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## Moderation in Proportional Systems: Coalitions Matter

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**Abstract:**

This paper examines the role of the coalition formation process in a proportional system. It models its impact on the voters (who maximize their expected utilities) and the parties (who choose their platforms in a Nash game). In contrast with the intuitive idea that proportional systems represent "proportionally", I show that a proportional system with minimal range coalitions leads to party convergence towards the median of the political spectrum. Indeed, a political party's prospects of power are better when it is more likely to find ideological partners, i.e. when it is not ideologically isolated. In contrast, if coalitions are formed according to a minimum winning coalition rule a la Riker, any policy can be implemented in equilibrium.

**Keywords:** electoral system, outcome simplex, strategic voting, pivot probability, positional equilibrium, minimal range coalition, minimal winning coalition, median voter theorem.

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### *1. Introduction*

This paper examines the role of the coalition formation process in a proportional system. It models its impact on the behaviors of the voters and on the strategies of the parties. In its prediction of policy outcomes, it confronts and contrasts two simple coalition settings: minimal range coalitions, where coalitions are thought to group political parties in terms of ideological proximity, and minimum size coalitions, where parties care more about theirs shares of the cake of power than about policy platforms per se.

According to the former theory (*minimal range theory*, a special case of *policy theory*, Leiserson (1966) and Axelrod (1970)), a party wants to belong to a coalition whose ideal policy is as close as possible to its own ideology. It predicts that the winning coalition is the coalition reaching more than 50% whose ideological distance between the more different parties (i.e. whose range) is as small as possible.

I show that minimal range coalitions lead to party convergence towards the median of the political spectrum. This median voter result is in sharp contrast with the commonsensical idea that proportional systems represent "proportionally", i.e. that the major ideological groups are represented by ideologically diverse parties, indeed one of the main arguments in favour of this type of systems.

To the contrary, I show that proportional systems can be used to guarantee moderate outcomes, but this at the expense of political diversity. The logic behind the result is as follows. Unless it does not need to form a coalition – meaning it can appeal to an absolute majority of voters, i.e. it occupies the median position – a political party's power prospects are good only when it is likely to find ideological partners with whom to form an absolute majority, i.e. when it is not ideologically isolated.

The key to the result lies *not* in proportionality in itself but in the coalition rule and the incentives it gives to the parties. It is in anticipation to the way the winning coalition is going to emerge in the different ideological party configurations that the parties all end up picking the middle-of-the-road policy. An isolated party wants either to move to the center to obtain an absolute majority or move closer to another party to form a winning coalition with it after the election.

Indeed, if coalitions are formed according to a different rule, convergence to the median is not guaranteed. I show that many configurations and policy outcomes are possible in a minimum winning coalition rule a la Riker. This rule was developed with *size theory* (Von Neumann and Morgenstern (1947) and Riker (1962)). It asserts that parties want to have a maximal influence in the decision making and therefore want to team with a party whose score is as small as possible. In that case, there is no special advantage to being gregarious and focal parties can win even if extremist as in other electoral systems.

These results are striking, both in their contrast to each other and in their contrast to common wisdom. They also fit with moderation observed in many countries with proportional systems. As closed minimal range theory is empirically the most satisfying of all existing theories in this field and is in line with recent emphasis on the partisan behavior of parties (see Alesina and Rosenthal (1995)), the median voter result seems especially appealing.[1]

I believe that the set-up in which I obtain these results is both quite general and convincing.

The model mainly uses a geometric interpretation of Myerson and Weber (1993). The general technical model can be found in a companion paper. Here we adapt it specifically to proportional systems. Here are the key assumptions.

The set of parties is given and their ideologies are endogenous. They choose their platforms in a Nash game. Voters are rational even if unable to directly interact with each other. They care for the policy outcomes, not for candidates in themselves, and therefore vote by taking into account the relative probabilities of being pivotal between outcomes. Which outcomes are possible and which outcomes can compete against each other for victory depend on the rule of coalition building.[2] Geometric analysis shows what pivot probabilities make sense. The relative weights of these probabilities are given exogeneously by social perceptions, forming common beliefs.

The rest of the paper is organized as follows. Section 2 presents an informal model for a proportional system with minimal range coalition and its concept of positional equilibrium. Section 3 derives results a median voter result. Section 4 considers minimal winning coalitions and derives a very different result. Section 5 concludes and discusses related literature.

## *2. The Set-Up*

The political space is a one-dimensional [0, 100] segment. The electorate's bliss points are uniformly and discretely distributed on {0, 1, ... 100}. Voters vote for one party. There are three parties, who choose their positions in order to maximize their chances of winning the election.

