Durable-Goods Monopoly with Privately Known Impatience∗
– A theoretical and experimental study–

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Abstract

We analyze a durable-goods monopoly which sells a single unit of a good to a buyer whose value of the good is private information. The discount factors of the buyer and the seller may differ and they are private knowledge. We solve for the closed-form solution of a two-period game and compare this solution with the behavior observed in laboratory experiments. The data are to a large extent consistent with the predictions.

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1 Introduction

Ever since Plato (1941)\textsuperscript{1} people seem to be aware that they may suffer from rational anticipation of own future behavior. A very prominent intra-personal decision conflict is one faced by a durable-goods monopolist (Coase, 1972). In a market with a durable good, a monopolistic seller could easily collect the monopoly profit by excluding any future price cut. Buyers will, however, anticipate that future prices are opportunistically chosen by the monopolist; in particular, that the good will be sold cheaper in later periods. For this reason, the monopolist loses market power. Coase conjectured that this can even lead to competitive and thus efficient market results.\textsuperscript{2}

Much of the literature on durable-goods monopoly has focused on the question under which conditions the Coase conjecture proves to hold and under which conditions it does not hold. For example, Stokey (1981) and Gul, Sonnenschein, and Wilson (1986) show that, with an infinite number of successive sales periods, there is an equilibrium in which the price is (arbitrarily) close to marginal cost. Others have shown that product durability does not necessarily reduce the monopolist’s market power (Ausubel and Deneckere, 1989; Bagnoli, Salant, and Swierzbinski, 1989). Güth and Ritzberger (1998) show that a durable-goods monopolist may even increase its profits when the model allows for a difference between the discount factor of the monopolist and that of the potential buyers. Under this assumption, Güth and Ritzberger (1998) show that even over a finite number of periods the monopolist may significantly increase market power, provided the buyer has a lower discount factor. This is the so-called Pacman Conjecture (Bagnoli et al., 1989). If the seller has a lower discount factor, he loses profits compared to a one-period monopolist.

In this paper, we follow Güth and Ritzberger (1998) in that we allow for a difference in discount factors. The usual assumption is that players have identical discount factors. However, there is ample evidence that discount factors may be highly idiosyncratic in

\textsuperscript{1}See Frank (1996) for a modern analysis.

\textsuperscript{2}A similar example of intra-personal decision conflict arises in vertically related markets. An upstream monopoly selling to multiple downstream firms may significantly lose its market power because of the opportunism resulting from downstream competition (for experimental evidence, see Martin, Normann, and Snyder, 2001).
social environments. In addition, we assume that discount factors are private knowledge. Commonly known impatience of players seems unlikely—at least, it requires further justification. How eager sellers and buyers are to obtain monetary rewards over time is presumably difficult to observe for others. So the assumption of privately known discount factors seems less restrictive. More specifically, we assume that discount factors can be either high or low, for both the monopolist and the buyer. Which state is realized is private information. For this scenario, we analyze a two-period game with one seller and one buyer who’s valuation is also private knowledge, and we derive the closed-form solution.

In addition, we provide experimental evidence. Experimental data may reveal to what extent subjects’ behavior conforms to (rational expectations) theory but it may also show that bounded rationality limits the predictive power of standard theory in durable-goods games. Theory has a number of interesting implications in our market. Will sellers with a high discount factor charge higher prices as predicted? Similarly, will buyers with a high discount factor refuse to purchase in period one more often? Considering bounded rationality, two kinds of behavior may be important. Firstly, because of fairness reasons, buyer subjects may withhold demand, that is, they may reject profitable purchases. Such behavior may soften the monopolist’s pricing behavior and may generally limit the predictive power of standard theory in durable-goods games. Secondly, it seems possible that seller subjects might feel committed by mere intentions about their future behavior—even when there is no formal commitment device. This again could limit the predictive power of the theory. The conflict of a durable-goods monopolist between avoiding the effects of intra-personal price competition and reacting opportunistically and how this enters the price expectations of the buyer seems an exciting topic of experimental analysis.

Previous experimental papers on durable-goods monopoly include Cason and Sharma (2001), Reynolds (2000), and Güth, Ockenfels and Ritzberger (1995). Supporting the predictions, there is strong evidence that monopolists indeed lose monopoly power when selling a durable good. However, a large number of observations have been made which indicate that subjects’ behavior is inconsistent with the predictions. Reynolds (2000)

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3There is the substantial “myopia” or “short-terminism” literature. Take-over threats, career concerns and risk considerations can induce managers not to maximize the presented discounted value of the firm but to choose projects with a high return (inefficiently) early. Such factors are likely to differ across managers. Thus, managers ultimately operate with different discount factors. See, e.g., Stein (1989), or Palley (1997) containing more references.
observed that initial prices were higher in multiperiod experiments than in single-period monopoly experiments. In all experiments, there is more demand withholding than predicted. For example, Cason and Sharma (2001) observed more trading periods than predicted due to higher demand withholding. Finally, durable-goods experiments seem to require a number of repetitions due to their complexity. In Güth et al. (1995), there was no opportunity for learning. Prices failed to conform to comparative statics predictions and were often higher than predicted. With experienced subjects, observed prices were closer to the prediction but participants still had serious difficulties to understand the crucial aspects of such dynamic markets.

In view of these previous experiments and their results it seems important to limit attention to the simple case of markets with two periods. We also have provided ample opportunities for learning by letting participants play the same market repeatedly in our computerized experiment. This allows us to incorporate a further complexity, namely that relative impatience is private information.

In section 2 we derive the game-theoretic solution for two-period markets. Section 3 explains the design of the experiment whose results are described and statistically analyzed in section 4. We summarize in section 5.

