in period $t = 0$, only means that in the future the candidates will be of the "type" initially chosen, not that they will be the same individuals.

⁵In terms of our model this means that the party will never choose the same candidate if he has lost elections. With this assumption, we are in fact restricting the party strategy space. However, if we consider a large population, where many other candidates of the same type are available, the choice to not reappoint a candidate who has previously lost elections, would indeed be an equilibrium strategy. Hence, supposing that $N \rightarrow \infty$, we can simplify the strategy space of parties with no loss of generality.

ties' strategies consists in the candidate choice. We assume that the candidate choice takes place by majority voting inside the party.

Citizens vote for candidates and they base their choice on the policy choice of the incumbent (*retrospective voting rule*). However, since the voters and candidates interact repeatedly, the voting strategy will also depend on actions undertaken by legislators in previous periods. Let j^t be the legislator in place in period t and let (P, C_t, a_{jt}) be the policy chosen in period t . Let's define h^t the sequence of policies $\{(P, C_0, a_{j^0}), (P, C_1, a_{j^1}) \dots (P, C_{t-1}, a_{j^{t-1}})\}$ chosen up to period $t - 1$. For each citizen i voting for a candidate j we define a voting strategy, $\sigma_{ij}^t(h^t)$. Where $\sigma_{ij}^t(h^t)$ can take values 0 or 1. $\sigma_{ij}^t(h^t) = 1$ means that the incumbent j in place in period $t - 1$ is reappointed in period t . On the other hand, $\sigma_{ij}^t(h^t) = 0$ means that the incumbent in place in period $t - 1$ is replaced in period t by the challenger. Therefore, in every period t the strategy $\sigma_{ij}^t(h^t)$ prescribes whether to reelect or not the legislator in place in period $t - 1$ (incumbent legislator).

The legislator in place in period t chooses the policy (P, C_t, a_{jt}) . Legislators interact repeatedly with voters in elections. Therefore when they choose policies, they will also consider the outcomes of past elections. Let's denote e_{j-j}^t the sequence of past electoral outcomes up to the period $t - 1$, where j and $-j$ have been the two candidates. We define $c_j^t(e_{j-j}^t)$ the strategy of the legislator j in period t prescribing a policy (P, C_t, a_{jt}) , given the history of electoral outcomes e_{j-j}^t . Since in the first period $t = 0$, the legislator is exogenously selected, the game become with the null history e^0 , since no previous election has ever been held.

To summarize, the strategy $\sigma_{ij}^t(h^t)$ is a voting rule that determines the identity of the policy maker j in each period $t \geq 1$. The strategy $c_j^t(e_{j-j}^t)$ is a policy rule which determines the policy choice (P, C_t, a_{jt}) in each period $t \geq 0$. In fact, the ideological component of the policy, a_j , only depends on the identity of the policy maker j . Hence, the policy rule $c_j^t(e_{j-j}^t)$ for the legislator j only concerns the variable $C \in \{C^L, C^H\}$.

In terms of outcomes, having denoted j^t the legislator in place in period t and (P, C_t, a_{jt}) the policy chosen by the legislator j^t in period t , we can say that the strategies of the two players finally generate a sequence of legislators and policies denoted p^t , where $\{p^t = [j^t, (P, C^t, a_{j^t})]\}_{t=t+1}^\infty$.

Since we have outlined the strategies and outcomes of the game, we can now write the payoffs of the players. The total intertemporal payoff of each player is the sum of their per period utility, where the future is discounted according to the discount factor $\delta < 1$. Let's denote $V_k^t(\sigma_{ij}^t(h^t), c_j^t(e_{j-j}^t))$ the expected

payoff in period t of the generic player k . Let $v_k(p^t)$ be the utility the individual k associated to the outcome p^t induced by the pair of strategies $(\sigma_{ij}^t(\cdot), c_j^t(\cdot))$. The expected payoff of each player can be written in the following form:

$$V_k^t(\sigma_{ij}^t(h^t), c_j^t(e_{j-j}^t)) = \sum_{\tau=t+1}^{\infty} \delta^\tau v_k(p^\tau)$$

Having described strategies and payoffs, we can conclude the description of the political game introducing the concept of equilibrium. Voters and legislators choose their strategies so as to maximise their intertemporal payoff. As far as parties are concerned parties, the objective of party members is to choose candidates maximising their total expected ideological payoff. Hence the main difference between parties, citizens and legislators is that parties have an objective function that does not depend on the monetary dimension of the policy⁶.

We require the candidate choice be a Nash equilibrium of the one shot game where parties select candidates. For the voting and policy rules in the repeated game, we require the equilibrium voting rule and the equilibrium policy to be subgame perfect Nash equilibrium. We restrict our analysis to pure strategies.

We define the equilibrium of this game as follows:

Definition - Political Equilibrium:

A political equilibrium consists of the following elements:

- (a) a pair of candidate that are best response to each other, given the equilibrium voting strategy and the equilibrium policy choice*
- (b) an optimal voting strategy, given the equilibrium policy choice*
- (c) an optimal policy, given the equilibrium voting strategy*

To summarize, a political equilibrium consists in a *pair of candidates* appointed by the two parties to participate to the political race, in an *equilibrium voting strategy* for the citizens that elect the legislator and in a *policy* chosen by the legislator .

⁶In the next section we discuss in details our assumptions on political parties and their role.

3.3. Parties

The choice of the candidate that will run for the elections is determined in each party by majority voting. Therefore, *de facto* the median party member in each party chooses the party's candidate. Since the winning candidate will choose his most preferred policy on the ideological dimension, parties choosing candidates have a direct control on the ideological dimension of the policy. On the other hand they do not have control on the monetary dimension of the policy.

We assume that party members choose candidates maximizing only the expected value of their ideological payoff⁷. This assumption reflects the idea that parties are associations of citizens bound together by ideological concerns. Hence, the objective of parties is to pursue the goal justifying the foundation of the association. Any other issue that is not the primary goal is not directly included in the objective function of the party. However, insofar as other issues may indirectly affect the achievement of the primary goal of a party through the electoral process, parties will also consider other issues. In other words, if other issues are politically salient, these issues will be "indirectly" considered by parties⁸. In our model, since the reelection of the incumbent depends both on the ideological and monetary dimension of the policy, then party members will be concerned by the choice made by the incumbent on the monetary dimension of the policy (although the monetary dimension of the policy is not the reason for the constitution of the party). This is due to the fact that in the political equilibrium, party members choose candidates *given* the equilibrium voting strategy and the equilibrium policy. More precisely, parties⁹ chose candidates taking into account the effect of their choice on the total intertemporal ideological payoff. This implies that each

⁷For each party member i the intertemporal payoff is simply the discounted sum of the utility from the ideological component of the policy chosen by the sequence of legislators j^t , where this payoff can be written as $\sum_{t=0}^{\infty} \delta^t a_{ij^t}$.

⁸In this model we do not consider the case where parties can change their agenda modifying their chart. This is an interesting question for further research since historically parties of modern democracies have changed their constitution. On the other hand, these are historical changes usually followed by long periods of stability. Hence, the objective of this paper is to investigate the effect of parties as stable institutions. Also, it is important to notice that there is a difference between party chart and party manifestos. New issues are more often introduced in the party agenda with party manifestos rather than changing the party chart. Hence, it would be interesting to analyze the conditions that may induce a party to opt for a change of their constitution instead of using different manifestos.

⁹In the rest of the paper we use the term parties and party members as synonymous

party chooses his own candidate taking for given the candidate choice of the other party, the equilibrium voting strategy of voters and the equilibrium policy choices of legislators.

4. Policy preferences, political salience and policy outcomes

The distribution of policy preferences affects crucially the candidate choice of the two parties and the final policy chosen by the incumbent legislator. Remember that the median party member selects the candidate, while the median voter M determines the result of the election. The median voter unambiguously prefers a low cost policy to an high cost policy. On the other hand, in terms of ideology he prefers a candidate that is as close as possible to himself. Ideally, a median voter would prefer a median candidate *type* choosing a low cost policy since his most preferred policy is (a_{MM}, C^L) . On the other hand, a median party member would like to have his own policy type a_{m_j} chosen with a low cost. Hence, the median voter and the median party member agree on the monetary dimension of the policy, but have different preferences on the ideological dimension.

The ideological dimension is determined by the median party member. However the median party member choosing the type of candidate needs to take into account the preference of the median voter, M , that ultimately decides whether to reappoint or not the incumbent legislator. Clearly, since the policy is multidimensional, the political equilibrium depends crucially on the salience of the two issues. Let's assume that the cost of the policy is politically salient. Formally:

Assumption 1 (political salience)

$$a_{Mj} - C^L \geq a_{MM} - C^H$$

In words, assumption 1 says that the median voter M prefers a legislator j choosing the low cost policy to a legislator M choosing the high cost policy. Given this assumption¹⁰, we will characterize the political equilibrium. In particular we will characterize the distribution of preferences supporting an equilibrium where the incumbent legislator is accountable to voters.