The utility for a voter at position t if policy $x_i$ is implemented takes the usual quadratic form: $u_i(t) = -(x_i - t)^2$.

In a purely proportional system, representation of the parties in the Parliament is proportional to their scores in the election. We assume no minimum threshold of representation.

*Institutional assumption*: in order for a policy to be implemented, it must be approved by an absolute majority of votes in the Assembly.

A *proportional system* is an electoral system in which a party forms a government alone if it gets more than half of the votes. If no party passes the critical score of 50%, a coalition of parties whose scores sum to more than 50% forms a government.

As there are three parties, if no party gets more than half of the votes, any coalition of two parties is a winning coalition (as the complement of the third party, which has less than 50%). To find out which coalition will be in power and which policy will be implemented, we must make assumptions on the process of coalition formation and on the position of a coalition[3].

This section considers a *proportional system with ideological coalitions*: the two parties whose ideologies are the closest form the winning coalition.

Regarding the *policy positioning* of the coalition, we assume that the coalition policy will be at the middle of the positions of the two winning parties, regardless of their respective scores[4].

The voter does not only take her preferences into account when voting, but also beliefs about the serious races for victory in order not to waste her vote. Pivot-probability $p_{ij}$ is the probability that outcomes i and j may be in a sufficiently close race for first place that her ballot alone could swing the election from one to the other. The $p_i$s are common beliefs formed via the pre-election polls (their exact formation process is exogenous). Pivot-probabilities are normalized so as to sum up to 1.

What are the relevant pivot-probabilities in this setting? The probability that parties 1 and 2 be in contention for first place, $p_{12}$, is not an appropriate concept: the question is not to know which party comes first, but whether any party will be strong enough to obtain an absolute majority or whether there will be a minimal range coalition.

An example of an appropriate pivot probability regards the race between party 1 reaching 50% and winning alone and a coalition government, leading to the minimal range coalition. Possible races are thus not between parties, but between outcomes. Here there are four possible outcomes: 1 wins alone (outcome 1), 2 wins alone (outcome 2), 3 wins alone (outcome 3), and the coalition of the ideologically closest two parties wins (mrc for minimal range coalition).

This can be seen geometrically by noticing that the percentage scores of the candidates sum up to one. If $S_i$ denotes the percentage score of party i, then $S_1 + S_2 + S_3 = 100\%$: the set of all possible results is a two-dimensional simplex.

In a proportional system with ideological coalitions, for any positioning of the candidates, there are four outcome zones, as illustrated in figure 1.

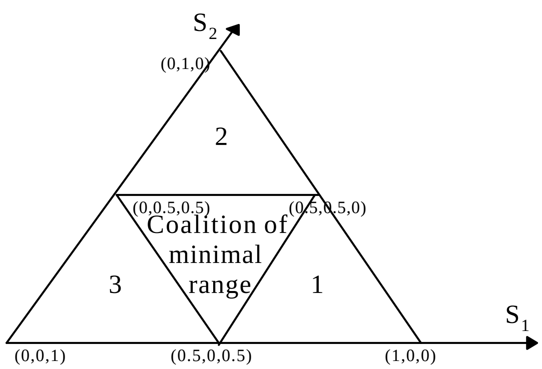


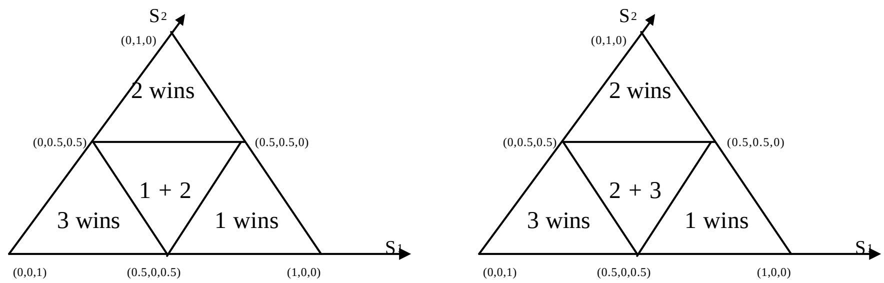

**Figure 1: The outcome simplex when coalitions are of minimal range**

The winning coalition includes the closest two parties.

What outcomes can be in close races for implementation? As just explained, $p_{1,mrc}$ should be considered. We exclude $p_{12}$, as both parties being in contention for 50% is not possible unless party 3 gets no vote at all. The three possibly positive pivot-probabilities will be $p_{1,mrc}$, $p_{2,mrc}$ and $p_{3,mrc}$.