### 2 The basic model

The monopolistic seller has an indivisible commodity which he evaluates by 0 whereas the only buyer evaluates the commodity by $v \in [0,1]$. The value $v$ is, however, the buyer’s private information. The distribution of $v$ is uniform over the unit interval $[0,1]$ and this is commonly known.

We consider two successive sales periods. The discount factor $\zeta \in (0,1)$ represents the seller’s weight for future (period $t = 2$) versus present (period $t = 1$) profit. Similarly, $\delta$
reflects the buyer’s impatience where $\delta \in (0, 1)$.\footnote{Only the assumption $\delta < 1$ is actually necessary for deriving a well-defined solution. The boundary case $\delta = 1$ can only be analyzed via $\delta \nearrow 1$ (see Guth and Ritzberger, 1998). Note that $\delta = 1$ renders buying in period $t = 1$ or $t = 2$ as homogeneous trades in view of the buyer. The fact that $\delta = 1$ cannot be solved directly provides an example that price competition for homogeneous products should be solved as the limiting case of such competition for heterogeneous products when heterogeneity vanishes.} We denote by $p_1$ the price in period $t = 1$ and by $p_2$ the price in period $t = 2$.

The decision process is as follows:

**Period $t = 1$:**

- The seller chooses his sales price $p_1 \in [0, 1]$ for this period.
- Knowing $p_1$ and her value $v$, the buyer decides whether or not to buy. If she does, this ends the interaction; otherwise period $t = 2$ follows.

**Period $t = 2$:**

- The seller chooses his sales price $p_2 \in [0, 1]$ for this period.
- Knowing $p_2$ and her value $v$, the buyer decides whether or not to buy. This ends the interaction.

The profit of the seller is $p_1$ if there is trade in period $t = 1$, it is $\zeta p_2$ if trade occurs in period $t = 2$, and it is 0 if there is no trade. For the buyer, the payoff is $v - p_1$ for trade in period $t = 1$, $\delta (v - p_2)$ for trade in period $t = 1$, and 0 in the case of no trade.

If both discount factors are commonly known, and if the seller is risk neutral, the solution prices $p_1^*$ and $p_2^*$ depend on the discount factor $\zeta$ of the seller and $\delta$ of the buyer as follows:\footnote{The general case of finitely many sales periods can be solved via backward induction and the infinite horizon via approximation by letting the number of sales periods approach $\infty$ (see Guth and Ritzberger, 1998).}

$$p_1^* = \frac{(2 - \delta)^2}{2 \left[4 - 2\delta - \zeta\right]} , \quad p_2^* = \frac{2 - \delta}{2 \left[4 - 2\delta - \zeta\right]} . \tag{1}$$
Note that, with just one trading period, the monopoly price\(^6\) would be \(p^* = \frac{1}{2}\), implying a profit of \(\frac{1}{4}\). The polar cases of relative impatience correspond to

- \(\zeta \searrow 0\) and \(\delta \nearrow 1\) with \(\lim p_1^* = \frac{1}{2} = \lim p_2^*\): as only buyers with \(v \geq \frac{1}{2}\) buy in period \(t = 1\), the seller earns only half of what he would earn as a usual monopolist, namely \(\frac{1}{4} \left(\frac{1}{2}\right) = \frac{1}{8}\);

- \(\zeta \nearrow 1\) and \(\delta \searrow 0\) with \(\lim p_1^* = \frac{2}{3}\) and \(\lim p_2^* = \frac{1}{3}\): the (extremely patient) seller engages in price discrimination over time by collecting \(p_1^* = \frac{2}{3}\) whenever \(v\) is in the interval \(1 \geq v \geq \frac{2}{3}\) and \(p_2^* = \frac{1}{3}\) when \(\frac{2}{3} > v \geq \frac{1}{3}\). This yields an expected profit of \(\left(\frac{2}{3} + \frac{1}{3}\right) \cdot \frac{1}{3} = \frac{1}{3}\), more than the static monopoly profit.

We assume that discount factors are private knowledge. In addition to information about their discount factors, players observe the following. In period \(t = 1\) the buyer is informed about his valuation and the seller’s price offer. If there is trade in period \(t = 1\), the seller learns that there is trade. If there is no trade in period \(t = 1\), the buyer additionally observes the price \(p_2\) and the seller learns whether or not she sold the commodity in period \(t = 2\). In order to simplify the analysis, we assume that the discount factors of buyers and sellers can adopt only two values, low or high. That is, we assume

\[
0 < \delta < \overline{\delta} < 1 \text{ and } 0 < \zeta < \overline{\zeta} < 1
\]

where the probability for \(\overline{\delta}\) is \(w \in (0, 1)\) and the one for \(\overline{\zeta}\) is \(\omega \in (0, 1)\). To allow for a clear-cut benchmark solution\(^7\) we assume that all the parameters \(\delta, \overline{\delta}, \zeta, \overline{\zeta}, w, \text{ and } \omega\) are commonly known.

\(^6\) Resulting from maximizing \(p(1 - p)\) where \(p\) is the unique sales price and \(1 - p\) the probability by which the buyer expects his price \(p\) to be accepted due to \(1 - p = \int dv\).

