PREDATORY PRICING IN AN OLIGOPOLISTIC FRAMEWORK

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ABSTRACT

In this paper we study the nature of predatory behavior in an oligopolistic framework. We use the long-purse story of financial vulnerability to demonstrate that predatory behavior is less likely to occur in an oligopoly than in a monopoly. We show the nature of the free-rider problem, and illustrate the range of multiple equilibria that may exist in this situation. We also show how small firms may be less likely targets for predatory attacks than their larger, more efficient rivals, examine the model with uncertainty added, and discuss the model's application to antitrust.

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I. INTRODUCTION

This paper examines predatory pricing in an oligopoly. Predatory behavior has been extensively studied in economics[1], but always from the perspective of one firm attempting to drive out another. Few industries are characterized by such structures, and this paper is the first to study whether predatory behavior is more or less likely in an oligopolistic market. We find that in a differentiated products, price-setting model, the incentive to engage in predation is less than in a monopoly. The more realistic set up of the model may help explain why predation is so rarely proven to exist in actual markets, and we suggest that the US courts sceptical attitude towards predation can be partly justified by a realization that most markets are characterized by oligopolies where predation is unlikely.

Ordover and Saloner (1989) divide theories of predatory behavior into three main areas, and we have to chosen to concentrate on the “long purse” story[2]. This has a long informal history in the literature, but we base our analysis upon Bolton and Scharfstein (1990) and Fudenberg and Tirole (1989). In such models a “deep pocketed” firm may attempt to drive a financially constrained competitor out of the market by engaging in a price war. The financial constraint arises due to capital market imperfections, and, if the price war lowers the constrained firm’s profits sufficiently in the first period, it will be forced to leave the industry.

We extend this type of analysis to a location model with three firms distributed around a circle. We show that there can be free riding in predation and that the incentives to prey may be lower than conventionally supposed. We also show that the presence of debt may lead to lower prices[3]. Thus, the model may help explain why apparently vulnerable firms are not attacked more often. Consider, for example, the growth of the Fox network in US television broadcasting. Despite the need for large, up-front investment, a new network - perceived to be financially constrained - was not seriously “attacked” by any of the incumbent networks. Fox has survived and has grown into an almost equal player in the industry. Alternately, existing firms may attempt to co-ordinate their efforts and prey on a financially vulnerable enterprise - an alleged example of this being the case of the Channel Tunnel (Chunnel) between England and France. Two major ferry companies began a price war when the Chunnel opened. The price war continues today; in March 1996 the Chunnel announced losses for the year of nearly £900 million, leaving their total debts at over £6 billion. This loss was blamed by the management upon the "malicious" attitudes of the ferry and hovercraft companies, whose "unrealistically" low prices were responsible for the financial distress of the Chunnel[4].

The model also applies to industries which have a number of well-established incumbents, one of which, for whatever reason, has (or is perceived to have) "shallower pockets" than its rivals. An example of this can be found in the newspaper industry in the UK, where the centre-right daily *The Times* - which enjoys considerable cross-subsidization from the rest of Rupert Murdoch's News International Corporation - has lowered prices aggressively in an attempt to drive out the centre-left *Independent*. The latter's sales have dropped by over 15 % in the last year leaving it close to bankruptcy. Part of the reason it still exists is the fact that the *Guardian*, which occupies the product space to the *Independent's* left, has refused to cut its prices. There has been considerable attention focused on this case, and Parliament has debated the issue formally, but no charges have been made against the *Times*. These three stories illustrate two types of situation: the first is a case of the incumbents refusing to prey upon the financially vulnerable entrant, and the second and third are cases where predation does take place. In this paper we obtain multiple equilibria, with and without predation, thus allowing for both outcomes in a single model.

Another alleged example of multi-firm predation, which has actually come before the US courts, is the 1986 *Matsushita v Zenith* case, where a number of US companies accused a small group of Japanese firms of collusive price-cutting in the television industry. The US firms alleged that the Japanese firms deep pockets came from collusive monopoly profits garnered in their home markets rather than from the US firms financial weaknesses, but the basic principle is similar. This could be considered a predatory equilibrium, but the US courts found insufficient evidence to support the plaintiff's claims.

The literature on predation is closely related to the literature on entry-deterrence[5]. The majority of such models assume quantity competition, where capacity accumulation has a commitment value. This credible threat of high future production discourages entry. Again, most of this literature assumes a single incumbent, with Gilbert and Vives (1986), Waldman (1987), and Kovenock and Roy (1997) being notable exceptions. Gilbert and Vives find that there might be *too much* entry deterrence from an industry perspective. If entry deterrence is taking place, every firm wishes to be the primary focus of deterrence and to have as high an output as possible, leading to industry output being higher than in the case of no entry-deterrence taking place. Their results depend upon the assumptions of Cournot competition and constant marginal costs. Kovenock and Roy show that when products are sufficiently differentiated, equilibria exist where some *under*investment in entry prevention can occur, but they did not find that an oligopoly would deter entry less than a monopoly.

