

# **Human Capital Evolution and Economic Crisis: Minding ‘The Gap’\***

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Abstract: When an economy experiences crisis that reduces the level of human capital stock, such a change should be accompanied by a change in the investment level. Is this always the best thing to do? In an optimal growth model of human capital evolution this paper shows that cutting down investment during crisis may not always be the optimal response to crisis. At certain levels of human capital, maintaining the pre-crisis level of investment is optimal and may be crucial for economic success.

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## *Introduction*

In the course of history, various nations of the world have at one time or the other experienced shocks or crisis situations. In the last century alone, the great depression of the 1930s, the oil price shock in the late 1970s and the subsequent foreign debt crisis that rocked emerging nations come readily to mind. Common in many of these situations is the coping strategy employed by the affected countries. Countries in crisis situations often cut back on investment and smooth consumption as much as possible until the crisis subsides. Although consumption takes some of the impact, we observe that investment bears the major burden as shown on table 1.

**Table 1 - GDP per capita, investment share of GDP, consumption share of GDP (in 1985 international prices)**

<b>Country</b>	<b>GDP Per Capita (1979)</b>	<b>Investment share of GDP(1979)</b>	<b>Consumption share of GDP(1979)</b>	<b>GDP Per Capita (1984)</b>	<b>Investment share of GDP(1984)</b>	<b>Consumption share of GDP(1984)</b>
<b>Mexico</b>	17744	19.4	73.8	16837	14.4	72.2
<b>Brazil</b>	11285	21.1	70.8	10478	14.3	75.5
<b>Guatemala</b>	9003	11.0	84.9	7687	7.9	83.0
<b>India</b>	2164	15.3	57.4	2611	12.9	59.6
<b>Philippines</b>	5104	20.5	65.1	4564	15.6	68.2
<b>Egypt</b>	5595	6.5	69.7	6984	6.3	65.6
<b>Nigeria</b>	3483	15.9	46.5	2846	8.7	65.5
<b>Kenya</b>	2198	13.0	67.8	2085	10.8	65.9

\*Penn World Table - NBER Web site( [www.nber.org](http://www.nber.org))

The table shows the trend of GDP per capita in constant 1985 U.S. Dollar terms, Gross investment as a share of GDP, and Gross consumption as a share of GDP for some of the most heavily indebted countries across the world at the time. These were mainly third world countries that had borrowed a lot during the years of the oil price boom at concession rates of interest. With the on-set of the oil price shock these countries were worst hit by the subsequent debt crisis of the mid 1980s. In the worst year of the crisis (1984) we see that there were major cuts in investment relative to 1979. The data suggests that when hit by unexpected shocks, countries tend to reduce their rate of investment as a coping strategy. The reason for this is simple: at low levels of income, consumption is usually at a relatively low level. When crisis hits, individuals are often unwilling to take cuts in consumption, other variables especially investment bear the greater burden of adjustment. Even when the countries involved are not so poor, for example the Southeast Asian financial crisis in the last couple of years, we observe that many financial institutions closed down and the cost of credit went up. This had a negative effect on the level of investment and aggregate demand. Ferri and Tae (1999) argue that this over-reaction by financial institution was responsible for worsening the crisis and if uninterrupted would have plunged the economies further into depression.

In standard optimal growth models such as the Ramsey-Cass-Koopman model, optimal investment levels and growth rates can be deduced from the first order conditions. Given some initial conditions, the behaviour of the economy can be mapped out straight away. Built into the model is the premise that output ( $Y_t$ ) is shared between consumption ( $C_t$ ) and investment ( $I_t$ ), that is  $Y_t = C_t + I_t$ . Applying actual data to this framework we see that if a crisis were to reduce  $Y_t$ ,  $C_t$  and  $I_t$  would

also fall. As can be seen from table 1 above,  $I_t$  would take a greater cut because of the need to smooth consumption. The aim of this paper is to determine if this strategy is optimal for developing economies when faced with shock or crisis situation. The crisis could be strictly economic (e.g. resulting from a debt burden, poverty, e.t.c.), or precipitated by political instability, natural disaster, wars and other factors which affect a country's ability to retain or fully utilise its stock of capital. Is it always the best thing to cut down on investment? This paper aims to find an answer to this question using a simple human capital evolution model.

We begin by describing what a crisis situation is and the possible effect on the level and evolution of human capital, output and welfare, and we examine the reactions to economic shocks (section 1). Using an optimal growth model, we analyse what the optimal investment strategy should be (section 2). Based on the relevant equations of the model we simulate the model behaviour so as to make meaningful deductions (section 3). We then present the model results in section 4 and take a closer look at the implied investment behaviour during crisis. We show that maintaining the existing level of human capital is crucial to attaining an economy's full potential and conclude that reducing investment when crisis hits is not always the optimal strategy.