¹⁰This assumption has precise consequences in terms of distribution of the population on the ideological dimension of the policy. We will discuss later these implications.

5. Political equilibrium

In this section we solve the model to verify if the game has equilibria where an incumbent legislator chooses a low cost policy and is therefore accountable to the voters. We have assumed that a legislator can be in office only for two periods and if he loses election cannot re-run for office. However, when the last round of elections is reached, the party can replace the incumbent with another candidate of the same type. Hence, there is no limit to the number of electoral rounds where a given "type" of legislator can run for elections. Nevertheless, since a legislator can serve only a limited number of mandates, there is a limit to the number of terms where an incumbent legislator can obtain private benefits from the public policy.

Remember that in our political game we have a one shot game where parties choose candidates, followed by a game where voters and legislators interact repeatedly for an infinite number of periods. To solve this game we proceed as follows. First, we solve the repeated game for any possible candidate location. Then, given the solution of the repeated game, we find the equilibrium candidate location of the one shot game.

5.1. Voting rule and policy choice

In this section we characterize the equilibrium of the repeated game taking place between the legislator and the voters. We will look for voting strategies and policy choices that are subgame perfect. Suppose that L is the incumbent party and R is the challenger. The incumbent party selects the incumbent legislator j knowing that at the end of the mandate this candidate will face a challenger $-j$ named by the opponent party. Both the incumbent and the opponent party can choose any type of candidate. In particular, their location with respect the median voter M can be either symmetric (i.e. the two candidates j and $-j$ are equally distant from M) or asymmetric (i.e. one candidate is closer to the median voter than the other). We can summarize the possible candidates locations in three cases:

- *case 1*: the challenger $-j$ is closer to the median voter than the incumbent j . For simplicity, let's assume that $-j = M \neq j$
- *case 2*: the incumbent j is closer to the median voter than the challenger $-j$. For simplicity, let's assume that $j = M \neq -j$
- *case 3*: the incumbent j and the challenger $-j$ are equally distant from the median voter.

Given these candidates' locations, let's solve the repeated game. We are interested in one particular equilibrium, i.e. an equilibrium where the incumbent legislator is accountable to voters. Hence, in the rest of this section we will look for the conditions supporting an equilibrium where the incumbent legislator chooses the low cost policy, for any possible location of the opponent.

We illustrate here in detail how to solve the game in *case 1*. Then the same logic will be used for the other two cases¹¹. The incumbent legislator $j \neq M$ has to decide whether to choose the high cost policy or the low cost policy. He chooses the policy maximising his total expected utility. The utility in period $t = 0$ only depends on his current policy choice¹², while the utility for the periods $t \geq 1$ will depend on the sequence of legislators and policies, $\{p^t = [j^t, (P, C_t, a_{jt})]\}_{t=1}^{\infty}$, induced by the current policy choice.

If the legislator j is reappointed, he will again choose the policy in the next period. If he is not reelected, the challenger with different ideological preferences will determine the policy. Hence, the reappointment can be seen as a "reward" for the previous policy choice, and the replacement as a "punishment" for the previous policy choice. Since the median voter prefers the low cost policy to the high cost policy, to obtain the low cost policy he can follow a strategy that prescribes reelection of the incumbent when he has chosen the low cost policy and no reelection if he has chosen the high cost policy. However, since the policy is multidimensional, this reelection rule is not necessarily optimal when the incumbent legislator has ideological preferences different from the median voter. On one hand the median voter would like to reward the incumbent to obtain the low cost policy, on the other he may be tempted to replace him with a challenger with policy preferences closer to himself. In other words, although the median voter may want to punish or reward the incumbent on the cost dimension of the policy, these punishment and rewards may be costly to carry on because of the ideological component of the policy. Therefore, we need to verify if rewards and punishments of this type are credible, or put it differently, subgame perfect.

We can represent this game with a game tree where in each period t a legislator decides whether to choose C^L or C^H and the identity of the legislator at each decision node from period $t = 1$ onward can be either $j = M$ or $j \neq M$, depending on the equilibrium voting strategy of the median voter M .

¹¹Importantly, to solve the game we assume that the game is stationary.

¹²Remember that in the first period there is no election.

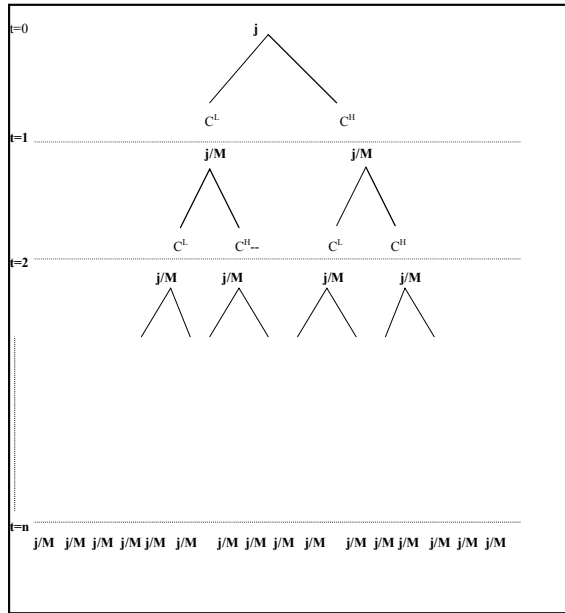


figure 1

Suppose that the median voter M follows the strategy "reappoint the incumbent if he has chosen the low cost policy, replace the incumbent with the challenger if he has chosen the high cost policy". For sub-game perfection to hold, we have to verify if this voting strategy is an equilibrium voting strategy in each stage game.

Let's start with the part of the game three following C^H in the $t = 0$. First, we show that the voting rule "replace the incumbent with the challenger if he has chosen the high cost policy" is an equilibrium voting strategy in $t = 1$. The argument proving this result goes as follows. In period $t = 0$ the incumbent legislator j has chosen C^H . When elections are called, the median voter M will face a legislator $j \neq M$ who has chosen the high cost policy and a challenger $-j = M$ who can choose any policy $C \in \{C^L, C^H\}$. Clearly, since $a_{MM} - C > a_{Mj} - C^H$ for any $C \in \{C^L, C^H\}$, the median voter can credibly carry on his threat and punish the incumbent.

Hence, we have shown that the following strategy:

$$\sigma_{Mj}^1(h^1) = 0 \text{ if } h^1 = (1, C^H, a_j)$$

is an equilibrium voting strategy.

Let's continue now along the game three and let's check if the same strategy is an equilibrium voting strategy for any $t \geq 1$. Given $\sigma_{Mj}^1(h^1)$, we know that in period $t = 1$ the incumbent will be replaced by the challenger $-j = M$. Assume again, now that in period $t = 2$, the voter follows the strategy $\sigma_{Mj}^2(h^2) = 0$ if

$h^2 = \{(1, C^H, a_j), (1, C^H, a_M)\}$. In other words, he punishes the incumbent M if he chooses C^H . We can immediately show that in period $t = 2$, the voter cannot credibly punish the incumbent M . The reason for this is that the two types of candidates, j and M , have both chosen C^H in the past¹³, but M is closer to the median voter on ideological ground. Hence, we can conclude that in $t = 2$, the strategy:

$$\sigma_{MM}^2(h^2) = 0 \text{ if } h^2 = \{(1, C^H, a_j), (1, C^H, a_M)\}$$

cannot be an equilibrium voting strategy. And indeed the opposite strategy :

$$\sigma_{MM}^2(h^2) = 1 \text{ if } h^2 = \{(1, C^H, a_j), (1, C^H, a_M)\}$$

is an equilibrium voting strategy¹⁴. Using the same argument, this is also an equilibrium voting strategy for any subsequent period $t > 1$. Also, if we consider our game three in *figure 2*, the strategy that prescribes to permanently replace the incumbent $j \neq M$ with the challenger $-j = M$ whenever j chooses C^H , is an equilibrium voting strategy. Therefore we conclude that if in $t = 1$ there are two candidates $j \neq M$ and $-j = M$, where j is the incumbent and $-j$ is the challenger, then the voting strategy prescribing the following sequence of actions:

$$\sigma_{Mj}^t(h^t) = 0 \text{ if } j \text{ has ever chosen } C^H \text{ in the past}$$

is an equilibrium voting strategy.