We use a price-setting, differentiated products model to examine similar issues. This seems a natural setting within which to analyze the case of preying upon an already existing rival. Prices are usually not considered as having any commitment value, but one typically considers a predatory episode as being initiated by a price cut rather than an increase in quantity. This is particularly apt in the case of an industry with differentiated products (eg. the British newspaper industry). We obtain stronger free rider results than the entry deterrence literature, as the circular city model, which, in common with most price setting games, has prices as strategic complements, has the private benefit from forced exit decreasing in prices. A decrease in a firm's price will raise its quantity sold, but the reduction in the price per unit sold will eventually be sufficient to lower its profits, leaving it with an incentive to make a smaller contribution to the lowering of prices necessary for successful predatory behavior. Under such circumstances, an individual firm would prefer one of its rivals to shoulder the responsibility for dropping its price in order to force the financially constrained firm out of the market. This contrasts with the case of entry deterrence - usually marked by strategic substitutes - where each firm wants to be the major force in a successful act of deterrence.

Gilbert and Vives identified two types of "underinvestment" in entry deterrence, and we consider their analogues in predation: (1) That either predation or no predation can be an industry equilibrium, but incumbents profits are higher when entry is prevented: (2) A monopoly preys in more situations than an oligopoly. Neither type of underinvestment occurs in Gilbert and Vives. In Kovenock and Roy, type (2) underinvestment does not take place, but - provided products are sufficiently differentiated - incumbents profits can be higher in entry prevention equilibria. We analyze both types of underinvestment in a predation framework, and find the same result for type (1) holds in our model, but also show that - with sufficient differentiation - type (2) also occurs; a monopoly will prey more than an oligopoly.

Following Waldman (1987), we also extend the model by analysing the situation where the "deep pocket" firms are uncertain about the exact financial target faced by the preyed-on firm. Given the extra complexity of the framework, we cannot obtain exact results, but it seems that outcomes where oligopolies prey less are possible, and, as was the case with certainty, this free-rider result is more likely to hold when products are highly differentiated. We also examine how adding to the number of firms affects the results, and also study how smaller, "weaker" firms might actually be preyed on less than larger ones

The layout of the paper is as follows: Section 2 outlines the financial contract framework, and sets up the model. Section 3 characterizes the equilibrium, while Section 4 looks at the public good problem. Section 5 discusses some comparative static issues and extends the model to the case of uncertainty, and Section 6 relates the results to public policy and concludes.

II. THE BASIS FOR PREDATION AND THE MODEL SETUP

We start this section by briefly sketching a modified version of the financial contracting model specified by Fudenberg and Tirole. Those interested in a detailed presentation should consult the original paper. Note that the Bolton and Scharfstein's model could equally weel be used to obtain the desired result. There are 3 firms in the industry, all competing in periods 1 and 2. At the end of the first period each firm must pay a fixed cost, K, to continue operations in the second period. We assume that all but one of the firms can finance this entirely through their own retained funds, but one firm, which we denote as firm N, must finance this cost through its first period profits and whatever it can borrow from a bank. Thus they must borrow an amount which we call $D = K - \pi_1$. In the second period we assume there is a disturbance term affecting profits, which we label θ, where θ takes on a random value in the interval $[\theta_L,\theta_H]$ with the mean of $\theta = 1$. In a deterministic environment, the second period would see the firm earning π_{2B} - the Bertrand-Nash level of profit -, but in this stochastic situation expected second period profits are now distributed randomly over some interval $[\pi_L,\pi_H]$. The firm must reimburse the bank D(1+r), where r is the rate of interest, at the end of the second period. If the realized level of profit $\underline{\pi}_2$ is greater than or equal to D(1+r), then the firm retains the remaining profits. If $\underline{\pi}_2$ is less than D(1+r), then the firm goes bankrupt and earns nothing. We assume there are bankruptcy costs of a legal and administrative nature, equal to B, so the bank retains $\underline{\pi}_2$ - B if bankruptcy occurs. Fudenberg and Tirole assume a competitive supply of banks, implying a zero-profit condition, and thus obtain an expression for the net benefit to the firm of paying the fixed cost and continuing operations in the second period.

$$\text{Net benefit} = [E(\pi_2) - (1+r)K] - [BF((1+r)(K-\pi_1))] \tag{1}$$

where E (.) refers to the expectation operator, while F(.) is the cumulative distribution function of θ. The first term shows the project's net present value in a world without financial imperfections, while the second term represents the cost of bankruptcy multiplied by the probability that it occurs.

It follows that $d(\text{benefit})/d\pi_1 > 0$, ie, the higher first period profits are, the higher the net benefit of the investment to the firm. High first period profits lower the probability of bankruptcy, reducing bankruptcy costs which are passed on to the firm, making the project look more attractive. Clearly the firm will only pay the cost, K, if the net expected benefit is greater than zero. This will only be the case if first period profits are above a certain level, and we can immediately see that there must exist some target level of first period profits, which we call π_E, below which the firm will exit the industry. This target provides an opportunity for the firms competitors to prey.

We model the market framework by considering a two-stage noncoperative game played by three firms (firms i, j, and N). In stage 1, all firms set prices p_1, and earn profits. Firms i and j play again in the second period. However, as shown above, firm N must earn a minimum level of profits, π_E, to participate in the second period, otherwise it exits the industry. In the second period, whatever firms are left all play the Bertrand game again, this time with a random element to demand, and earn their corresponding profit levels. Firms are assumed to be risk neutral and to maximize the expected value of their profits over the two periods.