How do these pivot-probabilities influence the strategic behavior of the voters? Consider for example $p_{1,mrc}$ with other pivot-probabilities negligible. Voters preferring outcome 1 to the minimal range coalition naturally vote for 1, while all others, in the hope to prevent 1 from reaching 50%, vote for either 2 or 3.

How do these pivot-probabilities vary? First, if a party moves, it can change the way it is perceived as a serious contender, exactly as in Myerson and Weber. But in this electoral system, a change of positioning can also change the structure of the pivot-probabilities themselves. Assume parties 1 and 2 are close ideologically, with 3 very far away. The minimal range coalition is the coalition including 1 and 2. Now if 3 moves very close to party 2, then 2 and 3 form the minimal range coalition.




$p_{1,1+2} + p_{2,1+2} + p_{3,1+2} = 1$ $p_{1,2+3} + p_{2,2+3} + p_{3,2+3} = 1$

$x_1 = 10, x_2 = 30, x_3 = 60$ $x_1 = 10, x_2 = 30, x_3 = 23$

**Figure 2: Example of how a party (here party 3) can change the belief structure by moving**

Using her preferences and beliefs, each voter can compute her expected gain for voting for each party.

*Voters' behaviors:*

1°) each t-type voter casts a ballot for a candidate maximizing her expected gain,

2°) in case of a tie he randomizes with a fair coin.

Let us insist on the fact that voters are not really playing a game, as the expected gains do not depend on the individual behaviors of the other voters (and does not even require them to know the voters' distribution).

The behavior of the voters then determines the expected scores of the parties and their probabilities of being in power (alone or with another candidate). These probabilities are endogenous and should not be confused with the exogenous pivot-probabilities.

Let us examine the strategies and actions of the parties. A party's only strategic choice is the position it chooses. Its choice depends on the positions of the other parties and on the state of beliefs. We assume the parties play a Nash game.

We also assume that being in power is a cake of size one, shared equally by the winning candidates. Thus if $U_i$ denotes utility of party i, then $U_1 + U_2 + U_3 = 1$.

A situation is a *positional equilibrium* if there exists a state of beliefs function such that

1°) the voters vote for the party maximizing their expected gains;

2°) the positioning of the parties is a Nash equilibrium.

Note that our definition of positional equilibrium is very weak in the sense that we assume no further restriction on the pivot-probabilities. Myerson and Weber demand that beliefs be in accordance with the outcome they imply and introduce the so-called *ordering condition*. We don't impose any such (adapted) ordering condition. Our results are robust to it.

We do not impose either any restriction that would link beliefs across positions: as in Myerson and Weber, independent sets of pivot-probabilities are associated to each positioning vector by the three parties. A candidate, considered as a very serious contender under some positioning, might become a sure loser if she, or another candidate, moves by a tiny amount.[5]

We made the choice to be as permissive as possible, since our concept is sufficient to really discriminate across voting systems, as the next two sections will show.

## *3. A Median Voter Result with Ideological Coalitions*

**Proposition 1: Under ideological coalitions, the only positional equilibrium shows total convergence of the parties at the median position.**

**Lemma:** Assume $x_1 = x_2 \neq x_3$. Then $U_1 = 1/2 = U_2$ if $d(x_1, 50) \leq d(x_3,50)$ and $U_3 = 1$ if $d(x_1, 50) > d(x_3,50)$.

Proof of lemma: The only relevant pivot probability is $p_{1+2,3}$, since 1, 2 and a coalition of 1 and 2 lead to the same outcome. Therefore any voter's expected gains for voting for 1 or 2 are equal: EG(2) = EG(1) to be compared to EG(3). Voters who prefer policy $x_1$ randomize between 1 and 2, voters who prefer $x_3$ vote for 3; Equidistant voters randomize between the three parties.

Proof of proposition 1:

Let us show that A) any situation without total convergence of the three parties is not an equilibrium, B) a situation where $x_1 = x_2 = x_3 \neq 50$ is not an equilibrium and C) a situation where $x_1 = x_2 = x_3 = 50$ is an equilibrium.

A) Consider any situation where $x_1 \leq x_2 \leq x_3$ with at least $x_1 < x_3$.