Given this equilibrium voting strategy, clearly the equilibrium policy choice for the legislator M will be to choose the high cost policy, since he cannot be punished for this. Hence, we can conclude that from period $t = 1$ onward the policy $(1, C^H, a_M)$ will be chosen. This implies that, in period $t = 0$ the expected payoff of the incumbent j choosing the policy C^H can be written as follows:

$$V_j^0(1, C^H, a_j) = a_{jj} + B + \sum_{t=1}^{\infty} \delta^t (a_{jM} - C^H)$$

In words, in the first period $t = 0$, the incumbent j choosing the high cost policy obtains the utility a_{jj} and the benefit B . In $t = 1$, he will be permanently replaced by the opponent M choosing the high cost policy. Therefore, from period $t = 1$ onwards as a citizen he will receive the utility, $(a_{jM} - C^H)$.

¹³In this model candidates do not have any other individual characteristics than their ideological preferences. Hence, two individuals with same ideology are identical in the voter's perspective.

¹⁴More rigourously we should say that:

$$\sigma_{ij}^2(h^2) = 1 \text{ if } h^2 = \{(1, C^H, a_j), (1, C^H, a_M)\}$$

$$\sigma_{ij}^2(h^2) = 1 \text{ if } h^2 = \{(1, C^H, a_j), (1, C^L, a_M)\}$$

is an equilibrium voting strategy.

We can illustrate the game with the a figure. It is useful to call G_t the game starting after any deviation from the low cost policy, since we know the solution to this game and we can easily compute the payoff of each player in this game. In the game three of the figure 2, the bold line illustrates the equilibrium path whenever the incumbent j chooses C^H .

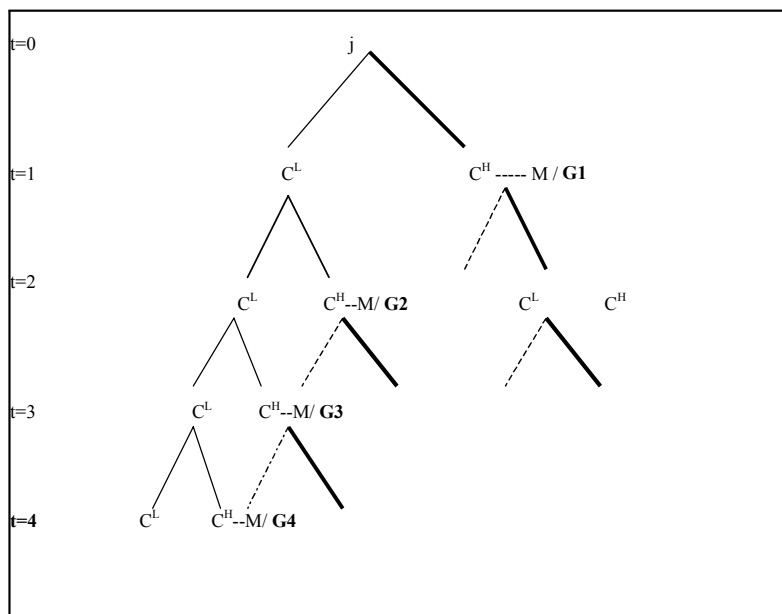


figure 2

Let's consider now consider the game three following the low cost policy in the first period. In period $t = 0$ the incumbent legislator j chooses C^L . When elections are called, the median voter M will face a legislator $j \neq M$ who has chosen the low cost policy and a challenger $-j = M$, who could choose any policy $C \in \{C^L, C^H\}$. Suppose that the voter follows the strategy "reappoint the incumbent if he has chosen the low cost policy, replace the incumbent with the challenger if he has chosen the high cost policy". We have already shown that the voter can credibly punish an incumbent $j \neq M$ for choosing the high cost policy. Can the voter credibly reward an incumbent legislator $j \neq M$ who has chosen the low cost policy? The argument proving the subgame perfection of this "reward" is more subtle then the argument proving the subgame perfection of the "punishment" in the case the incumbent has chosen the high cost policy. Hence, to solve the game in this case, we first conjecture that rewarding the incumbent with reelection if he has chosen C^L is an equilibrium voting strategy, and then we

show that this conjecture is correct under some restrictions on the parameters of the model.

Let's conjecture the following equilibrium voting strategy:

$\sigma_{Mj}^t(h^t) = 1$ if the legislator j has always chosen C^L in the past;

$\sigma_{Mj}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past;

Remember that after any deviation from the low cost policy, the equilibrium path will be represented by the bold line in *figure 2*, where the high cost policy is chosen forever by the legislator M . Given this conjectured voting strategy, let's compute the expected payoff of legislator j when he chooses C^L in $t = 0$. Since a deviation from the low cost policy may occur at any period t in the future, the main difficulty in computing the expected payoff of the legislator choosing the low cost policy in period $t = 0$ is the infinite number of possible paths associated to possible deviations in $t \geq 1$. However, among all the possibilities, we can identify the path that keeps the legislator j at his lowest possible payoff. Therefore, if we can find a condition supporting the policy C^L when the legislator obtains his lowest payoff, then this condition will guarantee that the legislator will choose C^L also in any other case where he can obtain a higher payoff.

To find the equilibrium policy choice for the incumbent j , note that we know that the legislator can be in place for two mandates only. Hence, knowing the last period where he will choose a policy, we can use backward induction to determine his equilibrium policy choice. Suppose that in period $t = 1$ the legislator j is in place¹⁵ and he has to decide whether to choose C^L or C^H .

We know that if he chooses C^H his expected payoff from period $t = 1$ onwards will be:

$$V_j^1(1, C^H, a_j) = a_{jj} + B + \sum_{t=1}^{\infty} \delta^t (a_{jM} - C^H)$$

If he chooses C^L , his expected payoff will depend on the policy choice of future legislators¹⁶. In particular, under the conjectured voting strategy, he will be replaced by a legislator with same policy preferences who could choose C^L in both mandates, or C^L in his first mandate and C^H in his second mandate or finally C^H in his first mandate. Clearly the best scenario for an incumbent choosing the low cost policy in his last mandate is when the first successor j will

¹⁵Remember that under our conjectured voting strategy this means that the legislator has chosen the low cost policy in $t = 0$.

¹⁶Remember that the incumbent j has to leave office after two mandates but can be replaced by a new legislator of his same policy type.

choose C^L in both mandates and also all future successors will do the same. There are however many other cases where, at a certain period $t = n > 1$, a successor may deviate and choose the high cost policy. Suppose that we are in the worst scenario, i.e. the incumbent j chooses C^L and the first successor of the same type of the incumbent immediately chooses C^H . Let's denote \underline{V}_j^1 the lowest payoff for the incumbent legislator j in his last mandate $t = 1$. This payoff can be written as follows:

$$\underline{V}_j^1 = a_{jj} + \delta (a_{jj} - C^H) + \sum_{t=2}^{\infty} \delta^t (a_{jM} - C^H)$$

In words, in $t = 1$ the incumbent choose the low cost policy obtaining the utility a_{jj} . Then in period $t = 2$ a legislator of the same type is reelected who chooses the high cost policy. Hence, the previous period incumbent as a citizen gets the utility $\delta (a_{jj} - C^H)$. Finally, the legislator j will be permanently replaced by M choosing C^H . Hence, the incumbent of period $t = 1$, as a citizens will obtain the utility $(a_{jM} - C^H)$ from the third period onwards.

Comparing the payoffs $V_j^1(1, C^H, a_j)$ and \underline{V}_j^1 we can state the following result:

Lemma 1

Given the candidates M and j and given the conjectured voting rule voting strategy $\sigma_{Mj}^(\cdot)$, in period $t = 1$ the incumbent legislator j will choose the policy $(1, C^L, a_j)$ if and only if $(a_{jj} - a_{jM}) \geq \frac{B}{\delta}$*

Note that the polarization of the political race, $(a_{jj} - a_{jM})$, and the discount factor δ play a crucial role. In fact, the benefit for the legislator j choosing a low cost policy in his last mandate is the election of his successor j which insures that in the future his most preferred policy a_j will be chosen. Clearly, the bigger the ideological distance between the incumbent legislator j and the challenger M , the more the incumbent legislator will care about the election of his successor j . The discount rate also plays an important role because if δ , for example, is closed to zero, it is unlikely that the incumbent j forgoes the current benefit B from a high cost policy to obtain his most preferred type a_j in the future. Finally note that, since *lemma 1* gives the condition for the legislator to be accountable in $t = 1$ given that the first successor will deviate in $t = 2$, then this condition will be sufficient for the legislator to be accountable in $t = 1$ if deviation will occur at any other date $t > 2$.