\(^7\) Except for highly special games, e.g., when all players have unique undominated strategies, game-theoretic analysis requires commonly known rules of the game.
3 The solution

Our first point is immediate but useful to note. Whenever \( p_2 \geq p_1 \) the buyer would not buy in period \( t = 2 \) as \( \delta < 1 \). We therefore obtain

**Proposition 1:** The solution of the two-period game involves a price decrease, that is, \( p_1 > p_2 \).

Given the buyer’s discount factor \( \delta \in \{\delta, \tilde{\delta}\} \), when will she buy the commodity? Consider the decision to buy in period \( t = 1 \) or \( t = 2 \). If a type \( v \in [0, 1] \) has not bought in period \( t = 1 \) at price \( p_1 \), she will buy in period \( t = 2 \) at price \( p_2 \) whenever \( v \geq p_2 \). Assume now a type \( v \geq p_2 \) who anticipates the actual solution prices \( p_1 \) and \( p_2 \). Since buying in period \( t = 1 \) yields \( v - p_1 \) whereas delaying it yields \( \delta (v - p_2) \), type \( v \) prefers to buy early if

\[
  v - p_1 \geq \delta (v - p_2) \quad \text{or} \quad v \geq \frac{p_1 - \delta p_2}{1 - \delta}.
\]

This establishes

**Proposition 2:** According to the solution play with sales prices \( p_1 \) and \( p_2 \),

(i) sale occurs in period \( t = 1 \) if

\[
  v \geq \begin{cases} 
    v = \frac{p_1 - \delta p_2}{1 - \delta} \quad \text{for} \quad \delta = \hat{\delta} \\
    v = \frac{p_1 - \tilde{\delta} p_2}{1 - \tilde{\delta}} \quad \text{for} \quad \delta = \tilde{\delta}
  \end{cases}
\]

and in period \( t = 2 \) if

\[
  \bar{v} > v \geq p_2 \quad \text{for} \quad \delta = \hat{\delta} \quad \text{(4)}
\]

\[
  \bar{v} > v \geq p_2 \quad \text{for} \quad \delta = \tilde{\delta} \quad \text{(5)}
\]

whereas

(ii) \( v < p_2 \) implies no sales at all.
Note that, from Proposition 1, the two thresholds \( \underline{v} \) and \( \bar{v} \) in Proposition 2 satisfy \( \underline{v} < \bar{v} \).

Next, we discard the possibility that the seller serves only the \( \bar{\delta} \)-buyer types in period \( t = 2 \). Assume, by contrast, that this is true. Then the \( \bar{\delta} \)-buyer would only switch between buying at price \( p_1 \) in period \( t = 1 \) and not buying at all, implying that only \( \bar{\delta} \)-buyers with \( v \geq p_1 \) buy in period \( t = 1 \). But, since \( p_1 > p_2 \), \( \bar{\delta} \)-buyer types \( v \) with \( p_1 > v \geq p_2 \) would like to buy in period \( t = 2 \), contradicting the assumption that only \( \bar{\delta} \)-buyer types are served in period \( t = 2 \). Thus we have proved

**Proposition 3:** Trade in period \( t = 2 \) involves both buyer types \( \delta \in \{ \underline{\delta}, \bar{\delta} \} \) with positive probability, i.e., \( \underline{v} > p_2 \).

We can now proceed to derive the full solution of the game. We start by solving the last period. Note that, in period \( t = 2 \), the seller knows that the \( \underline{\delta} \) \( \bar{\delta} \)-buyer has no value \( v \geq v(\bar{v}) \). Thus his posterior probability of trade in period \( t = 2 \) at price \( p_2 \) is

\[
D (p_2) = \frac{(1 - w) (\underline{v} - p_2) + w(\bar{v} - p_2)}{(1 - w)\underline{v} + w\bar{v}}
\]

where, in view of Proposition 3, both terms of the numerator on the right hand-side above are positive. Maximization of \( p_2 D (p_2) \) yields

\[
p_2 = p_2 (\underline{v}, \bar{v}) = \frac{(1 - w) \underline{v} + w \bar{v}}{2}.
\]

Substituting \( p_2 \) in (4), the equations for \( \underline{v} \) and \( \bar{v} \), yields a system of two equations with two unknowns

\[
\underline{v} = \frac{2p_1 - \delta \bar{v}w}{1 - \delta}, \quad \bar{v} = \frac{2p_1 - \underline{\delta}wv}{1 - \delta}.
\]

This system can readily be solved as

\[
\underline{v} = 2p_1 - \frac{\delta(1 + w) - \bar{\delta}w}{4 - 2(\bar{\delta} + \underline{\delta})(1 + w) + \underline{\delta} \bar{\delta}(1 + 2w)},
\]

\[
\bar{v} = 2p_1 - \frac{\delta(1 + w) - \underline{\delta}w}{4 - 2(\bar{\delta} + \underline{\delta})(1 + w) + \underline{\delta} \bar{\delta}(1 + 2w)}.
\]
Since the optimal price $p_2 = ((1 - w)v + w\overline{v})/2$ depends on $v$ and $\overline{v}$, it can be expressed as a function of $p_1$ only:

$$p_2(p_1) = p_1 \frac{2 - \overline{\delta} - 2\overline{w}}{4 - 2(\overline{\delta} + \overline{\delta})(1 + w) + \overline{\delta}(1 + 2w)}.$$  \hspace{1cm} (11)

With the help of these derivations, the expected profit from trade over the two sales periods can be defined as a function of $p_1$, the price of period $t = 1$, namely

$$p_1 [(1 - w)(1 - v(p_1)) + w (1 - \overline{v}(p_1))] + \zeta p_2(p_1) [(1 - w)(v(p_1) - p_2(p_1)) + w (\overline{v}(p_1) - p_2(p_1))],$$  \hspace{1cm} (12)

where, $\zeta \in \{\zeta, \overline{\zeta}\}$. Maximizing this function with respect to $p_1$ yields

$$p_1(\zeta) = \frac{[4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w)]^2}{4[2 - \overline{\delta} - 2\overline{\delta}w][4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]},$$  \hspace{1cm} (13)

and thus

$$p_2(\zeta) = \frac{4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w)}{4[4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]},$$  \hspace{1cm} (14)

\[ \begin{align*}
\overline{v}(\zeta) &= \frac{2[2 - \overline{\delta}(1 + w) - \overline{\delta}w][4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]}{2[2 - \overline{\delta} - 2\overline{\delta}w][4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]}, \\
\overline{\overline{v}}(\zeta) &= \frac{2[2 - \overline{\delta}(1 + w) - \overline{\delta}w][4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]}{2[2 - \overline{\delta} - 2\overline{\delta}w][4 - 2(\overline{\delta} + \delta)(1 + w) + \overline{\delta}(1 + 2w) - \zeta(1 - \overline{\delta}/2 - \overline{\delta}w)]}. 
\end{align*} \]