$U_2 < ½$ is not possible in equilibrium since 2 could deviate and obtain $U_2 = ½$ by the lemma. Therefore $U_2 \geq ½$, implying that $U_1 + U_3 \leq ½$ and therefore $U_1 \leq ¼$ or $U_3 \leq ¼$. This is not possible in equilibrium since either 1 or 2 would deviate and insure utility of at least 1/3.

B) Assume $x_1 = x_2 = x_3 \neq 50$.

Then $U_1 = U_2 = U_3 = 1/3$. Any party deviating to 50 would get utility 1 by lemma.

C) Assume $x_1 = x_2 = x_3 = 50$.

Then $U_1 = U_2 = U_3 = 1/3$. Any party deviating would get utility 0 by lemma.

The idea of the proposition is that an isolated extremist party cannot win the election while by joining the most centrist party it would belong to a minimal range coalition. The minimal range coalition is always a serious contender. As the most extreme two parties cannot win simultaneously, in equilibrium no party will be more extremist than the others. Thus, they finally all hold the same position at 50.

This median voter result is important, as minimal range theory is empirically the best theory of coalition formation. Note that it is robust to coalitions being simply connected (rather than necessarily of minimal range) and as long as the position of a coalition is anywhere strictly between the positions of the parties (rather than exactly at the middle).[6]

Proposition 1 is quite intuitive and quite close to the reality of countries where proportionality is applied. Germany fits very well the assumptions and the results. Belgium is another good example with three major centrist parties.

We did not impose any restrictions on the pivot-probabilities. Despite this, beliefs appear to be powerless in this system: they cannot exclude a party from the race if it is not ideologically isolated. Indeed, the set of pivot-probabilities that should be considered is restricted endogenously by the rule of coalition formation. And a world with ideological coalitions disadvantages lonely extreme candidates.

In the next section, we consider a proportional system when coalitions are formed according to size theory and obtain a strikingly different result.

### *4. A Proportional System with Minimum Size Coalitions*

In *a proportional system with coalition of the smallest two parties* there are six outcome zones: the coalition zone is split in three zones as shown in the following figure.

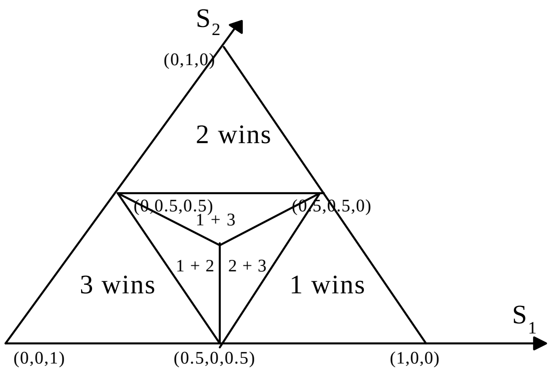



**Figure 3: The outcome simplex according to size theory**

The zone where a party wins alone is adjacent with the zone where the other two parties share the power. The possibly positive pivot-probabilities are of the form $p_{i,j+k}$ or $p_{i+j,i+k}$ where i,j,k represent different parties.[7]

**Proposition 2: If the winning coalition is the minimum size one, then in equilibrium, the winning policy can be anywhere strictly between 0 and 100.** [8]

Proof of proposition 2:

By symmetry, it is sufficient to show that the winning policy can be anywhere between 1 and 50. We consider a situation where $x_1$ is anywhere between 1 and 50 and both 2 and 3 are positioned at 100 - $x_1/2$. Consider any state of beliefs such that

$$p_{1,2+3}(x_1, 100 - x_1/2, 100 - x_1/2) = 1 \quad (A1)$$

$$p_{1+3,2+3}(x_1, 50 - x_1/4, 100 - x_1/2) = 1 \quad (A2)$$

$$p_{1+2,2+3}(x_1, 100 - x_1/2, 50 - x_1/4) = 1 \quad (A3)$$

$$p_{2+3,2+1}(x_1, s, 100 - x_1/2) = 1 \text{ for any } s \neq 100 - x_1/2 \text{ and } s \neq 50 - x_1/4 \quad (A4)$$

$$p_{3+1,3+2}(x_1, 100 - x_1/2, s) = 1 \text{ for any } s \neq 100 - x_1/2 \text{ and } s \neq 50 - x_1/4 \quad (A5)$$

In such a situation, by (A1), a majority of voters prefer 1 to a coalition of 2 and 3 and cast a ballot for 1. The outcome of the election is $x_1$.

Party 2 is dissuaded from moving to 50 - $x_1/4$ because 1 would win by beliefs (A2), or anywhere else because 1 or 3 would win by (A4).

Symmetrically, 3 is dissuaded from moving by (A3) and (A5).