### ***1. Economic Crisis and Human Capital Evolution.***

Standard of living and welfare are measured by consumption, which is determined by the level of income. Income/output levels depend on how much productive resource an economy has accumulated and how well it uses the resources. The presence and quality of human capital applied to physical capital is a major determinant of output

levels and standards of living. The effect of crisis on the evolution of human capital is therefore important.

Standard of living is usually associated with the level of human capital attained. Robert Lucas (1988) uses this idea in his work. He modelled the evolution of human capital both as a function of current investment and the level/quality of human capital already attained. The build up of human capital determines the level of output in the economy and therefore the level of welfare. There are however special cases where this does not necessarily hold true. For example, Russia's level of human capital is comparable with levels in western European countries but its standard of living is much lower and technological development remains well below world standards (Overland and Spagat, 1996). The reason for this is not far fetched. The marginal product of a well educated labour force working with poor technology and low quality physical capital will tend to be low. It may also be that the quality of human capital may be low such that income will be low, and standard of living poor. The economic environment is an important factor in determining how well and how quickly human capital is built up and therefore how higher standards of living can be achieved. For example when competitive factor and product markets are absent, there are no incentives for managerial innovations. This, together with other factors such as underdeveloped legal systems, weak property rights, corruption in the civil service and government, presents a fragile environment that cannot support a competitive market economy. The economic environment could be so unhealthy that the productivity of the people and the skills embodied in them is significantly impaired.

To deal with the problem and eventually improve the situation we will show that countries should, in times of crisis, invest to prevent loss of their existing stock of human capital. The reverse is however what we have observed. When the economic environment is fragile or in crisis, the much-needed investment cannot be easily attracted from abroad, yet economies reduce their rate of capital investment. This coupled with already low income in an unfavourable economic environment will have a negative effect on human capital stock, output, and welfare. We therefore propose that reducing investment may not be an optimal strategy because the welfare gain from sustaining the pre-crisis level could outweigh the gains from smoothing consumption.

Spagat (1995) with similar concern for Russia and other countries of the former Soviet Union in his work advocated for early intervention by full investment in human capital. In a 'learning or doing' type framework, political instability is used to explain why the environment is not attractive to investment and it is suggested that maintaining the existing stock of human capital would be a sound policy. This may require borrowing since resources are very scarce but such borrowing would be profitable in the long-run to both the borrower and lender countries. In this paper, political instability among other factors precipitates what we call a crisis. There is no borrowing but the idea of maintaining the stock is very much at play. We show that by systematically investing more units of resources, the problem can be solved as the economy proceeds on an optimal path. Murphy, Vishny and Shleifer (1997), with simultaneous industrialisation of sectors, create a 'big push' that shifts the economy out of the low productivity trap. Here, we follow a different approach and show that

each additional unit of capital invested can be viewed as a 'small push' towards the eventual take-off of the economy.

The importance of economic reaction to shocks cannot be overemphasised. Private individuals as well as corporate investors tend to cut back on investment into human capital when hit by crisis. UNICEF's<sup>1</sup> (1997) report on the relationship between education and child labour tells us that unexpected economic downturn is the major reason for the increasing participation of children in the labour force. In times of crisis, children are pulled out of school to work in order to supplement family income. Very many of them do not return to school as shown by the low completion rates in most affected countries. Even when they do return, they lag behind their peers. The World Bank<sup>2</sup> in their report on the social issues arising from the East Asian economic crisis also say that, "Families tend to withdraw their children from school due to falling incomes and an inability to pay school fees or other attendant costs (uniforms, school meals, textbooks, and 'voluntary contributions,' for example). Likewise working age children confront immediate opportunity cost of education versus income generating activities." These represent a decrease in investment in human capital and a deliberate policy by the government to reduce public investment can only worsen the situation, and result in significant human capital loss. Thus, the report expressed as one of its major concerns the need to sustain investment in human capital and to prevent leakage and efficiency loss.

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<sup>1</sup> Also see Basu and Pham.(1988) who proposed that child labour occurs not because parents are selfish or lazy, but as a result of parental concern for the family's survival under conditions of stark poverty.

It is also not uncommon in crisis situations for trained personnel to emigrate given half a chance and to find qualified professionals under-employed as they abandon their professions and take up other vocations. This could be either because they cannot find jobs for which they are trained or for example, driving a taxi yields more total income than practising medicine. When people do not practice the profession for which they are trained, the quality of their skills diminishes. This will have an adverse effect both on the present and future stock of human capital. For the present stock, the rate of depreciation experienced by the economy would conceivably exceed its natural rate. This is due to the fact that apart from losses due to death at old age, retirement and other such factors that constitute the natural rate of depreciation, the economy could suffer significant losses from emigration of skilled individuals, as was the case in many developing economies in the 1980s. Skilled personnel migrated from the countries worst hit by the oil crisis to countries such as Saudi Arabia and the United States of America. The term *Brain Drain*<sup>3</sup> was used to describe their exit because it represented not just a loss in the present stock of raw labour, but also the knowledge and skills embodied in them. The exit of medical personnel, for example, represented both a loss to the stock of doctors/professors at the time, and an even greater loss to the next generation of physicians-to-be.