We thus have derived the solution\(^8\) play described by

**Proposition 4:** For $\zeta \in \{\zeta, \overline{\zeta}\}$, the solution play of the two-period game is as follows

- In period $t = 1$, the price is $p_1(\zeta)$ which induces all buyer types $v \geq \overline{v}(\zeta)$ and $\delta = \overline{\delta}$ as well as $v \geq \overline{v}(\zeta)$ and $\delta = \overline{\delta}$ to buy.

- In period $t = 2$, all buyer types $\overline{v}(\zeta) > v \geq p_2(\zeta)$ and $\delta = \overline{\delta}$ as well as $\overline{v}(\zeta) > v \geq p_2(\zeta)$ and $\delta = \overline{\delta}$ buy whereas

\(^8\)A pooling equilibrium, based on the ex ante expected impatience parameter $\zeta = (1 - w)\zeta + w\overline{\zeta}$, would not satisfy sequential rationality since both seller types would like to deviate from the common price $p_1(\zeta)$ as shown by our derivation.
According to $p_1 (\zeta)$ the seller with time preference $\zeta \in \{ \zeta, \bar{\zeta} \}$ reveals his impatience by his first period price $p_1$. Therefore, the buyer can rationally anticipate $p_2 (\zeta)$ after observing $p_1$. The seller in turn only learns after the first sales period whether or not the buyer has bought in this period. Thus his demand expectations for the second sales period are as expressed by $D (p_2)$.

4 Experimental design

Our experimental design exactly matches the above setup of the durable goods monopoly with privately known impatience. We employ the parameters $\bar{\delta} = \zeta = 0.3$, $\bar{\delta} = \bar{\zeta} = 0.7$, $w = w = 0.5$. These parameters imply the solution values in figure 1. If, as assumed in the theory section, the buyers’ valuations are drawn from the unit interval, the two columns on the left apply. In the experiment, we took buyers’ valuations from the interval $[50, 150]$. Therefore, the absolute price prediction is according to the two columns on the right of the figure 1. For the sake of plausibility of the frame, we introduced a production cost of 50. Sellers had to choose prices from the interval $[0, 200]$.

<table>
<thead>
<tr>
<th>$v \in [0, 1]$</th>
<th>$v \in [50, 150]$</th>
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<tbody>
<tr>
<td>$\zeta$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$p_1 (\zeta)$</td>
<td>0.47</td>
</tr>
<tr>
<td>$p_2 (\zeta)$</td>
<td>0.33</td>
</tr>
<tr>
<td>$v (\zeta)$</td>
<td>0.53</td>
</tr>
<tr>
<td>$\pi (\zeta)$</td>
<td>0.79</td>
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</tbody>
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Table 1: Experimental Parameters.

We ran six sessions, each consisting of two matching groups. Each round was conducted exactly as follows. One group consisted of three sellers and three buyers. Within the groups, sellers and buyers were randomly rematched after every round. Subjects

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9For the more patient seller it does not pay to mimic the price $p_1 (\bar{\zeta})$ since the additional revenue in period $t = 1$ is overcompensated by the $\zeta$-weighted revenue loss in period $t = 2$. For $\zeta = \bar{\zeta}$ the opposite is true.

10See Appendix 6 for the translated instructions.

11Participants were not informed about the restriction of rematching within matching groups what should have further discouraged repeated-game effects.
learned their role, seller or buyer, only after they had read the instructions, and they did not switch roles during the experiment. In order to allow for learning, we decided to run the experiment over 40 rounds.\textsuperscript{12}

Sellers learned their discount factor, then they had to choose their price. Knowing their discount factor, buyers had to decide whether or not to buy at the period one price, $p_1$. If they decided not to, period two would commence and so forth. At the end of each round, subjects were informed about their private earnings in the previous round as well as their cumulative earnings up to this round.

The experiments were conducted at Humboldt University, Berlin, in December 2001 and January 2002. The 72 participants were mainly business and economics students who were recruited via email and telephone. Payments were 16 euros on average, including a show-up fee of 2.5 euros. Sessions lasted roughly 90 minutes.

5 Experimental results

5.1 Qualitative consistency of decisions

Let us first check whether buying and pricing behavior is consistent with a few qualitative theoretical implications. It seems worth emphasizing that consistency even with very basic principles cannot be taken for granted in a complex durable-good setting. For example, Güth et al. (1995) report a surprising amount of inconsistency in a durable-goods experiment. Similarly, Reynolds (2000) emphasizes the necessity of experience with the trading environment. Therefore, we find it useful to do a consistency check first.