Let's move now back to period $t = 0$. When *lemma 1* holds, we can easily compute the expected payoff of the incumbent legislator if he chooses C^L in period $t = 0$. In fact, under the conjectured voting strategy, supposing that the first successor j deviates to the high cost policy in $t = 2$, the expected payoff for the legislator j will be:

$$V_j^0(1, C^L, a_j) = a_{jj} + \delta a_{jj} + \delta^2 (a_{jj} - C^H) + \sum_{t=3}^{\infty} \delta^t (a_{jM} - C^H)$$

Comparing this payoff with the payoff from the high cost policy:

$$V_j^0(1, C^H, a_j) = a_{jj} + B + \sum_{t=1}^{\infty} \delta^t (a_{jM} - C^H)$$

We can state the necessary and sufficient conditions for the incumbent legislator j to choose the low cost in $t = 0$. Let $\sigma_{Mj}^*(\cdot)$ be the *conjectured voting strategy* that we have already described:

Lemma 2

Given the candidates M and j and given the conjectured voting rule voting strategy $\sigma_{Mj}^(\cdot)$, in period $t = 0$ the incumbent legislator j will choose the policy $(1, C^L, a_j)$ if and only if $(a_{jj} - a_{jM}) \geq \frac{B - \delta C^H}{\delta(1 + \delta)}$*

Note that, the ideological distance between the incumbent and the challenger, $(a_{jj} - a_{jM})$ and discount factor δ are again the crucial elements to determine the equilibrium policy¹⁷. If the benefit enjoyed by the legislator choosing the high cost policy is bigger than the discounted value of the high tax he will pay as a citizen in the subsequent period, i.e. if $B - \delta C^H > 0$, then for the legislator to be accountable to voters on the cost dimension of the policy in his first mandate, the political race must be sufficiently polarized.

Note also that the threshold level for the polarization in *lemma 1* is higher than in *lemma 2*, meaning that for the legislator to stay accountable in the second mandate, the polarization needs to be bigger. Therefore, we can conclude that

¹⁷The proof of the lemma is straightforward since we simply need to impose that the following inequality holds: $V_j^0(1, C^L, a_j) \equiv a_{jj} + \delta a_{jj} + \delta^2 (a_{jj} - C^H) + \sum_{t=3}^{\infty} \delta^t (a_{jM} - C^H) \geq V_j^0(1, C^H, a_j) \equiv a_{jj} + B + \sum_{t=0}^{\infty} \delta^t (a_{jM} - C^H)$

when lemma 1 holds, i.e. when $(a_{jj} - a_{jM}) \geq \frac{B}{\delta}$, then the incumbent legislator j will choose the low cost policy in both mandates.

Clearly, given the stationarity of the game, this result holds for any incumbent legislator in place for the first time at any period t . Therefore, we can conclude that under the *conjectured voting strategy*, when the polarization of the political race $(a_{jj} - a_{jM})$ is above the threshold level $\frac{B}{\delta}$, then a legislator j in place for the first time in period t will be accountable in both mandates t and $t + 1$.

Finally, we can prove that under the same parametric restrictions, the conjectured voting strategy is an equilibrium voting strategy¹⁸. To prove this, we have to verify that in each period t the median voter will carry on the reward conjectured by the legislator. Intuitively, when the conjectured strategy implies an equilibrium where the incumbent j is accountable in every period, for the voter it is optimal to vote according to the conjecture since if he doesn't, in any future period accountability will be lost. Note that the assumption of political salience is crucial to obtain this result, since when C is politically salient the median voter prefers an equilibrium where $j \neq M$ is accountable to an equilibrium where $j = M$ is not accountable. Clearly, the conjectured voting strategy is not the unique strategy that may sustain an equilibrium where the legislator is accountable. One might propose many different reward-punishment schemes that may sustain the same equilibrium. However, since we have necessarily to select some particular scheme, we have chosen a very simple reward-punishment scheme prescribing that an incumbent is reelected if he chooses the low cost policy and his permanently voted out of office if he doesn't, and we have shown that this simple voting strategy can sustain an equilibrium where incumbent legislator is accountable.

As a final but important remark, all the results crucially depends on the possibility of cooperation of future legislators. Even if a future legislator might deviate from the low cost policy and choose the high cost policy, cooperation cannot be excluded. This simple possibility is sufficient, under given parametric restrictions, to sustain a cooperative equilibrium. In fact, if the game had a final period, the conjectured voting strategy which sustains an equilibrium where the incumbent is accountable, could not be an equilibrium voting strategy. In fact, if the last period of the game is known, the voter cannot credibly reward the incumbent for choosing the low cost policy. This can be easily shown simply using backward induction. Suppose that t is the last period of the game. In the last mandate t , because no future election will be called, any j legislator would choose the high cost policy. If this is the case, an incumbent legislator in his last mandate in

¹⁸proof in appendix.

period $t - 1$, knows that the median voter will never "reward" him for choosing a low cost policy¹⁹ since he will certainly prefer the challenger M choosing C^H to the successor j choosing C^H . As a consequence, the incumbent j in his last mandate $t - 1$ will choose C^H . However, if in his last mandate, $t - 1$, j will choose C^H , then the median voter will not find optimal to reelect him at the end of the first mandate, even if he has chosen C^L . Hence, moving to the first mandate, $t - 2$, the incumbent j knowing that he cannot be rewarded for C^L , will certainly choose C^H . Hence, proceeding backward up to period $t = 0$ we can conclude that the policy C^H is the equilibrium policy in every period $t \geq 0$.

Having characterized the equilibrium of the repeated game when the two candidates are asymmetric and the challenger is closer to the median voter than the incumbent (*case 1*), we can now easily solve the repeated game in the other two cases where the incumbent is closer to the median voter than the challenger (*case 2*) and where candidates are symmetric (*case 3*).

From *lemma 1* we know that when candidates are asymmetric, there is a level of polarization which insures that an incumbent who is not located on the median voter, is accountable to voters. Hence, we can represent the game of *case 2* with the following picture :

¹⁹The reward here consists in the appointment of the successor j with same policy preference of the incumbent j who cannot re-run for office.

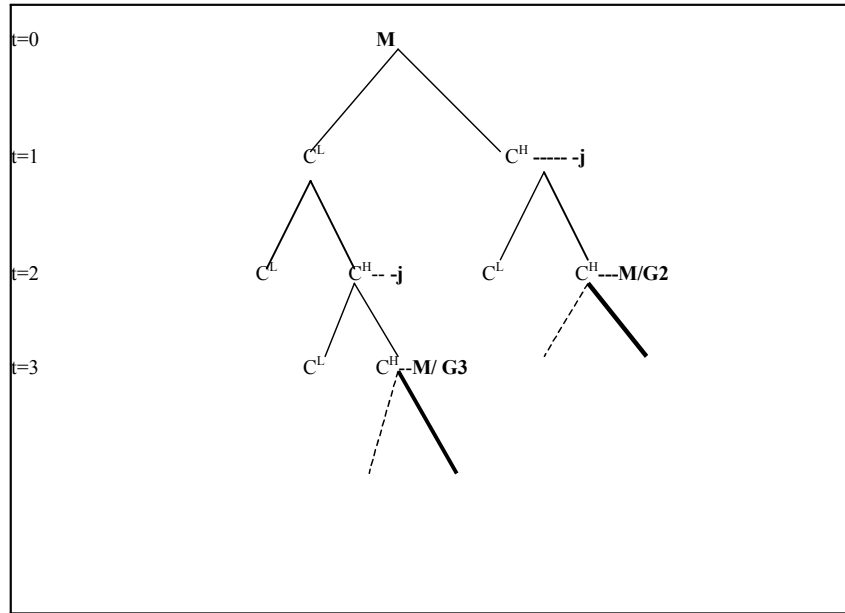


figure 3

Note that, the game starting after each deviation of the incumbent legislator M is identical to the game we solved in *case 1*. Therefore, as we can see from the game three, if the parameters of the model are such that *lemma 1* holds, clearly whenever the legislator M chooses the high cost policy C^H he will be permanently replaced by the challenger²⁰ $-j$ choosing the low cost policy C^L . Therefore, again to verify if for the incumbent M it is better to choose the high cost policy or the low cost policy, we can proceed backward determining his policy choice in the second mandate.

Lemma 3 states the conditions for the legislator M to be accountable to voters in his second (and last) mandate $t = 1$ ²¹:

Lemma 3

Given the candidates M and $-j$ and given the conjectured voting rule voting rule $\sigma_{Mj}^(\cdot)$, the incumbent legislator M in $t = 1$ will choose the policy $(1, C_1^L, a_M)$ if and only if $(a_{MM} - a_{Mj}) \geq \frac{B + \delta C^H}{\delta}$.*

²⁰We prove in the appendix that, when *lemma 2* holds, the equilibrium voting strategy in case 2 coincide with the equilibrium voting strategy $\sigma_{Mj}^*(\cdot)$ in case 1.

²¹For simplicity, here and whenever else it will appear, the low cost policy is normalized to zero, $C^L = 0$.