Furthermore, under-employment also became a serious problem. Professionals left their jobs or cut back on the number of hours worked. Many took up other commercial activities to supplement the income from their professional jobs thus their

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<sup>2</sup> *The report is a work-in-progress for discussion at a meeting of Development Co-operation Ministers in Sydney (March 5, 1999). It is available on the World Bank website for Social Crisis.*

<sup>3</sup> *Michael P. Todaro. (1992. pp. 132-133; 352-354); Ritterband, Paul. (1978. pp.2-4).*

skills could not be fully utilised while they are engaged with other activities. A chemical engineer, with all his training will probably not be as productive working as a security guard as he would be if he were working on an oil rig. The long run effect of this trend could be devastating as the gap between actual and natural rate of depreciation increases over time.

The problem of losing human capital is made more serious if we consider that the loss may not be temporary, as is the case with conscription into the army for the duration of a war. People who emigrate because the environment is not conducive are unlikely to return unless the situation that precipitated their exit has improved; hence the situation could well be permanent. Investing in order to maintain the pre-crisis level of human capital is important. We will show that the long run gain outweighs the short run sacrifice, because investing in human capital would not only maintain the existing stock, but it would also serve as an additional incentive to preventing excessive depreciation through underemployment and emigration. Inevitably, human capital stock in an improving environment would produce and attract the required technology. Hence the investment made can be viewed as an indirect investment in technological advancement.

## 2. *Optimal Growth Model*

The standard optimal growth model that readily comes to mind is the Ramsey-Cass Koopman model. A special case of it with a multi-period objective function of variables subject to a budget constraint (i.e. Dynamic Optimal growth model), presented by Gregory Chow<sup>4</sup> (1997. pp. 10-12) is followed here. Using the method of Lagrange multipliers, the objective function is maximised and the optimal path for the model is traced by backward induction using the first order condition. The model has the key implication that consumption is a constant fraction of output.

### *'The Gap' Model*

The model follows Chow's method of dynamic optimisation using Lagrange multipliers. It is an optimal growth model focusing on the optimal path of human capital. It employs a Cobb-Douglas production function with constant returns to scale:

$$Y_t = AH_t^\alpha \dots\dots\dots (1)$$

where  $0 < \alpha < 1$ ;  $Y$  = output;  $H_t$  = Human Capital which can be thought of as human beings (raw labour) and the skills embodied in them.  $A$  is a scale parameter representing the level of technology and we assume that there is no technological progress. The time subscript  $t = 0, 1, \dots, T$  shows that the model is in discrete time. For simplicity, physical capital has been normalised to unity, output is therefore a function of human capital only. Human capital evolves according to:

$$\mathbf{H}_{t+1} = \delta_- \mathbf{H}_t + (\delta^+ - \delta_-) \mathbf{H}_t f(\mathbf{I}_t) + \mathbf{I}_t \quad \dots\dots\dots(2)$$

Where  $\delta^+$  and  $\delta_-$  are actual and natural depreciation factors respectively (with  $0 < \delta_- < \delta^+ < 1$ ) such that  $(\delta^+ - \delta_-) \mathbf{H}_t$ , called ‘the gap’, captures the excess depreciation in human capital. When crisis results in the outflow of skilled individuals, or the inability to fully utilise the skills embodied in human beings, there is an element of loss that could significantly affect the growth rate and structure of the economy. In standard capital evolution equations, losses are viewed as depreciation to existing stock. When there is a crisis that wipes out a significant proportion of existing stock of human capital, depreciation rate would exceed what it ordinarily should be (i.e. its natural rate), the excess is what we call ‘the gap’. We will show that the investment strategy employed in the presence of ‘the gap’ is crucial to economic survival. The function  $f(\mathbf{I}_t)$ , indicates how productive a unit of investment is with respect to ‘the gap’ and therefore governs its closure.