The oligopolistic framework we use is the circular city model developed by Salop (1979). The three firms are located at equidistant points around a circle of unit perimeter. Consumers are located around the circle with uniform density, buy one unit of the good, and face a unit transportation cost t. The firms have identical cost functions with constant marginal costs, $C(q) = cq$, for $q \geq 0$. Each firm is in competition with both of its neighbours, and a consumer located at any distance, $x \in (0,1/3)$ from firm i, is indifferent between purchasing from firm i and from another firm if:

$$p_i + tx = p + t(1/3 - x) \tag{1}$$

Single period profits for firm i can thus be written as:

$$\pi_i = (p_i - c)\,[\{p_j + t/3 - p_i\}/2t) + (\{p_N + t/3 - p_i\}/2t)] \tag{2}$$

Firm N must reach the target profit level, π_E, in order to be refinanced, leaving the other two firms to trade off the gain in second period profits resulting from forced exit, against the cost of predation in the first period[6]. Prices are strategic complements in this game, implying upward sloping reaction functions. The symmetry of the problem means that, in terms of the prices necessary to force out firm N, only the aggregate of p_i and p_j matters. This allows us to write $\pi_N(p_N,p_i,p_j)$ as $\pi_N(p_N,P)$, where $P = p_i + p_j$.   We summarize these results in lemma 1.

Lemma 1: (a) a firm's best response function can be written as: $p_N = (p_i + p_j)/4 + c/2 + t/6$

(b) The symmetric Nash equilibrium of the 3-firm game sees each firm earning profits of t/9. The equilibrium of the 2-firm game sees profits equal to t/4. The equilibrium price in a symmetric equilibrium, called p_B, sees firms setting price equal to the sum of c and t/3.

(c) For any combination of prices, $\{p_a,p_b\}$, $\pi_N(p_N,p_{1a} + p_{2a}) = \pi_N(p_N, p_{1b} + p_{2b})$ if

$p_{1a} + p_{2a} = p_{1b} + p_{2b} = P$

Proof: See appendix

Part (c) is useful, as it allows us to replace π_E (the target profit level) with P. The "deep pocket" firms must set prices such that their sum is less than or equal to P. The relationship between π_E and P is as follows:

Lemma 2: For a given π_E any level of the aggregate price, P, equal to or less than $[4\sqrt{\pi_E}\sqrt{t} + 2c - 2t/3]$ will drive firm N out of the market.

Proof: See appendix.

This implies that the higher the target profit level, π_E, the higher the aggregate price required may be. For instance, if the firm requires a profit level of 2 to be refinanced, this implies that if c is equal to 5 and t is equal to 18, P must be less than or equal to 22.

III. NASH EQUILIBRIUM

We now look for the subgame perfect Nash equilibria (SPNE) for the model. We denote total expected profits over the two periods by Π. We define $\varphi(p_N,p_j)$ as the set of first-period best responses for firm i given the other two firms prices:

$$\varphi(p_N,p_j) = \{p_i > 0: \Pi(p_i,p_N,p_j) \geq \Pi(y,p_N,p_j) \text{ for all } y > 0\} \tag{3}$$

Thus, (p_i) is a SPNE if $p_i \in \varphi(P_{-i})$ for all i. Given the levels of both p_j and p_N, firm i may decided to engage in predatory behavior and induce exit by charging $p_i \leq P - p_j$. Then it earns:

$$\Pi_{iP} = \pi_{i1}(p_i,p_N,p_j) + E(\pi_{i2D}) \tag{4}$$

We call Π_{iP} the optimal level of profits over the two periods which can be earned through predatory behavior. The second term, $E(\pi_{i2D})$ refers to the expected value of the duopoly profits earned in the second period after firm N has exited. Alternately, firm i may choose not to prey and to set $p_i \geq P - p_j$. Then profits are equal to:

$$\Pi_{iP} \;=\; \pi_{i1}(p_i,p_N,p_j) + E(\pi_{i2T}) \qquad (5)$$

Here, Π_{iP} refers to total two-period profits earned when no predation takes place. The term $E(\pi_{i2T})$ refers to the expected value of the triopoly profits earned in the second period. Looking at these two equations, we see firm i's profits have a discontinuity at $p_i = P - p_j$.

We let $r(p_N,p_j) = \text{argmax}\ \Pi_{iNP}(p_i,p_N,p_j)$; this is the best response of firm i to the other prices if predation does not occur. This is just the Bertrand-Nash response of firm i, for if no predation is to take place, the firm chooses its optimal response based on its rivals prices. When $p_j + r(p_N,p_j) \leq P$, firm i preys by pricing according to its Bertrand best response function; it does not deliberately lower prices and yet exercises a predatory function. From lemma 1, we know that $p_j + r(p_N,p_j)$ is increasing in p_j, so, assuming that firm N is playing a best response to the aggregate of the other two prices, we can be sure there is a unique solution to $p_j + r(p_N,p_j) = P$. We call this solution $\underline{p}_j(P)$. Thus if $p_j \leq \underline{p}_j(P)$, firm i sets price equal to $r(p_N,p_j)$ and predation and exit take place. This is analogous to the case of entry being blockaded in entry-deterrence models. A firm that finds itself sufficiently financially constrained will thus be preyed upon, yet it would not be "predation" in any legal sense as the other firms are not attempting to price below their usual level[7].