Note again²² that a legislator M can be accountable to voters in his last mandate if the polarization of the political race, $(a_{MM} - a_{Mj})$, is bigger than the threshold level, $\frac{B+\delta C^H}{\delta}$. Also, the threshold level when the incumbent is $j = M$ is bigger than the threshold level when the incumbent is $j \neq M$. This result is due to the fact that the incumbent M choosing C^H is permanently replaced by the challenger j choosing C^L . Therefore, although the incumbent M in the future (as a citizen) will suffer a loss on the ideological dimension, still on the monetary dimension he will get his most preferred policy C^L . In other words, the current choice of C^H generates an ideological loss but does not imply any future monetary loss. As a consequence, to be willing to choose C^L and forgo current private benefits B , the ideological loss must be important.

Moving now to the first period, $t = 0$, we can state the conditions supporting the low cost policy choice:

Lemma 4

Given the candidates M and $-j$ and given the conjectured voting rule voting rule $\sigma_{Mj}^(\cdot)$, the incumbent legislator M in $t = 0$ will choose the policy $(1, C_0^L, a_M)$ if and only if $(a_{MM} - a_{Mj}) \geq \frac{B+\delta^2 C^H}{\delta(1+\delta)}$.*

Again we can see that²³ the accountability of the incumbent legislator depends on the polarization on the political race, $(a_{MM} - a_{Mj})$ and on the discount rate δ . Therefore, for the incumbent to be accountable the polarization of the political race must be higher than a threshold level given by the expression $\frac{B+\delta^2 C^H}{\delta(1+\delta)}$. Note also that, the threshold level required in the first mandate (*lemma 4*) is lower than the threshold level required in the second mandate (*lemma 3*). Therefore we can conclude that when *lemma 4* holds, the incumbent M will choose C^L in both mandates.

To conclude, we analyze the *case 3*, where the candidates j and $-j$ are symmetric with respect to the median voter. In this case the equilibrium voting

²²The proof of the lemma is straightforward since we simply need to impose that the following inequality holds:

$$a_{MM} + \delta(a_{MM} - C^H) + \sum_{t=2}^{\infty} \delta^t a_{M-j} \geq a_{MM} + B + \sum_{t=1}^{\infty} \delta^t a_{M-j}$$

²³The prove of the lemma we we simply verify that the following inequality holds:

$$a_{MM} + \delta a_{MM} + \delta^2(a_{MM} - C^H) + \sum_{t=3}^{\infty} \delta^t a_{M-j} \geq a_{MM} + B + \sum_{t=1}^{\infty} \delta^t a_{M-j}$$

strategy is the same as in the asymmetric case where the incumbent is $j \neq M$ and the challenger is $j = M$ (*case 1*)²⁴.

Consequently, the condition insuring that the incumbent legislator will choose the low cost policy in both mandates is the same of *case 1* :

$$(a_{jj} - a_{j-j}) \geq \frac{B}{\delta}$$

As a final remark, since $\frac{B}{\delta} > 0$, then again the political race needs to be polarized for the incumbent legislator to be accountable to voters. Therefore, if the two legislators are symmetric but exactly located on the median voter, i.e. $j = M = -j$, the incumbent legislator cannot be held accountable.

6. Candidate selection

At this stage we have identified the equilibrium voting strategy and the equilibrium policy choice. The last step for the full characterization of the political equilibrium, is to find the equilibrium *candidate choice* of the two parties competing for elections.

From the previous analysis remember that when ideological distance between the incumbent j and the challenger $-j$, is strictly bigger than the threshold level $\frac{B+\delta C^H}{\delta}$ defined in *lemma 3*, then the incumbent legislator is accountable to voters in both mandates for any candidates' location.

Suppose that the incumbent party is L and the opponent is R . Using the previous lemmata, we can now state the following result:

Proposition 1

If $(a_{m_L m_L} - a_{m_L M}) \geq \frac{B+\delta C^H}{\delta}$, then the incumbent party L runs the median party member m_L , wins the election against any candidate of the opponent party R , and chooses the policy $(1, C_t^L, a_{m_L})$ in any period $t \geq 0$.

The previous proposition characterizes the political equilibrium highlighting the role of policy preferences. When the distance between the median party member and the median voter is sufficient to induce the incumbent legislator to choose the low cost policy, then the incumbent can be held accountable even in the event the opponent party would converge to the median voter in his candidate choice!

²⁴Proof in appendix.

Hence, in this case the polarization of policy preferences has two implications. The median party member will realize his most preferred policy. The median voter will not receive his most preferred policy on ideological grounds but he will discipline the incumbent on the monetary dimension of the policy.

If we assume that, although the cost is politically salient, the ideological distance between the median party member of the incumbent party, m_L , and the median voter, M , is not sufficient to guarantee accountability, then we obtain the following results:

Proposition 2

Suppose that $(a_{m_L m_L} - a_{m_L M}) < \frac{B + \delta C^H}{\delta}$ then the following holds:

- when $(a_{m_L m_L} - a_{m_L m_R}) = \frac{B + \delta C^H}{\delta}$, the incumbent party L runs the median party member m_L and the opponent party R runs the median voter M . The candidate of incumbent party, m_L , in $t = 0$ chooses the high cost policy and loses the elections. The candidate M chooses the high cost policy and stays in place in every period $t \geq 1$;
- when $(a_{m_L m_L} - a_{m_L m_R}) > \frac{B + \delta C^H}{\delta}$, let m_{L_2} be the first extreme candidate such that $|a_{m_L m_{L_2}} - a_{m_L M}| = |a_{m_L m_L} - a_{m_L m_R}|$. Then the incumbent party L runs the extreme candidate m_{L_2} and the opponent party R runs the median voter M . The candidate of the incumbent party, m_{L_2} , chooses the low cost policy and wins the elections in every period $t \geq 0$;
- when $(a_{m_L m_L} - a_{m_L m_R}) < \frac{B + \delta C^H}{\delta}$ and the discount factor is $\delta \leq \frac{1}{2}$, the incumbent party L runs the median party member m_L and the opponent party R runs the median party member m_R . The candidate m_L in $t = 0$ chooses the high cost policy and is permanently replaced by the challenger m_R choosing the high cost policy in any period $t \geq 1$.
- when $(a_{m_L m_L} - a_{m_L m_R}) < \frac{B + \delta C^H}{\delta}$ and the discount factor is $\delta > \frac{1}{2}$ two cases arise. If $\frac{\delta}{1 - \delta}(a_{m_L M} - a_{m_L m_R}) \leq (a_{m_L m_L} - a_{m_L m_R})$, then the political equilibrium coincides with the previous case with $\delta \leq \frac{1}{2}$. On the other hand, if $\frac{\delta}{1 - \delta}(a_{m_L M} - a_{m_L m_R}) > (a_{m_L m_L} - a_{m_L m_R})$, then the incumbent party L runs the median candidate M and the opponent party R runs the median party member m_R . The candidate M chooses the high cost policy and stays in place in every period $t \geq 0$.

Intuitively, the proposition states that if the distance the median party member of the incumbent party, m_L , and the median voter, M , is not sufficient to guarantee accountability i.e. $(a_{m_L m_L} - a_{m_L M}) < \frac{B + \delta C^H}{\delta}$, but the distance between the median party member of the incumbent party, m_L , and the median party member of the opponent party m_R is sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L m_R}) > \frac{B + \delta C^H}{\delta}$, then the incumbent party can delegate the policy choice to a more extreme policy type than himself so as to obtain accountability. In other words, when the opponent party locates his candidate on the median voter M , the incumbent party, choosing a more extreme type than himself, can locate his own candidate sufficiently far away from the median voter to obtain an accountable candidate. However, this is an optimal strategy provided that the median party member is closer to the first extreme candidate that guarantees accountability than to the median voter M . In fact when m_L delegates to a more extreme candidate, he is facing an ideological loss, although this will guarantee him to win the elections. Clearly if the median party member is closer to the median voter than to the extreme candidate, than for him it is better to loose the election and have a median opponent in place than to win the election with an extreme candidate. Geometrically, we can show that the when the distance between the median party members is sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L m_R}) > \frac{B + \delta C^H}{\delta}$, then the distance between the median party member and the first extreme candidate that will be accountable is not bigger than the distance from the median voter. On the other hand, when the distance between the median party members either necessary or not sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L m_R}) \leq \frac{B + \delta C^H}{\delta}$, than the median party member is closer to the median voter than to the first extreme candidate that will be accountable. Therefore, in this case, delegation to a more extreme candidate to obtain accountability is not optimal.

Finally, let's consider the following case. Suppose that the distance between the median party member m_L and the median voter M is such that if m_L , choosing the low cost policy, could be reelected, then he would prefer to choose the low cost policy and be in office instead of obtaining the private benefits the high cost policy and loose the possibility to determine the ideological dimension of the policy. In other words, the incumbent legislator cares enough about the ideological dimension to forgo private benefits. However, suppose that, given the distance between m_L and M , the median voter M would rather prefer a median candidate choosing the high cost policy than a the more extreme candidate m_L choosing a

low cost policy. In this example, the distribution of preferences is such that the polarization is enough to provide incentives to legislators but the distribution is too polarized for the median voter to reward a legislator for choosing the low cost policy. Given this distribution of preferences, we can show the following result:

Proposition 3

Suppose that $a_{m_L M} - C^L < a_{MM} - C^H$ but there exist a candidate m_{L_2} with most preferred policy type a_{L_2} such that $a_{m_{L_2} M} - C^L > a_{MM} - C^H$.