In the evolution equation, investment directly affects the stock of human capital through  $\mathbf{I}_t$ , and indirectly through  $f(\mathbf{I}_t)$  which reduces loss of human capital as ‘the gap’ closes. The function  $f(\mathbf{I}_t)$  in equation 2 is defined as:

$$\begin{aligned} f(\mathbf{I}_t) &= a \mathbf{I}_t \text{ if } 0 < \mathbf{I}_t < \mathbf{I}^* \quad \dots\dots\dots(3) \\ &= 1 \text{ if } \mathbf{I}_t \geq \mathbf{I}^* \end{aligned}$$

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<sup>4</sup> See appendix 2 for further exposition.

where  $I^*$  is the minimum investment required to close ‘the gap’ and  $a = 1/I^*$ . Once  $I^*$  is reached,  $f(I_t)$  becomes unity. The first and second terms on the right hand side of equation 2 combine and the equation becomes the ‘no gap’ capital evolution equation:

$$H_{t+1} = \delta H_t + I_t .$$

This change in behaviour is important because of its implication for the marginal product of investment with respect to ‘the gap’. This we define as:

$$f'(I_t) = a \text{ if } 0 < I_t < I^* \quad \dots\dots\dots (4)$$

$$= 0 \text{ if } I_t \geq I^*$$

As long as the investment level is below  $I^*$ ,  $f(I_t) = aI_t$ , the derivative (equation 4),  $f'(I_t) = a$ , will be positive. We will show that this condition holds true when optimality conditions are satisfied.

Total output in each period is divided between consumption ( $C_t$ ) and investment( $I_t$ ) -  $Y_t = C_t + I_t$ . Therefore:

$$C_t = AH_t^\alpha - I_t \dots\dots\dots (5)$$

Utility derived from consumption is assumed to be a logarithmic function:

$$U[C_t] = \sum_{t=0}^T \beta^t \ln[C_t] = \sum_{t=0}^T \beta^t \ln[AH_t^\alpha - I_t] \quad \dots\dots\dots (6)$$

The discount factor  $0 < \beta < 1$  indicates the intertemporal nature of consumption choices. The closer it is to unity the more the agents prefer present to future consumption. Given the initial level of human capital, agents choose their consumption with the aim of maximising their utility over their entire horizon subject to the constraint presented by the capital evolution equation. The Lagrangian is therefore given by:

$$L = \sum_{t=0}^T \beta^t \ln[AH_t^\alpha - I_t] - \beta^{t+1} \lambda_{t+1} [H_{t+1} - \delta_- H_t - (\delta^+ - \delta_-) H_t f(I_t) - I_t] \dots\dots\dots (7)$$

The first order conditions yield:

$$I_t = AH_t^\alpha - 1/\beta \lambda_{t+1} [(\delta^+ - \delta_-) H_t f'(I_t) + 1] \dots\dots\dots (8)$$

$$\lambda_t = \alpha AH_t^{\alpha-1} / [AH_t^\alpha - I_t] + \beta \lambda_{t+1} [\delta_- + (\delta^+ - \delta_-) f(I_t)] \dots\dots\dots (9)$$

In the final period T, no investment is made i.e  $I_T = 0$ . From equation 8:

$$AH_T^\alpha = 1/(\beta \lambda_{T+1}) \text{ and}$$

$$\lambda_{T+1} = 1/(\beta AH_T^\alpha) \dots\dots\dots (10)$$

This can be substituted into equation 9:

$$\lambda_T = \alpha / H_T + \delta_- / (AH_T^\alpha) \dots\dots\dots (11)$$

Equations 8, 9, 11, and the evolution equation can be used to trace the optimal path for the models variables. Unlike Chow's model there are no simple analytical solutions from which we can make meaningful deductions. To see clearly what the models implications are, it was simulated using simple computer programming based on the relevant equations. The program was designed to solve the model incorporating all its features e.g. that below  $I^*$ ,  $f'(I_t) = a$  and investment is determined by equation 8. Once  $I^*$  is reached and the gap closes,  $f'(I_t) = 0$  and the investment equation becomes identical to that of the 'no gap' model:

$$I_t = \frac{A H_t^\alpha}{1 - \beta \lambda_{t+1}} \dots \dots \dots (12)$$

### 3. *Model Simulation*

In keeping with convention,  $\beta$  the discount factor and  $\delta$  the target depreciation factors were both set equal to 0.95. The minimum investment required to close 'the gap'  $I^*$  is assumed to be constant and exogenous. As a representative case, the exponent on human capital is set at 0.7. Recall that human capital here consists of labour and the skills embodied in them. Researchers have shown that the exponent on physical capital is in the region of 0.3. Mankiw, Romer, and Weil (1992), with their production function –  $Y_t = AK^\alpha H^\beta L^{1-\alpha-\beta}$ , estimate that the exponent on the two other factors in their model Labour (L) and Human capital (H) add up to 0.7 (in the constant returns to scale framework, the exponents must add up to unity).