Consider the buyers. Basic understanding of the situation implies that buyers would never purchase at a price above their valuation. It seems impossible that some argument based on repeated games or bounded rationality could plausibly support such loss inducing purchases. Out of 1440 possible sales, we observed 1037 actual purchases. In all but six

\textsuperscript{12}In the durable-goods experiment by Reynolds (2000), subjects interacted in 12 durable-goods markets.
purchases, buyers had valuations above the prices. That is, there are virtually no such loss-making purchases and we can conclude that basic buyer behavior was consistent in this sense.\footnote{In two cases buyers accepted a higher price than their valuation in period $t = 2$. The average loss, $-2.5$, was quite small suggesting the possibility that a preference for efficiency might explain these loss-making decisions; in particular, as they occurred in later rounds (16, 38). By contrast, three of the four cases in which buyers accepted a price higher than their valuation in first period occurred early (rounds 1, 1 and 7). Here, the average loss was $-27$. Rather than efficiency seeking behavior, these cases can be seen as mistakes.}

Buyers knew that profits from sales made in period $t = 2$ are discounted. This implies buyers with $\overline{\delta} = 0.7$ should reject a profitable purchase in period $t = 1$ more often than a buyer with $\overline{\delta} = 0.3$. Given any path of (expected) seller prices $\{p_1, p_2\}$, the impatient buyer has to purchase early more often as her second period opportunities are less attractive. Even if we take repeated game effects like demand withholding into account, it seems implausible for the impatient buyer to reject more often because it is more costly for her to reject in periods with low discount factors. Confirming this, the data\footnote{Because of possible dependence of observations within groups, we count group averages including all periods as one observation. Unless mentioned, all tests reported in this paper are therefore based on matching group averages.} show that in period $t = 1$ buyers with $\overline{\delta} = 0.7$ reject profitable offers (that is, offers with $p_1 \leq v$) significantly more often than buyers with $\overline{\delta} = 0.3$. Relative acceptance rates are lower with $\overline{\delta} = 0.7$ for all groups, the according non-parametric test is highly significant (one-sided Wilcoxon, $p = 0.0002$). We conclude that buyers do understand the basic impact of discounting.

Now consider the sellers. Did they understand the implication of discounting? If so, sellers with a high discount factor should charge a higher price in both periods than sellers with a low discount factor. As shown above (see figure 1) this is the prediction for the solution prices. Even if subjects do not play the solution, it should be apparent to them that a high discount factor makes it relatively more attractive to charge a high price in period $t = 1$ as there is still another profitable opportunity to come. As both types of sellers should (and indeed did) reduce their price in $t = 2$, a higher period $t = 1$ price for high discount factor types also implies higher period $t = 2$ prices. By contrast, the impatient seller has to make his sales early and, therefore, charges also a lower period $t = 2$ price. The data show that average prices in period $t = 1$ were higher than period $t = 2$ prices in all
groups and in both periods. The according test is highly significant (one-sided Wilcoxon, $p = 0.006$). It appears that sellers understood the impact of their own discount factor.

Proposition 1 states that sellers should charge lower prices in period $t = 2$ compared to period $t = 1$. The intuition is that a discounting buyer has no incentive to buy at a higher price in period $t = 2$. If sellers want to exploit the opportunity to sell in period $t = 2$, they should lower the price. However, the prediction of a price decrease over the two periods is not the only plausible behavior. Boundedly rational sellers may refuse to charge a lower period $t = 2$ price in an attempt to solve the commitment problem.

In 750 cases, there is no trade in period $t = 1$ and therefore a period $t = 2$ price is observed. In the vast majority of these cases, sellers indeed charged a lower price in period $t = 2$. Over the entire course of the experiment, 33 out of 750 period $t = 2$ prices were strictly higher than $p_1$. This figure gets relatively smaller over time. Over the last 10 rounds, only 3 out of 155 period $t = 2$ prices were strictly higher than $p_1$. In many cases (13 out of 33 and 3 out of 3 cases, respectively), we observe the maximum price of 200 in period $t = 2$, and all but one of these 13 observations were caused by a single seller.\textsuperscript{15} In these cases, the higher price does not appear to be a mistake but a signal. In addition, there are another 33 observations (7 over the last 10 rounds) in which the price was constant over the two periods. Out of these 33 observations, 27 can be attributed to four sellers which followed this pricing policy four or more times. Note that we never observed a seller who regularly behaved as a one-period monopolist in the sense of $p_1 = p_2 = 100$. To summarize, we find only few violations of Proposition 1. A few subjects occasionally charged $p_2 = p_1$ or $p_2 = 200 > p_1$. This may be interpreted as attempts to solve the durable-goods monopolist’s commitment problem. The remaining number of inconsistencies is small and scattered over time and subjects.

\textbf{Result 1:} Subjects’ behavior is consistent with several qualitative predictions. Buyers virtually never make unprofitable purchases. Almost all sellers systematically lowered prices in period $t = 2$. Patient buyers reject profitable purchases in period $t = 1$ more often. Patient sellers charge higher prices in both periods.

\textsuperscript{15}This seller followed a pricing policy of $p_1 = 75$ and $p_2 = 200$ in many rounds. With an expected value of $v$ of 100, this splits the surplus of 50 evenly in period $t = 1$. If this price is not accepted, this seller refused to transact at all by offering a price above the buyer’s value ($p_2 = 200 > 150 \geq v$).
5.2 Buyer behavior

Let us now compare the data to the exact predictions of \( p_1, p_2, \overline{v} \) and \( v \). Consider buyer behavior first. Buyers withhold demand whenever an offer \( v > p \) is rejected. The prediction is that any price offer smaller than \( v \) (in period \( t = 2 \)) or smaller than \( \overline{v} \) or \( v \) (in period \( t = 1 \)) should be accepted independently of the history of the game. There can be rational and boundedly rational (or irrational) demand withholding. When \( v \) is larger than \( p_1 \) but smaller than \( \overline{v} \) or \( v \) (in period \( t = 1 \)), a rejection is rational. In period \( t = 2 \), there is no rational demand withholding. While demand withholding as part of boundedly rational strategy has been frequently observed (see, e.g., Ruffle, 2000), note that, in this experiment, demand withholding in order to establish a reputation for aggressive buyer behavior is particularly difficult. First, there is the random matching scheme and the design does not allow to identify buyers. Moreover, sellers do not know whether their offer was rejected because of boundedly rational demand withholding or because it was simply not profitable. By contrast, in many posted-offer experiments, buyers’ evaluations are known and demand withholding can serve much better as a signal.