If $(a_{m_{L_2}} - a_{MM}) \geq \frac{B + \delta C^H}{\delta}$, then the incumbent party L runs the candidate m_{L_2} and the opponent party R runs the median party member m_R . The candidate m_{L_2} chooses the low cost policy and wins the elections in every period $t \geq 0$.

Intuitively the proposition says that if the median party member, m_L , is "too far" from the median voter, M , and there is a less extreme candidate m_{L_2} that can guarantee accountability, then the median party member prefers to delegate to the less extreme candidate.

7. Polarization of preferences, political parties and polarization of policies

Using the main propositions, we can try to analyze the effects of party competition on the different dimensions of the policy, given the underlying preferences of the individuals. The main objective of parties is to control the ideological dimension of the policy choosing a candidate whose ideological preferences represent the preferences of the majority of the party members. To accomplish this goal, parties rely on majority voting to select candidates. Hence, the median party member has the possibility to choose a candidate with his same preferences. However, as the control of the ideological dimension of the policy over time depends on the elections, when parties choose candidate they have also to consider the preferences of the median voter. The median voter decides whether to reappoint an incumbent candidate evaluating his policy choice on the ideological dimension and on the monetary dimension. The weight given to these two dimensions is crucial for the equilibrium policy outcome. In particular, if the ideological dimension is politically salient, the median voter will always obtain his most preferred type of policy but he will always pay a high cost for it. Hence we say that in this case the party competition cannot help to solve the accountability problem. On

the other hand, if the monetary dimension of the policy is politically salient, i.e. if the median voter is willing to sacrifice part of his utility coming from the ideological dimension to obtain a low cost policy, then the party competition can help to discipline legislators on the monetary dimension. However, the ability of the electoral mechanism to discipline legislators depends on the distribution of preferences on the ideological dimension. In particular, from *proposition 1* we learn that, if the distance between the median party member and the median voter is sufficiently high to provide incentives to legislators to forgo monetary transfers in order to maintain the control of the ideological dimension of the policy, then accountability can be achieved. On the other hand, if the distance between the median party member and the median voter is not sufficient to induce an incumbent legislator to forgo private benefits, the median party member may solve the accountability problem choosing a more extreme policy type than himself. In other words, to gain an everlasting control on the ideological dimension of the policy it is necessary to discipline candidates. Therefore, if the median party member cannot be held accountable because he is too closed to the opponent to be willing to forgo private benefits and stay in office, then for the median party member it may be optimal to delegate the policy choice to a more extreme candidate. In this case therefore we may say that parties use polarization of candidates as a *discipline device*. However polarization as a discipline device may work only if the median party member gains more from having in power his candidate (although more extreme than himself) than he would from the candidate of the opponent party member. In fact, if he is closer to the median voter than to the first extreme candidate that can guarantee accountability, then he would rather prefer the median candidate of the other party than the extreme candidate of his own party. This concept is summarized by the *proposition 2*, where we say that the median party member decides to candidate a more extreme candidate that guarantees accountability when the distance between the median party member and the more extreme candidate is smaller than the distance between the median party member and the median voter.

Finally, when the preferences are very polarized so that the median voter is not willing to reelect the median party member of the incumbent party, the incumbent party in order to win the elections is again willing to sacrifice partly his ideological preferences choosing a less extreme candidate than himself.

As we have noted already, our result depends crucially on the fact that the game is repeated. This characteristic of the game also provides useful insight on the function of parties. In our model, the role of a party as a long-lived institutions

is to insure the selection of a precise type of candidate, i.e. the implementation of a given ideology. Hence, even if candidates are short-lived and cannot stay in place for ever, still because they are policy motivated on the ideological dimension of the policy, they can be loyal to the party so as to obtain a candidate with their same preferences in place (at least for one period) in the future. If an incumbent legislator in his last mandate does not believe that choosing a low cost policy he can enhance the electoral prospects of the candidate belonging to his party, then he will never choose the low cost policy. On the other hand, we have shown that if the low cost policy is not chosen in the second mandate, then this policy will not be chosen in the second mandate neither. Therefore, we conclude that party loyalty is an key factor for legislators' accountability.

An important *caveat* is necessary at this point: we assume the existence of parties. In other words, this is not a model of *party formation* but a more simple model of *party strategy*. This is a limit of our model since party formation is the first mechanism of aggregation of individual preferences and therefore an important part channel through which individual preferences are translated into policy choices. However, party formation is not the unique mechanism through which individual preferences are represented in the political arena. Once formed parties are relatively stable institutions that can generate diverse candidate locations due to their internal democratic processes. Therefore, the focus of this paper is precisely on the effects of these internal mechanism of candidate selection on final policy outcomes. We have seen in our model that even assuming that parties already exists, the choice of the party location in the policy space is not a trivial problem and the set of the equilibrium locations is quite rich.

8. Conclusions

Parties are associations of citizens that play a fundamental role in representative democracies. They are the primary actor of the electoral competition since they choose the candidates of the political race. Furthermore, choosing candidates, parties may influence policies. Clearly, how parties can actually affect policies through the selection of candidates crucially depends on the candidates preferences and on the relationship between parties and candidates.

In this paper we propose a model where legislators choose a multidimensional policy. We assume that candidates are policy motivated on the ideological dimension of the policy but they are not on the monetary dimension. In particular, a legislator can obtain private benefits from a policy that goes against the interests

of the citizens on the monetary dimension. In this set up, we show how parties as long lived institutions can provide incentives to short-lived candidates to reject lobby transfers and choose policies in the interests of the majority of their constituents.

The role of a party as a long-lived institutions is to insure the selection of a precise type of candidate, i.e. the implementation of a given ideology. Hence, even if candidates are short-lived and cannot stay in place for ever, still because they are policy motivated on the ideological dimension of the policy, they can be loyal to the party so as to obtain a candidate with their same preferences in place in the future. However, loyalty to the party is costly for incumbent legislators, since in order to enhance the reelection of future candidates of the same party they have to forgo private benefits. Hence, an incumbent legislator is willing to forgo current private benefits for future ideology provided that the ideological gain is bigger than the benefit loss. The ideological gain depends on the ideological distance between the incumbent legislator and the challenger of the opponent party, since the bigger the distance, the higher the loss from being replaced by the challenger. As a consequence, parties can use strategically the polarization of the political race to provide incentives to candidates.

The strategic use of party polarization implies that the polarization of *preferences* and the polarization of the *political race* do not necessarily go in the same direction. For a medium degree of polarization, the political race tends to reflect the polarization of preferences. However, for more extreme distributions we can obtain different results. Hence for example, when the *polarization* of preferences is *low*, parties may choose to increase the polarization of the political race in order to provide incentives to candidate so that the candidate may be more extreme than the median party member. On the other hand, if the *polarization* of preferences is *high*, parties may choose less extreme candidates to get closer to the median voter. Hence, again the candidate will not represent the preferences of the median party member. In general, the polarization of the political race seems to be a compromise between policy preferences of party members and electoral goal and parties may use strategically polarization to provide incentives.

This model is a further step toward the understanding of the role of political parties in representative democracies. We propose a relatively simple model of party where the selection of the candidate relies on majority voting and party polarization is the only instrument that parties can use to provide incentives and win elections. However, parties can typically use other instruments and in particular campaign spending to influence the electoral outcome. Also parties are

complex institutions that require consistent funding for their ordinary functioning. Transfers from candidates to parties can be an important source of party financing. Therefore, as money seems to play an increasingly important role in politics, we may expect that parties may choose their candidates looking not only to their ideological preferences but also considering their ability to obtain transfers from the private sector. Hence, a natural extension of our model would be the introduction of transfers from candidates to parties. Insofar as transfers may be used by parties to increase their chances of winning elections, we may expect that parties can trade ideology for money.

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Appendix

Candidate choice: each median party member can choose a candidate of his own type or can choose either a more extreme or a less extreme type. Depending on the candidate locations, a candidate can be accountable or not and hence can be re-elected or not.

Proposition 1

Suppose that $|a_{m_L m_L} - a_{m_L M}|$ is sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L M}) \geq \frac{B + \delta C^H}{\delta}$. Let's define $n = \frac{1}{1 - \delta}$.