In this model, we adopt this value for  $\alpha$  as the representative case since labour and human capital are not separated. The parameters for the representative case are:

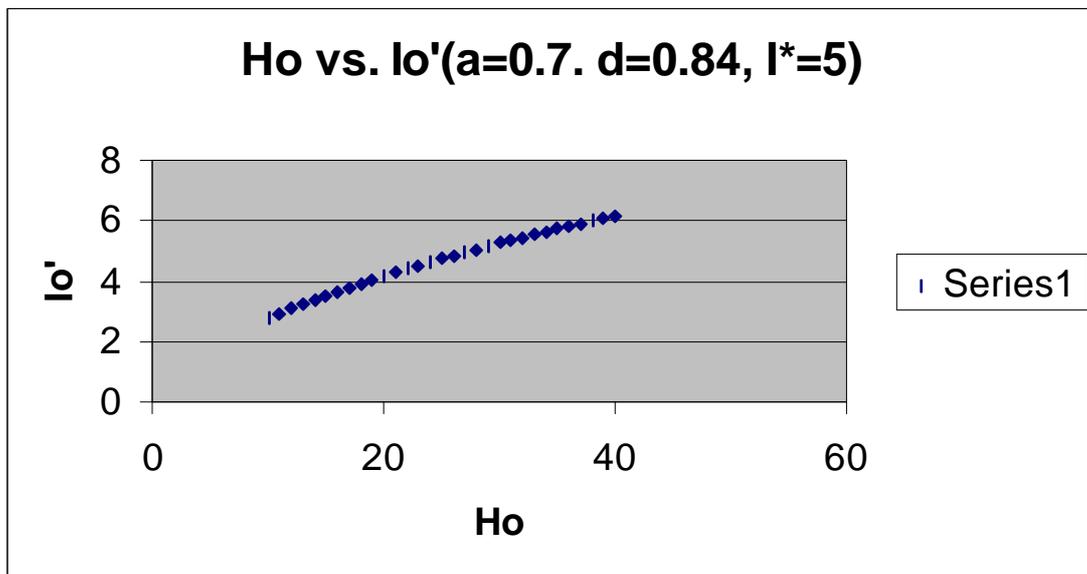
$$\begin{array}{lll} \alpha = 0.7 & \beta = 0.95 & \\ \delta^{\top} = 0.95 & \delta_{-} = 0.84 & \\ H_0 = 10 & I^* = 5 & A = 1 \end{array}$$

As a test for the robustness of the results, the model was simulated for  $\alpha$  ranging between 0.65 and 0.79 and the results qualitatively remained the same. When *the gap* was also varied in magnitude from as small as 0.03 to as large as 0.19 by varying  $\delta_{-}$  from 0.92 to 0.76, the results did not change qualitatively. The model results are therefore robust for any reasonable set of parameter.

#### **4. Results**

The simulation shows a distinct difference between ‘the gap’ and the ‘no gap’ model. Recall that in the ‘no gap’ model, consumption is a constant fraction of output. For every level of output, there is a corresponding level of consumption and investment. Simulating the ‘no gap’ model for various levels of initial human capital yields an upward sloping straight line showing that the absolute value of investment ( $I_0$ ) increases directly with the level of output. Fig. 1.1 below illustrates this behaviour:

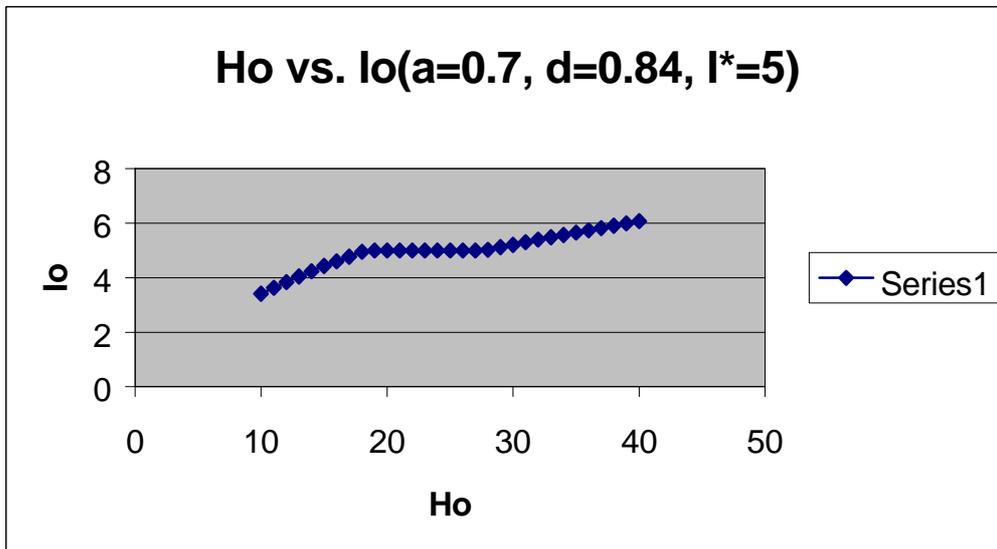
**Fig. 1.1: Human capital and investment levels - the ‘no gap’ model simulation.**



\*Io' is the predicted investment in the ‘no gap’ model simulation.