<table>
<thead>
<tr>
<th>rejected</th>
<th>( v &lt; p_1 )</th>
<th>( p_1 \leq v &lt; \overline{v} )</th>
<th>( \overline{v} \leq v )</th>
<th>all</th>
<th>( v &lt; p_2 )</th>
<th>( p_2 \leq v )</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>accepted</td>
<td>507</td>
<td>174</td>
<td>69</td>
<td>750</td>
<td>335</td>
<td>68</td>
<td>403</td>
</tr>
<tr>
<td>all</td>
<td>511</td>
<td>206</td>
<td>723</td>
<td>1440</td>
<td>337</td>
<td>413</td>
<td>750</td>
</tr>
</tbody>
</table>

Table 2: Acceptance numbers for different value classification concerning \( p_1 \) and \( p_2 \).

Buyer behavior in period \( t = 2 \) is simple to analyze as there is no dynamic effect of decision making any more. Buyers’ period \( t = 2 \) behavior is also independent of \( \delta \). Any \( p_2 \leq v \) should be accepted by all buyers. In the data, we find that 68 out of 413 offers (16.5%) with \( p_2 \leq v \) were rejected (see Table 2). These offers rejected typically left only a small profit margin for the buyers. This margin was \((v - p_2)/v = 0.0693\) on average of the rejections. Two thirds of all rejections involved a margin of less than 8%. Regarding accepted offers, buyers often were willing to accept even low margins and, in four cases, buyers accepted a period \( t = 2 \) price at which they just broke even. Two thirds of all accepted prices gave them less than 26% profit margin. Buyers never rejected margins of more than 25%.
Figure 1: Acceptance threshold for period $t = 2$ on group averages (only for $v \geq p_2$).

Figure 1 illustrates the acceptance and rejection averages of $(v - p_2)/v$ for the twelve groups (provided $v - p_2 \geq 0$). The lowest average of accepted $(v - p_2)/v$ at the group level is 13% and the highest average of rejected $(v - p_2)/v$ at the group level is 11%. As the acceptance and rejection average margins are not overlapping at the group level, this suggests a threshold below and above which offers are rejected and accepted respectively.

Recall that buyers knew the production cost of the seller (50). Therefore, besides the impact of the discount factor, they were able to identify the seller’s profit and compare it to their own. Take buyers’ reaction to the median period $t = 2$ price, $p_2 = 75$, as an example. Buyers with $v < 100$ knew that the seller would get a larger profit from the sale but they rejected only in 9 out of 38 cases (taking only buyers with $v \geq p_2 = 75$ into account). Thus, it seems that aversion against disadvantageous inequality played only a little role here.\textsuperscript{16} Nevertheless, there is demand withholding in period $t = 2$.

We turn to buyer behavior in period $t = 1$. The prediction is that, after observing the solution $p_1 (\zeta)$, buyers with $\underline{v}$ (or $\overline{v}$) $> p_1$ should accept. (Henceforth, we will refer to $\overline{v}$ whenever we want make a statement about “$v$ or $\overline{v}$”.) For out-of-equilibrium $p_1$, buyers with $\underline{v}(p_1) > p_1$ and $\overline{v}(p_1) > p_1$ should accept. The corresponding numbers are listed in figure 2. First, consider buyers with $\overline{v} > v > p_1$ which are predicted to reject (rational demand withholding). Out of 206 cases, buyers rejected in 174 cases (84.5%). That is, to a large extent, buyers’ behavior was in accordance with the theory. There are,\textsuperscript{16}This suggests that inequality aversion (Bolton, 1991) loses influence in situations where at least some individual payments are private information or difficult to guess.
however, some inconsistencies, namely the 32 accepted offers yielding a profit margin of \((v - p_1)/v = 0.159\). These buyers did not realize that a lower period \(t = 2\) price should have given them a higher discounted margin. Second, did buyers with \(v \geq \bar{v}\) accept? If \((v - \bar{v})/v > 0\), 90% of all offers were accepted (see Table 2 again). If \((v - \bar{v})/v > 0.1\), 96% of all offers were accepted. The average margin rejected was \((v - p_1)/v = 0.115\). Note that this margin is larger than the one in period \(t = 2\), so there is more demand withholding in period 1.

**Result 2:** Buyers’ behavior is to a large extent consistent with the prediction. Buyers usually accepted profitable offers in period \(t = 2\) while, in period \(t = 1\), they accepted only if the offer gave them a more than positive profit margin. Both in period \(t = 1\) and \(t = 2\), there is some irrational (or boundedly rational) demand withholding.

### 5.3 Seller behavior

Now consider seller behavior. We report deviations from the (conditional) predictions rather than absolute values because the optimal prices \(p_2\) depend on the realization of \(p_1\), and \(p_1\) is often different from the predictions \(p_1 (\zeta) = 90\) and \(p_1 (\bar{\zeta}) = 97\). Accordingly, we refer to \(p_2 (p_1)\) rather than \(p_2 (\zeta)\) and \(p_2 (\bar{\zeta})\), and we define \(\Delta p_1 (\zeta) = p_1 - p_1 (\zeta)\), \(\Delta p_2 = p_2 - p_2 (p_1)\). Note that \(\Delta p_2\) does only depend on \(p_1\) but not on the realization of \(\zeta\). We find that \(\Delta p_1 (\zeta) = -4.73\) (standard deviation: 5.52), \(\Delta p_1 (\bar{\zeta}) = -6.46\) (5.56), and \(\Delta p_2 = +0.89\) (5.88).\(^{17}\) In absolute terms, the average prices charged are \(p_1 (\zeta) = 84\) and \(p_1 (\bar{\zeta}) = 91\). These are lower than the predicted values. Concluding from the standard deviations, it appears that \(\Delta p_1 (\zeta)\) and \(\Delta p_1 (\bar{\zeta})\) are significantly below the prediction of zero.