Now we analyze what happens when $p_j > \underline{p}_j(P)$. Here firm i can prey by setting $p_i = P - p_j$ and earn Π_{iP}, or it can accommodate firm N and set $p_i = r(p_N,p_j)$ and earn Π_{iNP}. We know that at $p_j = \underline{p}_j(P)$, $\Pi_{iP} > \Pi_{iNP}$ at $p_i = P - p_j$. We show in the appendix that Π_{iNP} is continuously rising in p_j, as one would expect in a game with strategic complements. Characterizing the path of Π_{iP} is somewhat more difficult. Intuitively, the higher the price one's rival charges, the lower the price firm i has to charge, which should result in lower profits for firm i. However, it is possible that a high value of firm j's price could see firm i's price being lowered sufficiently to boost market share enough to raise profits. But we show in the appendix that even if profits rise temporarily, they will always fall eventually. The change in firm i's profits can be decomposed into (1) the markup multiplied by the change in quantity plus (2) the quantity times the change in markup. In the case of predation, when firm i is lowering the price on all units, she is losing money on the quantity she is already selling even if she is selling more units. This effect becomes more pronounced the larger firm j's price is relative to firm i's and thus drags profits down.

Lemma 3: $\Pi_{iP} > \Pi_{iNP}$ at $\underline{p}_j$; Π_{iNP} rises continuously for all p_j above $\underline{p}_j$, while Π_{iP} eventually falls for higher values of p_j. Thus there exists a value of p_j such that $\Pi_{iP} = \Pi_{iNP}$ and beyond which $\Pi_{iP} < \Pi_{iNP}$.

Proof: See appendix.

We call $p^*_j(P)$ the solution to $\Pi_{iP} = \Pi_{iNP}$ (we assume the solution is positive; otherwise let it be equal to zero). We show this in figure 1. Note that Π_B refers to firm i's maximum level of profits over the two periods when it plays a Bertrand-Nash response in both periods. We can now write the best reply correspondence of firm i:

$$\varphi(p_j,p_N) = \begin{cases} r(p_N,p_j) & \text{if } p_j \leq \underline{p}_j(P) \quad \text{(natural predation)} \\ P - p_j & \text{if } \underline{p}_j(P) < p_j \leq p^*_j(P) \quad \text{(predation)} \\ r(p_N,p_j) & \text{if } p_j \geq p^*_j(P) \quad \text{(no predation)} \end{cases} \tag{6}$$

This best reply correspondence is displayed graphically in figure 2. Note that when firms i and j have an aggregate price of P, firm N will always play the same best response regardless of the relative values of p_i and p_j. We show below that we will never see predatory equilibria where $p_i + p_j < P$. At $p^*_j(P)$ firm i is indifferent between preying upon the financially-constrained rival and pricing normally. We now develop a property of $p^*_j(P)$ in lemma 4:

Lemma 4: $p^*_j(P)$ is increasing in P.

Proof: See appendix.

The type of equilibria that will emerge depends upon where the φ's intersect in the various regions. We let p_B be the equilibrium prices chosen when predation does not occur; the aggregate of the firm i and j's Bertrand set of prices is $P_B = 2p_B = 2(c + t/3)$. If $P_B < P$ then predation occurs naturally at the Bertrand equilibrium. When this condition does not occur, we define P_H as the largest P such that both φ's intersect at p_B. Thus P_H solves $p^*_j(P) = p_B$. When firm j is setting price at p_B, firm i is indifferent between preying and tolerating firm N. Now let P_L be the smallest P such that the φ's intersect on $p_i + p_j = P$. In other words, P_L is the smallest P such that to prey is an equilibrium. P_L solves $p^*_j(P) = P/2$; at the smallest P at which predation can take place there should be equal prices chosen by each of firms i and j. Now, p_B must be greater than P/2, otherwise we would be in a case of natural predation. As we know that $p^*_j(P)$ is increasing in P, we have established that $P_H > P_L$. Thus:

Proposition 1: (i) If $P \geq P_B$, then each firm sets prices at the Bertrand level. Firm N earns profits below π_E and will not be refinanced for the second period.

(ii) If $P_B > P \geq P_L$, then any p_i such that $r(p_N,p_i) \geq p_i$, and $p_i + p_j \leq P$ is an equilibrium where firm N is preyed upon and will not be refinanced for the second period.

(iii) If $P \leq P_H$, then each firm sets price at the Bertrand level and firm N is not preyed upon. Firm N's profits are greater than or equal to π_E and it is refinanced for the second period.

We can summarize this by saying that if the aggregate of the Bertrand prices is sufficiently low, predation occurs naturally and exit ensues. When P is less than the Bertrand price but more than P_H, we are in a situation where true predatory pricing does take place, as at least one of the firms is reducing its price below what it would otherwise have been. There is a continuum of such predatory equilibria, with the burden of lower prices being shifted around among the incumbent firms. When P is less than P_L we are in a situation where predation will not occur, and where the firm will be refinanced and will become an established part of the market. When P is greater than P_H firm N is always forced out. Most interestingly, when $P_L \leq P \leq P_H$ we have both types of equilibria occurring; a similar outcome to Gilbert and Vives, and Kovenock and Roy. Figure 3 illustrates the various regions and outcomes that can occur.