The payoff matrix of the two party members can be written as follows:

	M	m_R	m_{R_1}
M	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$
m_L	$na_{m_L m_L}, na_{m_R m_L}$	$na_{m_L m_L}, na_{m_R m_L}$	$na_{m_L m_L}, na_{m_R m_L}$
m_{L_1}	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$

we can see that m_L is dominant strategy for the incumbent party, while the other median party member is indifferent among all the possible party locations. Since, for any challenger party location, the incumbent party choosing m_L will be accountable. Then we conclude that, the incumbent party chooses m_L who implements the policy $(1, C_t^L, a_{m_i})$ in every period t winning the election against any candidate of the opponent party.

Proposition 2

Suppose that $|a_{m_L m_L} - a_{m_L m_R}|$ is necessary for accountability.

First we prove that m_L is closer to the median voter, M , than to the first extreme party member that guarantees accountability, m_{L_1} . In fact, if $|a_{m_L m_L} - a_{m_L m_R}|$ is necessary for accountability, than for m_{L_1} to guarantee accountability it must be that:

$$|a_{m_L m_{L_1}} - a_{m_L M}| \geq |a_{m_L m_L} - a_{m_L m_R}| \quad (8.1)$$

we can show that inequality 8.1 implies that m_L is closer to the median voter, M , than to m_{L_1} .

claim 1: when $|a_{m_L m_{L_1}} - a_{m_L M}| = |a_{m_L m_L} - a_{m_L m_R}|$, then

$$|a_{m_L m_L} - a_{m_L m_{L_1}}| = |a_{m_L m_L} - a_{m_L M}|$$

proof of claim 1: given a symmetric distribution, we have that

$$|a_{m_L m_L} - a_{m_L m_R}| = |a_{m_L m_L} - a_{m_L M}| + |a_{m_R m_R} - a_{m_R M}|$$

and

$$|a_{m_L m_{L_1}} - a_{m_L M}| = |a_{m_L m_L} - a_{m_L M}| + |a_{m_L m_L} - a_{m_L m_{L_1}}|$$

if $|a_{m_L m_{L_1}} - a_{m_L M}| = |a_{m_L m_L} - a_{m_L m_R}|$, then

$|a_{m_L m_L} - a_{m_L m_{L_1}}| = |a_{m_R m_R} - a_{m_R M}|$ and by symmetry of the distribution

$$|a_{m_L m_L} - a_{m_L m_{L_1}}| = |a_{m_R m_R} - a_{m_R M}| = |a_{m_L m_L} - a_{m_L M}|$$

hence

$$|a_{m_L m_L} - a_{m_L m_{L_1}}| = |a_{m_L m_L} - a_{m_L M}|$$

claim 2: if $|a_{m_L m_{L_1}} - a_{m_L M}| > |a_{m_L m_L} - a_{m_L m_R}|$, then

$$|a_{m_L m_L} - a_{m_L m_{L_1}}| > |a_{m_L m_L} - a_{m_L M}|$$

proof of claim 2: in fact, using the proof of claim 1 it is immediate to verify that:

$$| a_{m_L m_L} - a_{m_L m_{L_1}} | > | a_{m_R m_R} - a_{m_R M} | = | a_{m_L m_L} - a_{m_L M} |$$

hence we conclude that if $| a_{m_L m_{L_1}} - a_{m_L M} | \geq | a_{m_L m_L} - a_{m_L m_R} |$ then $| a_{m_L m_L} - a_{m_L m_{L_1}} | \geq | a_{m_L m_L} - a_{m_L M} |$
(i.e. the median voter is closer to M than to m_{L_1} .)

The same logic applies when $| a_{m_L m_L} - a_{m_L m_R} |$ is not sufficient for accountability, with the difference that in this case m_{L_1} to guarantee accountability it must be that:

$$| a_{m_L m_{L_1}} - a_{m_L M} | > | a_{m_L m_L} - a_{m_L m_R} | \quad (8.2)$$

hence with *strict inequality* the previous proof holds and we can conclude that again the median party member is closer to the median voter than to the first extreme accountable candidate.

similarly , if $| a_{m_L m_L} - a_{m_L m_R} |$ is sufficient for accountability, then

$$| a_{m_L m_{L_1}} - a_{m_L M} | \leq | a_{m_L m_L} - a_{m_L m_R} | \quad (8.3)$$

therefore we can prove the following:

claim 3: if $| a_{m_L m_{L_1}} - a_{m_L M} | \leq | a_{m_L m_L} - a_{m_L m_R} |$, then $| a_{m_L m_L} - a_{m_L m_{L_1}} | \leq | a_{m_L m_L} - a_{m_L M} |$

proof of claim 3: given a symmetric distribution, we have that

$$| a_{m_L m_L} - a_{m_L m_R} | = | a_{m_L m_L} - a_{m_L M} | + | a_{m_R m_R} - a_{m_R M} |$$

and

$$| a_{m_L m_{L_1}} - a_{m_L M} | = | a_{m_L m_L} - a_{m_L M} | + | a_{m_L m_L} - a_{m_L m_{L_1}} |$$

hence,

$$| a_{m_L m_L} - a_{m_L m_{L_1}} | \leq | a_{m_R m_R} - a_{m_R M} | = | a_{m_L m_L} - a_{m_L M} |$$

therefore, the median party member is closer to the first extreme accountable candidate than to the median voter.

Now using the previous results we can determine the equilibrium party location from the parties' payoff matrix:

Case 1: suppose the following:

either $|a_{m_L m_L} - a_{m_L M}|$ **is not sufficient for accountability, i.e.** $(a_{m_L m_L} - a_{m_L M}) < \frac{B + \delta C^H}{\delta}$;

or $|a_{m_L m_L} - a_{m_L m_R}|$ **is necessary for accountability, i.e.** $(a_{m_L m_L} - a_{m_L m_R}) = \frac{B + \delta C^H}{\delta}$;

or $|a_{m_L m_L} - a_{m_L M}|$ **is necessary for accountability, i.e.** $(a_{m_L m_L} - a_{m_L M}) = \frac{B + \delta C^H}{\delta}$;

	M	m_R	m_{R_1}
M	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$
m_L	$a_{m_L m_L} + (n - 1) a_{m_L M},$ $a_{m_R m_L} + (n - 1) a_{m_R M}$	$na_{m_L m_L}, na_{m_R m_L}$	$na_{m_L m_L}, na_{m_R m_L}$
m_{L_1}	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$

by iterated dominance, we can eliminate strategy M for party L and strategies m_R and m_{R_1} for party R . Hence the equilibrium location can be either (m_L, M) or (m_{L_1}, M) .

Given that we have established that the median party member is closer to M than to m_{L_1} , we know that $a_{m_L M} - a_{m_L m_{L_1}} > 0$ which implies that

$$a_{m_L m_L} + (n - 1) a_{m_L M} > na_{m_L m_{L_1}}$$

Hence the equilibrium party location is (m_L, M) . In the first period $t = 0$, the incumbent m_L chooses the policy $(1, C_0^H \cdot a_{m_L})$ and from period $t \geq 1$ he will be permanently replaced by the challenger M choosing the policy $(1, C_t^H \cdot a_{m_L})$.

This also proves proposition 4 when $|a_{m_L m_L} - a_{m_L M}|$ is necessary for accountability.

Case 2: suppose the following:

$|a_{m_L m_L} - a_{m_L M}|$ **is not sufficient for accountability, i.e.** $(a_{m_L m_L} - a_{m_L M}) < \frac{B + \delta C^H}{\delta}$

and $|a_{m_L m_L} - a_{m_L M}|$ **is not sufficient for accountability and** $|a_{m_L m_L} - a_{m_L m_R}|$ **is not sufficient for accountability, i.e.** $(a_{m_L m_L} - a_{m_L m_R}) < \frac{B + \delta C^H}{\delta}$;

	M	m_R	m_{R_1}
M	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$
m_L	$a_{m_L m_L} + (n-1)a_{m_L M},$ $a_{m_R m_L} + (n-1)a_{m_R M}$	$a_{m_L m_L} + (n-1)a_{m_L m_R},$ $a_{m_R m_L} + (n-1)a_{m_R m_R}$	$na_{m_L m_L}, na_{m_R m_L}$
m_{L_1}	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$

by iterated dominance, we can eliminate strategies M and m_{R_1} for party R . Hence the equilibrium location can be either (m_L, m_R) or (m_{L_1}, m_R) or (M, m_R) .