Given that buyers charged prices in period \(t = 1\) partly far away from the prediction, it is more difficult to analyze period \(t = 2\) pricing behavior. If we interpret the \(p_1 \notin \{90, 97\}\) as

\(^{17}\) The reported numbers are group averages. Individual averages have the same means for \(\Delta p_1\). As the numbers of trades continued in period \(t = 2\) differs within groups, for individual observations the mean also slightly differs: \(\Delta p_2 = +0.79\).
as in equation (11).

As mentioned, we report the difference between actual price in $t = 2$ and this prediction:

$$\Delta p_2 = p_2 - p_2(p_1).$$

Now, $\Delta p_2 = 0.89$ is surprisingly small what we interpret as support of rationality theory whenever the situation is simple enough (in $t = 2$ sellers do not have to anticipate own future choices any longer). But there is much variability in individual decisions. Regarding group averages, figure 2 shows that there all except one group have a rather small $\Delta p_2$ while the $\Delta p_1$ observations are more dispersed and clearly negative. The fact that $\Delta p_2$ average is slightly positive does not mean that pricing behavior in period $t = 2$ changes qualitatively from that in period $t = 1$. Sellers start with a lower price and they reduce the price by the proportion predicted. Hence, whatever accounts for the lower prices in period $t = 1$, this behavior carries over to period $t = 2$.

**Result 3:** Sellers charge lower prices than predicted, both in period $t = 1$ and period $t = 2$. The reduction of period $t = 2$ prices is consistent with the (conditional) rationality.

To conclude the analysis of seller behavior, the only significant deviation from the prediction are the lower period $t = 1$ prices. This is a robust finding in that it is very similar for both discount rates $\zeta$ and $\bar{\zeta}$. Instead of a continuous demand function, we have assumed
a single buyer whose value is private information. The density of the value plays the role of the continuous demand function. Theoretically, this does not matter much for the outcome but this may matter behaviorally as in such bilateral encounters fairness concerns may become stronger, and this could account for low first period prices. Alternatively, risk considerations (an attitude of sellers to ensure trade) may explain the result. We did not control for fairness concerns or risk aversion of sellers. Because the buyer’s valuation is private knowledge, sellers only know the expected buyer profit. Though it is possible for buyers to make inter-personal profit comparison, it is relatively complex to do so and, regarding profits made in the second period, there is uncertainty about the discount factor. Therefore, compared to pure bargaining experiments, it seems less likely that fairness matters and the lower period \( t = 1 \) prices may rather reflect the risk attitude of sellers.

5.4 Impact of the discount factors

We finally analyze the impact of the distribution of the discount factors. It is a central feature of our model that the discount factor of the seller as compared to the buyer’s determines whether the seller suffers from intra-personal competition or gains by price discrimination. In this sense, a higher discount factor implies higher “power”, affecting both acceptance rates and profits. Above, we already reported the impact of discount factors, separately for buyers and sellers. Here, we compare acceptance rates and profits for all \((\zeta, \delta)\) seller-buyer combinations.

We start with the percentage of accepted offers. Let \( a_t(\zeta, \delta) \) denote the rate of acceptance for some \((\zeta, \delta)\) seller-buyer combination in period \( t \). Theory predicts that sellers with a high discount factor charge higher prices both in period \( t = 1 \) and period \( t = 2 \), and that buyers with a high discount factor reject profitable purchases in period \( t = 1 \) more often. This immediately implies that, in period \( t = 1 \), \( a_1(\zeta, \delta) \) should have the smallest acceptance rate and \( a_1(\zeta, \delta) \) should have the highest, while \( a_1(\zeta, \delta) \) and \( a_1(\zeta, \delta) \) should be intermediate. Deducing acceptance rates from figure 1, the prediction is \( a_1(\zeta, \delta) < a_1(\zeta, \delta) < a_1(\zeta, \delta) \). This turns out to hold in our data. The acceptance rates for the four combinations in figure 3 show that, indeed, \( a_1(\zeta, \delta) \) < \( a_1(\zeta, \delta) \) < \( a_1(\zeta, \delta) \) < \( a_1(\zeta, \delta) \).
Figure 3: Acceptance rates (period $t = 1$; $t = 2$) for all discount factor constellations.

$a_1(\zeta, \delta)$ with corresponding significance level of the one-sided Wilcoxon tests above the inequality signs. Intuitively, the acceptance rates in period $t = 2$ must exhibit the opposite inequality signs: if there are fewer acceptances in period $t = 1$, more buyers are left to accept in period $t = 2$. In accordance with this intuition, one can deduce $a_2(\zeta, \delta) > a_2(\zeta, \delta_2) > a_2(\zeta, \delta)$ from figure 1. We find that $a_2(\zeta, \delta) > a_2(\zeta, \delta)$ significantly (one-sided Wilcoxon test) as predicted, but neither $a_2(\zeta, \delta) > a_2(\zeta, \delta)$ (as predicted) nor $a_2(\zeta, \delta) > a_2(\zeta, \delta)$ (not predicted) were significant.