We finish this section by ruling out the existence of predatory equilibria where the aggregate of firm i and j's prices is actually less than the value of P, a situation which we would see a "wastefully" high amount of predation. If predation does occur, then the sum of prices will always exactly be equal to P.

Lemma 5: No predatory equilibria exist where $p_i + p_j < P$.

Proof: See appendix.

IV. PREDATORY BEHAVIOR AS A PUBLIC GOOD

We now come to the question posed in the introduction: will firms "underinvest" in predatory behavior ? We show that they will for a large range of parameter values in the model analyzed here. Following Gilbert and Vives, we associate "underinvestment" with either or both of the two types mentioned in the introduction: (1) Both predatory and non-predatory behavior are industry equilibria but profits are higher when predation takes place. (2) A monopoly would prey more than an oligopoly.

There is a counterforce to the possibility of the second of these phenomena, in that a firm will earn a lower level of profits in a triopoly than a duopoly. This means a cash-constrained firm is less likely to be blockaded automatically in a situation with one, rather than two, incumbents. But we show that provided the variable, t, is large enough, this effect is swamped by the predatory advantages of a monopoly, and we show that type 1 underinvestment always occurs. We summarize this in proposition 2:

Proposition 2: (i) When $P_L \leq P \leq P_H$, predation and non-predation are both equilibria, but the profits of "deep-pocket" firms are higher under predation.

(ii) As long as $\sqrt{\pi_E} < (5\sqrt{t})/16$, then a monopolist will prey whenever a duopoly would, and will prey in situations where a duopoly would not.

Proof: See appendix.

Note that actual prices are lower at the predatory equilibrium than they would be at the non-predatory equilibrium, which accords with the basic definition of predation: firms lower prices in order to hurt an opponent[8]. The fact that there is a continuum of predatory equilibrium means the degree to which an individual firm lowers its price is indeterminate. One could observe an outcome where both firms lower their prices by the same amount, or one could have an equilibrium where one firm maintains a relatively high price and its rival suffers a larger fall in profits. In either case the financially constrained rival is driven out of the market. The condition in part (ii) can be interpreted as saying that for a fixed value of t, the oligopoly will prey less than the monopoly as long as the target profit level is not too high. As t rises, the predatory advantage enjoyed by the monopolist increases[9].

These results can be contrasted with those of Gilbert and Vives. In their model each firm wanted to take on the burden of entry-prevention and to have as high an output as possible.

Here, each firm wants the other to be the predator, as the private benefit from forced exit is decreasing in the "deep-pockets" firm's price. The results are closer to Kovenock and Roy's who predominantly studied a differentiated-product quantity-setting game, though they still found that type (2) underinvestment did not occur. The key difference is that here firms compete in strategic complements rather than strategic substitutes. Kovenock and Roy did find that type (1) underinvestment would occur provided that firms were sufficiently differentiated. We find that type (1) underinvestment always occurs, regardless of the degree of product differentiation. We can also relate our results to the general analysis of Appelbaum and Weber (1992), who suggest that public good underinvestment tends to occur when an increase in a precommitment variable of firm i increases the profits of the other incumbents. Although we use a somewhat different framework to them - not having any precommitment variables - our strategic complements set-up is clearly analogous to the issues they discuss.

V. COMPARATIVE STATICS AND UNCERTAINTY

In this section, we look at some comparative static issues and add uncertainty to the model. We first examine whether the set of outcomes where predation occurs is smaller or larger when firm N has different parameters to the "deep pocket" firms. Here we refer to cost efficiency or some notion of size, looking at whether "deep pocket" incumbents will take advantage of a strong rival's financial weakness to try push them out of the industry, or whether they will be aggressive towards a weaker rival who will be more easily prevented from obtaining the required profit level. There is anecdotal evidence to suggest that small, seemingly inefficient firms are more likely to be allowed to remain unmolested within an industry than potentially large rivals, who may be preyed upon when vulnerable rather than being let become entrenched in the industry. Again, the Chunnel is an example of a potentially destructive competitor to the ferry companies; this awareness enticed them into attacking it when it was at its most financially vulnerable. This also relates to Gellman and Salop's (1983) paper on "judo economics", where an entrant deliberately limits its capacity in order to avoid a destructive response from its competitor.