There is a direct relationship between levels of human capital and investment ( $Io'$ ) in the ‘no gap’ model described by the upward sloping line. As the level of human capital increases, the corresponding optimal investment level rises (i.e. for any  $Ho_1 < Ho_2$ ,  $Io_1' < Io_2'$ ). At higher levels of human capital, investment levels are always strictly higher. In ‘the gap’ model, this not always the case. From fig. 1.2 below we see that there is segment of the curve that is flat. Fig. 1.3 plots both models on the same set of axis for easy comparison of their behaviour.

**Fig.1.2: Human capital and investment levels - 'The Gap' model simulation.**  
 (Segment 1:  $H_o = 0-18$ ; Segment 2:  $H_o = 19-27$ ; Segment 3:  $H_o = 28-40$ )

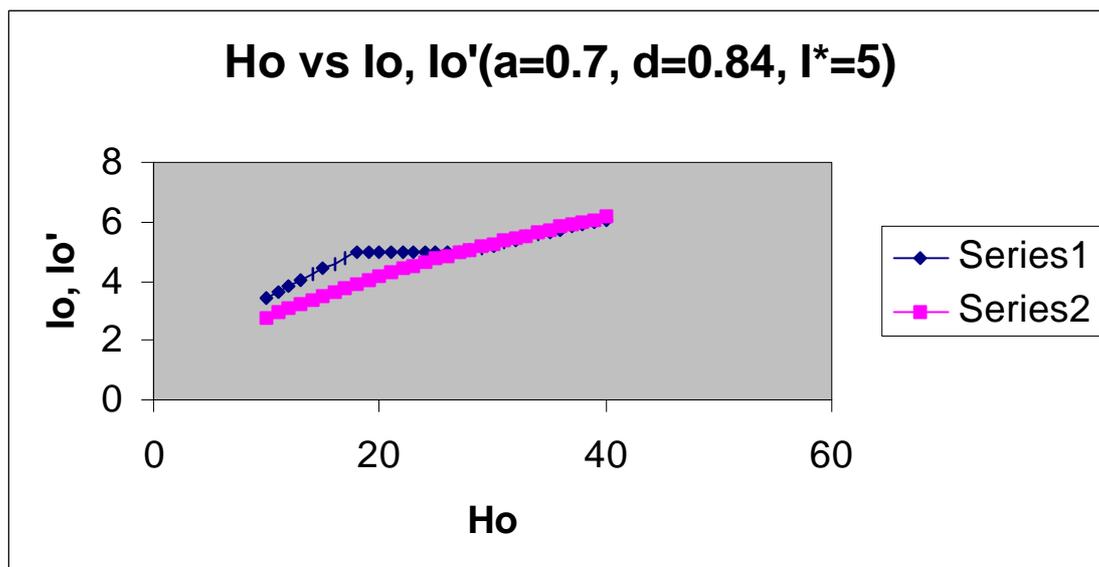


\*See Appendix for numerical results of model simulation.

When  $I_t < I^*$ , the marginal product of investment with respect to 'the gap' is positive.

**Fig.1.3 Human capital and investment levels for 'The Gap' and 'No Gap Models.**  
 (Segment 1:  $H_o = 0-18$ ; Segment 2:  $H_o = 19-27$ ; Segment 3:  $H_o = 28-40$ )

\*Series one( $H_o$  vs  $I_o$ ) represents the 'the gap' model and series two( $H_o$  vs  $I_o'$ ) represents 'no gap' model.



From fig.1.3 above we see that it is optimal to invest more factor units than the ‘no gap’ model implies. When  $I_t = I^*$ , the gap closes, and thereafter the model predicts levels of investment for which  $f'(I_t) = 0$  is satisfied. We observe that from this point ( $H_0 = 19$ ) the model does not behave in the same way for all levels of human capital. Increase in human capital does not always increase investment, and the reverse (i.e. decrease in human capital does not necessarily imply that investment should be decreased) is also true for a range of values ( $H_0 = 19$  to  $H_0 = 27$ ). At lower levels (the first segment of the curve) where  $I_t < I^*$ , higher human capital implies higher investment. At higher levels (the third segment) where  $I_t > I^*$ , the same is true. In the middle range (the second segment) where  $I_t = I^*$ , this does not hold.