Now consider profits. Predictions are simple. Given the discount factor of the other player, a high own discount factor implies a higher profit. Given the own discount factor, a high discount factor of the other player implies a lower profit. It turns out that this holds in the experimental data for all possible $(\zeta, \delta)$ combinations. That is, though high and low discount factor types can actually realize the same profit in period $t = 1$, high discount factor types make larger profits because of the trade shifted to period $t = 2$. Let $u_S(\zeta, \delta)$ and $u_B(\zeta, \delta)$ indicate the average profits made by sellers and buyers, respectively, in a $(\zeta, \delta)$ seller-buyer encounter. The average $u_S(\zeta, \delta)$ was roughly 19 and the average $u_B(\zeta, \delta)$ was about 21. The following inequalities are significant (with the corresponding significance level of the one-sided Wilcoxon tests above the inequality signs). We find that $u_S(\zeta, \delta) > u_S(\zeta, \delta_2) > u_S(\zeta, \delta)$, and $u_S(\zeta, \delta) > u_S(\zeta, \delta)$ for the seller, and $u_B(\zeta, \delta) > u_B(\zeta, \delta)$, and $u_B(\zeta, \delta) > u_B(\zeta, \delta)$ for the buyer. Further, we find $u_S(\zeta, \delta) > u_S(\zeta, \delta)$ and $u_B(\zeta, \delta) > u_B(\zeta, \delta)$ because of the high rejection rates which a $(\zeta, \delta)$ combination implies.
**Result 4:** High discount factors of either the seller or the buyer reduce the probability of a successful trade in period $t = 1$. Nevertheless, participants realize higher average earnings if their opponent has a low discount factor.

### 6 Conclusions

The literature substantiating the intuition of Coase’s (1972) durable-goods monopolist has inspired much theory but only few experiments. In this paper, we have extended both lines of research. We solve, for the first time, the simplest case where discount factors are private information. Secondly, by conducting a laboratory experiment, we provide a test of the theory.

Participants behaved rather reasonably according to qualitative predictions—possibly because we provided enough opportunity for learning. There are few unprofitable purchases and there are generally lower prices in the second period as predicted. Furthermore, participants reacted adequately to changes in discount factors (within-subject comparisons) and, as buyers, maintained higher acceptance thresholds in the first than in the second period. Ceteris paribus, a higher discount factor of at least one player shifts more trade to the second period. Whenever the situation becomes rather simple, as for instance in the second period, conditional rationality can account for most of the decision data.

It has already been indicated in the Introduction that we view durable-goods monopolies as very intriguing. They challenge the conventional wisdom that several competitors are needed to induce competitive outcomes; they are also philosophically challenging by claiming intra-personal price competition. After all, it is due to rational anticipation of own future behavior that the monopolist may earn so much less than a usual monopolist. It seems remarkable that such surprising insight seems to be well understood by the participants.
Appendix: Instructions

The experiment was conducted in German and the original experimental instructions were also in German. This is a shortened\textsuperscript{18} translated version of the instructions. Participants read the paper instructions before the computerized experiment started. In the beginning of the instructions, subjects were informed that the instructions are the same for every participant, they receive an initial endowment of DM 5, that wins and losses from all periods would be added, the exchange rate from ECU (Experimental Currency Unit) to DM: 30 ECU = DM 1, that communication was not allowed and questions would be answered privately and that all decisions will be treated anonymously. Then the main instructions started.

Two parties, a seller $S$ and a buyer $B$ negotiate in each period about the sale of a product. The buyer’s product value $v$ is $50 \leq v \leq 150$ (all in ECU). The valuation is the payoff a buyer receives if he purchases the product. In each period there will be a new $v$ drawn from this interval, with all values being equally likely. The seller has production costs of $50$ if he sells the good.

Whether you act as $S$ or $B$ is determined randomly at the beginning of the experiment. You will keep your role for the whole experiment. You will interact in total over 40 periods. Your bargaining partner will every time be randomly determined at the beginning of each period.

Trade takes place according to the following rules:

1. $S$ decides about the price $p_1$ with $0 \leq p_1 \leq 200$ within a first sales opportunity.

2. $B$ decides whether to buy and pay $p_1$ or not.

   (a) If $B$ purchase the product, $S$ receives $p_1 - 50$. $B$ receives $v$ and pays $p_1$, i.e., his profit is $v - p_1$.

   The period is over.

\textsuperscript{18}The complete German instructions are available at request.
(b) If $B$ does not purchase, there will be a second sales opportunity. In this case $S$ decides about a second price $p_2$ with $0 \leq p_2 \leq 200$. $B$ decides whether to buy and pay $p_2$ or not to buy at all.

i. If $B$ purchase the product, $S$ receives a discounted profit $\zeta (p_2 - 50)$. $B$ receives $v$ and pays $p_2$, i.e., his discounted profit is $\delta (v - p_2)$.

(The discount rates $\zeta$ and $\delta$ of the seller and the buyer, respectively, specify with which factor the profit from the second sales opportunity is multiplied.)

ii. If $B$ does not purchase (i.e., not to buy at all), both parties receive zero profits.

The period is over.

[At this point, the decision process is also graphically illustrated.]

There are only two values possible for both discount rates $\zeta$ and $\delta$, namely 0.3 and 0.7. Possible $(\zeta, \delta)$ constellations are therefore (0.3, 0.3), (0.3, 0.7), (0.7, 0.3), and (0.7, 0.7). The likelihood for both discount rates’ values is the same and are randomly determined at the beginning of each period independently for the seller and the buyer. All four constellations have the same probability. Only $S$ knows which of the two values $\zeta$ has. Correspondingly only $B$ knows his realized $\delta$ value.

At the beginning of each period you are, according to your role, informed about:

- As seller $S$: Your discount rate $\zeta$.
- As buyer $B$: Your discount rate $\delta$.

and your valuation for the product $v$.

At the end of each period you will be informed about your profit at each period and your total payoffs.

Thank you for participation!
References