We model size in terms of the cost function of the firm. If the new firm is a large rival then it has costs, $c_N < c$. If it is small, then $c_N > c$. If the Bertrand-Nash equilibrium is recalculated, the prices become:

$$p_{i,j} = t/3 + 4c/5 + c_N/5 \qquad p_N = t/3 + 2c/5 + 3c_N/5 \tag{7}$$

If firm N is weak (strong) and c_N is higher (lower) than c, profits will be higher (lower) for the incumbents than they would have been with a symmetric firm. This leads to two effects: (1) A high (low) cost firm will be easier (harder) to force out of the industry, as a higher aggregate price level, P, will drive out a firm with the same target profit level as before. (2) But the gains from driving out a high (low) cost firm will be smaller (larger) than if they had been symmetric to the incumbents. These two effects work in opposite directions, with the first implying a small firm is easier to prey upon, whereas the second implies the incentive to do so is less. In terms of the model, Π_{iP} and Π_{iNP} have a smaller jump than before, and with the functions converging at the same rate, p^*_j will occur at a lower value of p_j than before. Intuitively, the gains from preying have fallen due to the gains from ousting firm i not being as large, but the increased intrinsic vulnerability of firm N will mean that the zone of "natural predation" has increased. Here, P is larger than before, and $p^*_j(P)$ is increasing in P, so we know that P_L will rise. Similarly, p_B will be higher in the non-predatory equilibrium, so P_H will rise also. The zone of accommodation has increased, as has the zone of "natural predation". But the relevant P is now larger and more likely to fall into the zones of predatory behavior. We cannot be sure whether predation is more or less likely to occur, but it is possible that a small firm might not be preyed upon as much as a rival of equal efficiency to the incumbent[10].

We also look at what happens when the number of competing firms increases from 3 to 4. Here four firms are located around the circle, with the extra firm, firm k, not directly in competition with firm N. The Bertrand-Nash equilibrium is for each firm to set its price at p = c + t/4, with profit levels equal to t/16. The benefits of predation are less than in the case of 3 firms, with the gain from exit for the deep-pocket firms being only 7t/144 rather than 5t/36. However, the cost of preying is less, as firm k - not in competition with firm N - will not lower its price by as much as firms i and j. Thus firms i and j's profits do not fall as much as they would have with 3 firms, as their sales are not eaten into by firm k dropping its price as well. Thus Π_{iP} falls more slowly and Π_{iNP} rises more slowly. This is shown in figure 4, and formalized in Lemma 6:

Lemma 6: With four firms, Π_{iP} falls more slowly, and Π_{iNP} rises more slowly than with three firms.

Proof: See appendix

The outcome is ambiguous with regard to whether predatory behavior is more or less likely to occur. The costs of predation are lower, but so is the benefit, and it is possible that $p^*_j(P)$ can be greater or less than was the case with three firms. This analysis is predicated upon the assumption that all three "deep-pocket" firms costlessly relocate to optimal points on the circle after firm N has exited. In reality, if firm i and j surround firm N, it seems probable that they will gain most from forcing it out, thereby leading to greater benefits for them and a corresponding higher probability of predation. We cannot obtain exact results, but would suggest that as more firms are added these two conflicting effects remain: (1) the free rider problem worsens, as the burden of predation always falls on just the two firms surrounding the vulnerable firm, and the gain from exit falls with an increase in the number of firms. (2) the actual mechanics of predation are easier, as the other firms do not fully engage in the debilitating dropping of prices.

So far the model has been one of certainty. There are a number of ways of introducing uncertainty, but we think that having firms i and j uncertain about the target profit level, π_E, of firm N is the most plausible. We model this by letting π_E be a draw from a random variable with a cumulative distribution function, H(•). Firm i and j's beliefs about the target profit level thus imply a distribution function $H(\pi_E)$ which is defined on $\{\pi_E^L,\pi_E^H\}$. Given the relationship between π_E and P, this allows us to define a distribution function F(P) on $\{P^L,P^H\}$. This is assumed to satisfy the following restrictions: $F(P^L) = 0$, $F(P^H) = 1$, $F'(z) > 0$, and $F''(z)$ exists for all $z \in (P^L,P^H)$. Let F*(P) denote the realization of P such that firm N exactly earns its target level of profit. Thus predation is successful if and only if $F < F^*(P)$. We write E(P) to denote the probability of predation given the sum of prices, P, and, for simplicity, assume that $E(P) > 0$ for all cases we examine[11].

We want to see if the free-rider issue is more or less of a factor in this framework. We omit the details here but anyone interested should contact the author for a note on the derivation. Following Waldman (1987) and his extension of the Gilbert and Vives framework, we cannot prove the existence of equilibrium, but concentrate on whether a monopolist would prey more or less than an oligopoly in any equilibrium that does exist. We approach the problem by assuming a monopolist preys less, which would imply that $(p_i + p_j)$ - the sum of prices in a duopoly - is less than $(2p_m + t/3)$, the price level at which a duopoly would prey exactly as much as a monopoly. This gives us the result that a monopolist will prey more than a duopoly if and only if the following condition holds:

$$\pi_i(p_m+t/6,p_m+t/6,p_N) - \pi_i(2p_m+t/3-p_j,p_j,p_N) < [E(p_i+p_j) - E(2p_m+t/3)][17t/72] \qquad (8)$$

Note that the RHS of this expression is always positive, and becomes greater as t rises. Unfortunately, we cannot sign the LHS. If it is negative, then a monopolist always preys more and the addition of uncertainty has strengthened the free-rider problem. However, we note that the LHS is equal to zero when $(p_i + p_j)$ is equal to $(2p_m + t/3)$, and thus it is possible that the LHS will be greater than zero. Under these circumstances, we cannot be sure whether predation is more likely in a monopoly or not.