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If the winning coalition is of minimal size, the nature of the appropriate pivot-probabilities does not depend on the positions of the parties. Therefore and contrarily to the ideological coalition case, beliefs do not in essence favor moderate parties. Beliefs are able to support any party and make it win.

The contrast between propositions 1 and 2 shows that the process of coalition formation in proportional systems is crucial when trying to predict the outcome of such systems.

## *6. Discussion*

This paper adapts Myerson and Weber's model to study proportional systems with three parties. The overall results can be summarized as follows. The moderation capacity of a proportional system strongly depends on the coalition formation rules. Convergence to the median is achieved if coalitions are formed according to minimal range theory while multiple equilibria exist if coalitions are formed according to size theory.

These results should be compared to Austen-Smith and Banks (1988) who proposed the first formalization of a proportional system where both voters and candidates are strategic. A party's utility is defined by a combination of its share of power and the distance between its platform and the eventual outcome. A non-cooperative game describes the coalition formation process: first the biggest party can make a coalition proposal (including a division of the pie of power) to a party. If the party refuses, the second biggest party makes an offer and eventually the smallest.

The authors isolate one specific equilibrium where two major parties position symmetrically with respect to a small median party. Each of them makes with probability ½ a coalition proposal to the small centrist party so as to have it accept it. This equilibrium is sustained by a very specific assumption regarding the out-of-equilibrium rational expectations: the voting equilibrium anticipated if one of the two major parties moves is making this party worse off.

The coalition formation game chosen by Austen-Smith and Banks is therefore close to our minimal winning coalition model where extreme outcome equilibria can be sustained by explicit states of beliefs.

Our geometric framework could formalize their game, as well as alternative coalition formation rules, by integrating its coalition formation equilibria into the outcome simplex.

In a two-dimensional space, Baron (1993) also obtains equilibria where the parties do not converge to the center. We are unaware of any paper predicting full convergence.

Our results show the crucial importance of the coalition formation rule in predicting outcomes of proportional systems. They should encourage the already very active research on coalition formation, at both the empirical and theoretical levels.

The present paper was in fact an informal adaptation of Myerson and Weber (1993).

Their multiple equilibrium result in a plurality election inspired our proposition 2 directly.

Myerson and Weber's multiple equilibria result in a plurality election is easily interpreted in relative majority election where there are three parties: the largest party, whatever its score, even if it is lower than half of the total of the votes, gets the power.[9] In a companion paper we show that in a runoff system too, any policy can be sustained in equilibrium.

We therefore propose a striking comparative result: contrarily to majority systems, a proportional system can drive the parties toward moderation. This result seems in line with empirical reality in Europe, where coalition governments in countries like Belgium, Germany, the Netherlands, Spain, Italy, seem indeed to propose more moderate policies than the parties in France or the United Kingdom (at least before the New Labour).

## *Endnotes:*

1, This should be linked with the analysis by Alesina and Rosenthal (1995) on "moderating elections": in electoral systems with two polarized parties, moderate voters are able to vote strategically for their less preferred party in one assembly so as to moderate the overall policy.

2, Recently, the process of coalition formation in a legislature has been examined within a non cooperative sequential game framework, mainly by Baron. Parties are sequentially selected to make a coalition proposal to the others with some given probabilities (which may depend on their sizes). These papers show that the resulting coalitions depend strongly on the game and on whether or not a default outcome exists at some point. These models have not yet been tested empirically.

3, This means that when modeling a proportional system in what follows, we in fact model a proportional system *and* a government coalition process.

4, We thus implicitly assume that the parties have the same bargaining power (note that they all share a common Shapley-Shubik power index of 1/3). This assumption is not crucial.

5, Note that, in general, "out of equilibrium" pivot-probabilities are not negligible. Indeed, when optimally choosing its position, each party takes the positions of the other parties as given and computes the probability of being in power in each possible situation, which depends on the pivot-probabilities for each of his positions.

6, This means for example that a median voter result is obtained if the largest party is always in the winning coalition and chooses to form a coalition with the closer party. Or if the median party has all the bargaining power and forms a winning coalition with the smaller of the other two.

7, Note a funny thing about expected gains here: influencing the outcome in the direction of i+j means making the majority of i+j smaller than the majority of i+k, as the winning coalition is the smaller one.

8, The case of a proportional system with *maximum* size coalition would lead to similar results.

9, Myerson and Weber find a median voter theorem under approval voting. In such a system, the coordination problem is solved by the possibility to cast a ballot for several candidates.

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