Given that we have established that the median party member is closer to M than to m_{L_1} , we know that $na_{m_L M} - na_{m_L m_{L_1}} > 0$. This implies that we can eliminate the location (m_{L_1}, m_R) . Hence, the equilibrium locations can be either (m_L, m_R) or (M, m_R) . Note that m_L is preferred to M when the following inequality holds:

$$a_{m_L m_L} + (n-1)a_{m_L m_R} \geq na_{m_L M} \Leftrightarrow n(a_{m_L M} - a_{m_L m_R}) \leq (a_{m_L m_L} - a_{m_L m_R}).$$

Since $(a_{m_L m_L} - a_{m_L m_R}) > (a_{m_L M} - a_{m_L m_R})$, then for $n \leq 1$, clearly the previous inequality is verified. Therefore if $0 < \delta \leq \frac{1}{2}$, then m_L is preferred to M . On the other hand, if $\frac{1}{2} < \delta < 1$ since $n > 1$, then the previous inequality is not necessarily verified. Therefore, we conclude that:

if $n(a_{m_L M} - a_{m_L m_R}) > (a_{m_L m_L} - a_{m_L m_R})$ then the equilibrium location will be (M, m_R) .

if $n(a_{m_L M} - a_{m_L m_R}) \leq (a_{m_L m_L} - a_{m_L m_R})$ then the equilibrium location will be (m_L, m_R) .

Under both locations the high cost policy will always be chosen. However under the location (M, m_R) the incumbent M will be in place for ever, while under the location (m_L, m_R) , the incumbent m_L is in place in $t = 0$ and the challenger m_R will be in place for $t \geq 1$.

Case 3: Suppose the following:

$|a_{m_L m_L} - a_{m_L M}|$ is not sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L M}) < \frac{B+\delta C^H}{\delta}$

and $|a_{m_L m_L} - a_{m_L m_R}|$ is sufficient for accountability, i.e. $(a_{m_L m_L} - a_{m_L m_R}) > \frac{B+\delta C^H}{\delta}$;

	M	m_R	m_{R_1}
M	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$
m_L	$a_{m_L m_L} + (n-1)a_{m_L M},$ $a_{m_R m_L} + (n-1)a_{m_R M}$	$na_{m_L m_L}, na_{m_R m_L}$	$na_{m_L m_L}, na_{m_R m_L}$
m_{L_1}	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$	$na_{m_L m_{L_1}}, na_{m_R m_{L_1}}$

by iterated dominance, we can eliminate strategy M for party L and strategies m_R and m_{R_1} for party R . Hence the equilibrium location can be either (m_L, M) or (m_{L_1}, M) .

Given that we have established that the median party member is closer to m_{L_1} than to M , we know that $a_{m_L M} - a_{m_L m_{L_1}} < 0$ which implies that

$$a_{m_L m_L} + (n-1)a_{m_L M} < na_{m_L m_{L_1}}$$

Hence the equilibrium party location is (m_{L_1}, M) . Given this equilibrium location, the incumbent m_{L_1} will choose the policy $(1, C_t^L; a_{m_{L_1}})$ in every period $t \geq 0$ and will be in place for ever.

Proposition 3

Suppose that $a_{m_L M} - C^L < a_{MM} - C^H$ but there exist a candidate m_{L_2} with most preferred policy type a_{L_2} such that $a_{m_{L_2} M} - C^L > a_{MM} - C^H$.

Assume that $(a_{m_{L_2} M} - a_{MM}) \geq \frac{B+\delta C^H}{\delta}$.

	M	m_{R_2}	m_R
M	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$	$na_{m_L M}, na_{m_R M}$
m_{L_2}	$na_{m_L m_{L_2}}, na_{m_R m_{L_2}}$	$na_{m_L m_{L_2}}, na_{m_R m_{L_2}}$	$na_{m_L m_{L_2}}, na_{m_R m_{L_2}}$
m_L	$a_{m_L m_L} + (n-1)a_{m_L M},$ $a_{m_R m_L} + (n-1)a_{m_R M}$	$a_{m_L m_L} + (n-1)a_{m_L m_{R_2}},$ $a_{m_R m_L} + (n-1)a_{m_R m_{R_2}}$	$a_{m_L m_L} + (n-1)a_{m_L m_R},$ $a_{m_R m_L} + (n-1)a_{m_R m_R}$

by iterated dominance, we can eliminate strategy M and m_L for party L and strategy M and m_{R_2} for party R . The equilibrium location will therefore be (m_{L_2}, m_R) , the incumbent m_{L_2} will choose the low cost policy and will be in place for any $t \geq 0$.

Equilibrium voting strategy

• case 1

Let $j \neq M$ be the incumbent legislator in $t = 0$ and let $-j = M$ be the challenger.

We want to show that the conjectured voting strategy:

$\sigma_{Mj}^t(h^t) = 1$ if the legislator j has always chosen C^L in the past;

$\sigma_{Mj}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past;

is an equilibrium voting strategy. We have already shown in the paper that the punishment, $\sigma_{Mj}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past, is an equilibrium voting strategy.

Now we claim that if *lemma 1* holds, i.e. if $(a_{jj} - a_{jM}) \geq \frac{\beta}{\delta}$, then the conjectured strategy:

$\sigma_{Mj}^t(h^t) = 1$ if the legislator j has always chosen C^L in the past
is an equilibrium voting strategy.

Suppose that this is not the case and suppose that after the legislator j has chosen C^L in his second mandate, the voter does not carry on his reward. If this happens, the equilibrium will permanently switch to a legislator M choosing C^H for ever. Hence, the payoff for the median voter who does not reward a legislator j will be:

$$V_M^1 \left(\sigma_{Mj}^1(1, C^L, a_j) = 0, c_j^t(e^t) \right) = \sum_{t=0}^{\infty} (a_{MM} - C^H)$$

On the other hand, if the median voter carries on his reward, given that $(a_{jj} - a_{jM}) \geq \frac{\beta}{\delta}$, the equilibrium will preeminently be an incumbent j choosing C^L . The payoff from carrying on the reward is:

$$V_M^1 \left(\sigma_{Mj}^1(1, C^L, a_j) = 1, c_j^t(e^t) \right) = \sum_{t=0}^{\infty} (a_{Mj} - C^L)$$

therefore given the assumption of political salience the voter will prefer to carry on the reward in the second mandate. If we move to $t = 1$, under the same parameter restrictions, we have exactly the same payoffs from carrying on the reward or deviating. Hence we conclude that the conjectured voting strategy is an equilibrium voting strategy.

• **case 2**

Let $j = M$ be the incumbent legislator in $t = 0$ and let $-j \neq M$ be the challenger. We want to show that the conjectured voting strategy:

$\sigma_{MM}^t(h^t) = 1$ if the legislator j has always chosen C^L in the past;
 $\sigma_{MM}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past;
is an equilibrium voting strategy.

The proof of the first part of the conjecture is trivial since the median voter always prefer a legislator

$j = M$ who chooses C^L to any other legislator $-j \neq M$ choosing any policy C .

For the second part, i.e. $\sigma_{MM}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past, note that if in at the end of the second mandate the median voter does not carry on the punishment, then from $t \geq 2$ onwards the equilibrium will preeminently be the incumbent $j = M$ choosing C^H . On the other hand, if *lemma 3* holds, i.e. if $(a_{MM} - a_{M-j}) \geq \frac{\beta + \delta C^H}{\delta}$, then carrying on the punishment the equilibrium will be the

incumbent $j = M$ choosing C^L , hence clearly the median voter will prefer to carry on the punishment in $t = 2$. Similarly, if we move to $t = 1$, under the same parameter restrictions, we have exactly the same payoffs from carrying on the punishment or deviating. Hence we conclude that the conjectured voting strategy is an equilibrium voting strategy.

•case 3

Let j be the incumbent legislator in $t = 0$ and let $-j$ be the challenger, where j and $-j$ are symmetric with respect to M . We want to show that the conjectured voting strategy:

$\sigma_{Mj}^t(h^t) = 1$ if the legislator j has always chosen C^L in the past;
 $\sigma_{Mj}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past;
 is an equilibrium voting strategy.

The proof for the first part of the conjecture is trivial. Since the median voter is indifferent between the two candidates on the ideological dimension, he cannot lose from carrying on his reward.

Let's consider now the second part of the conjecture, i.e. $\sigma_{Mj}^t(h^t) = 0$ if the legislator j has ever chosen C^H in the past. This strategy prescribes to permanently replace the incumbent j with the opponent $-j$ if ever the incumbent deviates from the low cost policy. Note that the median voter is indifferent between the two candidates on the ideological dimension. Hence, the best strategy must be the strategy that is most likely to guarantee the policy C^L . The policy C^L can be obtained punishing the incumbent with no re-election. Note also that, the voting strategy can prescribe several type of punishment. In particular, our conjectured strategy prescribes the most severe punishment for the incumbent j since the incumbent j will be banned from office for ever after he has chosen C^H . Any other strategy prescribing the reelection of j in the future, would be a less strong punishment. For example, if the voting rule would be "don't reelect the incumbent if he has chosen the high cost policy", then the two legislators types j and $-j$ will alternate in office. Alternatively, if given the indifference, the median voter will randomize, then the incumbent type j will always have a chance to be in office in the future. Hence we conclude that if the median voter faces two candidates among which he is indifferent, then the best voting rule is to always reelect the last incumbent.