Kovenock and Roy criticize Waldman's approach for only being able to deal with symmetric equilibria, thus circumventing the issue of multiple equilibria and not allowing both predatory and non-predatory equilibria to exist simultaneously. We note the same problem here, and the lack of any formal proof that an equilibrium exists means that the results with uncertainty should be approached with some caution.

VI. CONCLUSION

This paper has analyzed predatory pricing in an oligopolistic framework, offering a break with the previous tradition of discussing the phenomenon in the context of a monopolist preying upon a new entrant. We motivated the idea by using the agency problems that can arise in financial contracting to arrive at a situation where one firm is vulnerable to predation by the others. This idea is particularly relevant when one considers the venture capital market, but is applicable to any firm experiencing financial distress. Within this framework there exist considerable free rider problems, and we showed that predatory behavior is less likely to occur than in a monopoly.

The welfare consequences of predation are not clear, as consumers will benefit from lower prices in the first period, but may face higher second period prices. However, the model of predation analyzed here considers lowering prices to be the only form of predatory behavior. In reality, as Ordover and Saloner (1989) point out, predation can include product proliferation, advertising, and other forms of non-price behavior, and a more complete analysis of the problem would take such concepts into account. In particular, the free rider problem might be accentuated if firms were allowed to relocate themselves in the product space to take customers away from the firm preying upon the financially constrained firm. Introducing switching costs would also be interesting, and could lead to a reversal of our conclusions, as the desire to create a large first period market share would neatly dovetail with the incentive to lower prices and force the newcomer out of the market.

The legal consequences of multi-firm predation are also unclear, but the judgment in the *Matsushita* case indicates the difficulty in identifying culpability if a combination of firms are involved. The judges made clear that they would require explicit evidence of collusion between firms in order to prove multi-firm predatory intent; in the absence of such evidence predation would have to be proved for individual defendants[12]. And in our examples, few individual firms would have enough market power to be considered able to prey on their own. In addition, a firm might participate in a predatory attack, yet still be pricing above cost, thus not satisfying any of the standard tests of predation. This might seem to indicate that multi-firm predation is possible, in that it would be difficult for a court to identify that predatory behavior actually occurred, but as Cabral and Riordan (1997) indicate, the American courts have taken a sceptical attitude over the general issue of predation in recent years. We should also point out that the predation will probably be even harder to sustain as a multi-firm equilibrium in both the signaling and reputation versions of the phenomenon. For two firms to convince a preyed-on firm that both incumbents costs were low would be difficult in the signaling story, and persuading a newcomer that both incumbents were “tough” would also be hard.

A final factor arising is a possible re-evaluation of McGee’s (1980) famous claim that predatory behavior was irrational, as the firm would always choose to merge with its opponent as a less costly method of obtaining monopoly power. When two or more firms are attempting to remove a rival, the option of merging may not exist, as that would concentrate power in the hands of the merging firm, unless we consider a highly unlikely "grand coalition" (which would probably be prevented on antitrust grounds anyway). In this situation, agreeing to prey rather than merge may be the only solution for a group of firms who wish to preserve their market power, and thus predation again becomes an important issue[13].

APPENDIX

Proof of lemma 1: (a) We need to show $p_i = (p_j + p_N)/4 + t/6 + c/2$

Firm i maximizes profits, which are: $(p_i - c)\,[(p_j + 2t/3 + p_N - 2p_i)/2t]$

$d\pi_i/dp_i = [p_j + 2t/3 + 2c + p_N - 4p_i]/2t = 0.$ This simplifies to:

$p_j + 2t/3 - 4p_i + 2c + p_N = 0$, which gives us: $p_i = (p_j + p_N)/4 + t/6 + c/2$

(b) We calculate profits in the symmetric one-shot Bertrand equilibrium. With three firms, we know that: $p_j = p_N = p_i = (p_i + p_i)/2 + t/6 + c/2.$ We get: $p_i/2 = t/6 + c/2$, which means that $p_i = c + t/3$. Thus, $\pi_i = (c + t/3 - c)\,[(t/3 + t/3)/2t]$, which simplifies to $\pi_i = t/9$.

Analogous reasoning in the two firm game produces: $\pi_i = t/4$.

(c) We need to show firm N's profits depend only on the sum of its opponents prices. Thus, $\pi_N = (p_N - c)\,[(p_1 + 2t/3 + p_2 - 2p_N)/2t]$, giving, $\pi_N = (p_N - c)\,[((p_1 + p_2)/2 + t/3 - p_N)/t]$.

Thus π_N is unaffected by the relative levels of p_1 and p_2. Q.E.D.

Proof of lemma 2: We want to connect the target profit level, π_E, with the target price level, P. From lemma 1, if we replace p_N with $(p_i + p_j)/4 + t/6 + c/2$, π_N can be written as:

$\pi_N = (((p_i + p_j)/4 + t/6 + c/2) - c)\,(((p_i + p_j)/2 + t/3 - (p_i + p_j)/4 - t/6 - c/2))/t$

Simplifying and taking square roots gives us: $\sqrt{\pi_N} = ((p_i + p_j)/4 + t/6 - c/2)/\sqrt{t}$

Thus, $\sqrt{t}\sqrt{\pi_N} - t/6 + c/2 = (p_i + p_j)/4$. If $p_i + p_j = P$, then $P = 4\sqrt{\pi_E}\sqrt{t} - 2t/3 + 2c$ Q.E.D.