### ***Optimal Investment Strategy during Crisis***

In this paper, we describe a crisis as any event be it political, economic or by nature, that wipes out a significant proportion of the existing human capital stock or impairs the ability to fully benefit from the built up stock. The implication of the results presented above for optimal behaviour during crisis is interesting. For a country whose level of human capital attained places it in segment one or three (i.e. poor and rich countries respectively), when a crisis that wipes out part of human capital stock hits, cutting back investment to match the new lower level of capital stock will be optimal. This behaviour is similar to the ‘no gap’ model. Comparing the slope of segments one and three, we find that segment one is steeper than segment three. When crisis hits, the model implies that poorer countries would cut down investment

more than richer countries. At lower levels of income, consumption is crucial to survival and hence investment tends to absorb more of the shock than would be the case if the country were richer. Table 2 below is a numeric illustration of this difference.

**Table 2. Change in investment corresponding to 15% cut in human capital**

	<b>Human Capital (Ho)</b>	<b>Investment (Io)</b>	<b>Percentage Change in investment</b>
<b>Segment 1</b>	<b>16</b>	<b>4.599547</b>	
<b>less 15% cut</b>	<b>13.6</b>	<b>4.23515</b>	<b>7.9%</b>
<b>Segment 2</b>	<b>26</b>	<b>5</b>	
<b>less 15% cut</b>	<b>22.1</b>	<b>5</b>	<b>0.0%</b>
<b>Segment 3</b>	<b>36</b>	<b>5.742357</b>	
<b>less 15% cut</b>	<b>30.6</b>	<b>5.301439</b>	<b>7.7%</b>

In segment two, the behaviour differs from the other segments and the ‘no gap’ model. Segment two represents a window within which cutting down investment when crisis hits is not the optimal strategy. For a country that falls into this range (possibly a middle income country), the optimal level of investment remains the same even in the wake of crisis. From table 2 above we see that in segment two, if a crisis wipes out 15% of human capital, the model predicts that the optimum level of investment should

remain the same as the pre-crisis level. Minding ‘the gap’ is crucial to economic survival. Reducing investment in this range would force the economy to a level that is incompatible with its level of human capital, and the condition  $f'(I_t) = 0$  which holds true at an optimum would be violated.

In segment 3 we observe that although both models use equation 12 to predict the post gap optimal investment level,  $I_o' > I_o > I^*$ . The difference is not significant but it is nevertheless interesting to know why  $I_o$  is not exactly equal to  $I_o'$ . Both models are beyond the range where minding ‘the gap’ is crucial but do not predict the same values for investment. The reason for this is not far fetched. Lagrange multipliers ( $\lambda$ ) give the rate at which the optimal value of the objective function increases per unit increment in the constraint if appropriate derivatives are defined. In a dynamic model, each multiplier contains information for the entire horizon of the economy (since they are arrived at by backward induction). In the ‘no gap’ model, there is no minimum investment necessary because of the gap, therefore as we approach the final period  $T$ , investment can optimally fall below  $I^*$  without re-opening the gap. For ‘the gap’ model this is not the case; in the final period  $I_T = 0 < I^*$  and ‘the gap’ is again open. This difference in the experience in the models feeds into each of the multipliers, hence the multipliers will differ and so will the predicted values of investment.

An important feature of this model is  $\alpha$ , the elasticity of output with respect to changes in human capital. We find that the closer it is to unity, the more responsive output is to changes in human capital stock. Change in human capital stock depends on investment, therefore every unit of investment is very important. With a high  $\alpha$ ,

output reacts robustly to changes in human capital, and the economy reaches  $I^*$  faster. If  $\alpha$  is too low, then output would be inelastic with respect to human capital. The returns to additional investment in human capital would be very small and it would take the economy a much longer time to close the gap. If this were the case and especially where resources are very scarce, the cost to of investing in human capital could outweigh the benefit. Using a realistic value for  $\alpha$  is therefore important.

### *Conclusion*

The model as simulated gives useful insights to the qualitative behaviour of economies in crisis, especially those that are neither rich nor poor. The key implication of this model is that for middle income countries, a cut in investment may not be the optimal strategy when faced by crisis. Richer economies may optimally do this but, as we have seen, minding ‘the gap’ makes it sub-optimal for middle income economies.

An important question, which arises from this implication of the model, is why economies cut down on investment when hit by crisis if it is not optimal to do so? A closer look at some economies that have gone through the different types of crisis described could perhaps proffer some explanation. One possible explanation for this is that the marginal product of human capital could be negative. This does not however seem plausible since in reality we do not observe this. We are then left with the alternative that the crisis economic environment is not competitive and gives no incentive to private entrepreneurs to invest in their workers. It may be the case that at the national level there are benefits to be reaped in the long run, from investing during

crisis as we have shown. If the individual entrepreneur who is fundamentally profit driven can not tap these benefits, then he has no incentive to invest. If all private entrepreneurs do not invest, the level of gross investment in the economy will be affected downwards; hence the behaviour observed from data of gross investment during crisis. An investigation into this possibility and how government can stimulate private investment especially during crisis situations by policy (e.g. tax reductions) should be (at least) informative and useful for economic policy making. Strategies employed by some of the East Asian countries (e.g. Thailand where education spending remained the same as the pre-crisis year level in real terms), suggests that this is the way to go.

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## Appendix 1

**Results for model simulation using base line set of parameters.**

<b>I*=5</b>	<b>a = 0.7</b>	<b>d_ = 0.84</b>
<b>Ho</b>	<b>Io</b>	<b>Io'</b>
10	3.410182	2.763222
11	3.627548	2.926958
12	3.842024	3.084091
13	4.042063	3.235345
14	4.23515	3.381314
15	4.429934	3.522495
16	4.599547	3.65931
17	4.77078	3.79212
18	4.944629	3.921235
19	5	4.046928
20	5	4.169437
21	5	4.288974
22	5	4.405726
23	5	4.519863
24	5	4.631535
25	5	4.740879
26	5	4.844019
27	5	4.953069
28	5.019258	5.056132
29	5.115029	5.157302
30	5.209064	5.256668
31	5.301439	5.354309
32	5.392475	5.450299
33	5.482125	5.544709
34	5.570283	5.637603
35	5.657009	5.72904
36	5.742357	5.819075
37	5.826377	5.907763
38	5.909118	5.995151
39	5.990625	6.081285
40	6.070939	6.166209

\*Io and Io` are the optimal investment level predicted by 'the gap' model and the 'no gap model respectively.

## Appendix 2

### Optimal Growth Model

In practice, there are two methods of solving dynamic models namely: Dynamic Programming and the method of Lagrange Multipliers. Both method apply optimality conditions to the objective function in the terminal period and then trace the optimal path for the model's variables by backward induction.

$$\text{Given: } Y_t = Z_t K_t^\alpha$$

where  $t = 0, 1, 2 \dots, T$  denotes time;  $Y = \text{Output}$ ;  $Z$  is a scale parameter for technology;  $K = \text{physical capital (Stock variable)}$ . Assuming total depreciation of capital in each period, the capital evolution equation is given by:

$$K_{t+1} = Z_t K_t^\alpha - C_t$$

where  $C_t = \text{Consumption (control variable)}$  in period  $t$ . The equation reads that next periods capital stock is equal to investment which is the difference between output and consumption. Change in capital stock occurs when actual investment exceeds break-even investment. However here there is total depreciation so  $K_{t+1}$  simply equals investment. Consumer utility function is assumed to be of the logarithmic form:

$$\sum_{t=0}^T \beta^t \ln(C_t)$$

where  $0 < \beta < 1$  is a discount factor that captures the intertemporal nature of consumption choices. Agents must choose the control variable in order to maximise their utility subject to the constraint given by the capital evolution equation. The objective function is given by:

$$L = \sum_{t=0}^T \beta^t \ln(C_t) - \beta^{t+1} \lambda_{t+1} [K_{t+1} - Z_t K_t^\alpha + C_t]$$

The first order conditions with respect to the control and state variables respectively yield:

$$1/C_t = \beta \lambda_{t+1} \dots\dots\dots(1)$$

$$\lambda_t = \beta \lambda_{t+1} \alpha Z_t K_t^{\alpha-1} \dots\dots\dots(2)$$

In the terminal period  $t = T$ , no further investment is made therefore consumption equals total output. That is, since  $I_T = 0$ ,  $C_T = Z_T K_T^\alpha$  and  $K_{T+1} = 0$ . Substituting this into equation 1 in the final period gives:  $1/ \beta Z_T K_T^\alpha = \lambda_{T+1}$ . Substituting into equation 2,  $\lambda_T = \beta \alpha Z_T K_T^{\alpha-1} / \beta Z_T K_T^\alpha$ . This yields:

$$\lambda_T = \alpha / K_T \dots\dots\dots(3)$$

Equations 1 - 3 and the capital evolution equation are used to trace the models optimal path by backward induction. When this is done, a pattern emerges such that if we define  $T - t = \tau$ , we can write:

$C_t = [1 + \beta\alpha + (\beta\alpha)^2 + \dots + (\beta\alpha)^t]^{-1} Z_t K_t$ . The denominator is an infinite sum, thus we can write :  $C_t = (1 - \beta\alpha) Z_t K_t$ . The process also yields:  $\lambda_t = [1 + \beta\alpha + (\beta\alpha)^2 + \dots + (\beta\alpha)^t] \alpha K_t^{-1}$ . This also can be written as:  $\lambda_t = (1 - \beta\alpha) \alpha K_t^{-1}$ . From these equations we deduce that the optimum consumption level is a constant fraction of output  $(1 - \beta\alpha)$ .