Proof of lemma 3: We characterize the change in overall profits as p_j rises, but with second period profits constant throughout, we can confine analysis to changes in first period profits.

We write: $d\pi_i/dp_j = [(p_i - c)dq_i/dp_j + q_i\,[dp_i/dp_j]$.

For π_{iNP}: $dq_i/dp_j = d(p_j + t/3 - p_i(p_j))/2t + (p_N(p_j) + t/3 - p_i(p_j))/2t]/dp_j$ which is equal to: $1/2t - 1/8t + 1/8t - 1/8t = 3/8t$. Note that $dp_i/dp_j = ¼$ (from lemma 1), giving us: $d\pi_{iNP}/dp_j = ((3/8t)(p_i - c) + q_i/4$. This must be positive for any p_j, as if $p_i < c$, negative profits are being earned and firm N would exit for any π_E.

For π_{iP}/dp_j: we let $(P - p_j) = p_i$; $dq_i/dp_j = d[(p_j + t/3 - (P - p_j))/2t + (p_N + t/3 - (P - p_j))/2t]/dp_j$ This gives: $d[(2p_j + t/3 - P)/2t + (p_N + p_j + t/3 - P)/2t]dp_j$ Note p_N is constant, thus: $dq_i/dp_j = 1/t + 1/2t$. Note $dp_i/dp_j = -1$. Thus, $d\pi_{iP}/dp_j = (3/2t)(p_i - c) - q_i$. This expression can be positive, but as p_j rises, with p_N constant, p_i must fall and q_i must rise. This implies that $d\pi_{iP}/dp_j$ must eventually fall continuously. So, given π_{iP} eventually falls as p_j rises, and π_{iNP} continually rises, we reach a point where $\pi_{iP} = \pi_{iNP}$ and beyond which $\pi_{iP} < \pi_{iNP}$. Q.E.D.

Proof of lemma 4: We know, from its definition, that $p^*_j(P)$ solves $(P - p_j) - \Pi_{iNP}(p_j)$. By the implicit function theorem $d(p^*_j(P))/dP = -[(P - p_j)\partial q_i/\partial P + q_i]/[- q_i - \partial\pi_{iNP}(p_j)/\partial p_j]$. The sign depends on $\partial\Pi_{iNP}(p_j)/\partial p_j$. By lemma 3, this is > 0. This gives $d(p^*_j(P))/dP > 0$ Q.E.D.

Proof of lemma 5: We start by assuming that predatory equilibria exist where $p_i + p_j = Q < P$. Here we know that $\underline{p}_i$, the critical price which leaves firm N exactly earning π_E, must be greater than $Q - p_j$. We know that firm N will just play a best reponse to the aggregate Q. Thus for any $p_j < Q$, our candidate equilibrium must be: $[Q - p_j, p_j, Q/4 + c/2 + t/6]$

Looking at possible deviations from this equilibrium, they can be either:

(a) $BR_i(p_j,Q/4 + c/2 + t/6) \geq \underline{p}_i(p_j,Q/4 + c/2 +t/6)$. In this case, by the concavity of firm i's profit function we know that $\underline{p}_i(p_j, Q/4 + c/2 + t/6)$ yields higher profits than any lower price. But we know that $\underline{p}_i$ is greater than $Q - p_j$, implying that a profitable deviation exists and we do not have an equilibrium. The same will be true for any $p_i < Q$. OR

(b) $BR_i(p_j,Q/4 + c/2 + t/6) < \underline{p}_i(p_j,Q/4 + c/2 + t/6)$. Here firm i can force firm N to exit by choosing the optimal one-shot price in period 1. But this implies: $BR_i(p_j,Q/4 + c/2 + t/6) = Q - p_j = p_i$. We know from (a) that if $BR_j(p_i,Q/4 + c/2 + t/6) \geq \underline{p}_j(p_i,Q/4 + c/2 + t/6)$ we do not have an equilibrium. And, if $BR_j(p_i,Q/4 + c/2 +t/6) < \underline{p}_j(p_i,Q/4 + c/2 + t/6)$ then $p_j = Q - p_i$. So the only way for this to be an equilibrium is for each firm to be playing its static optimum against the other's price. But this is just the case of natural predation dealt with already. Thus any "non-natural" predatory equilibria must have $p_i + p_j = P$. Q.E.D.

Proof of proposition 2: (a) We know that $P_H \geq P_L$. According to the definition of P_L, profits with and without predation are equal when $P = P_L$ if firm j prices at p_B and $p_i = P_L - p_B$. Here firm i is indifferent between, (i) earning t/9 in each period and, (ii) earning - t/36 in the first and t/4 in the second. When $P < P_L$, in a predatory equilibrium, firm i sets $p_i \geq P - p^*_j(P)$. But $P - p^*_j(P)$ must be $\geq P_L - p^*_j(P)$, as $p^*_j(P)$ increases in P. But $P_L - p^*_j(P_L) = P_L - p_B$. Thus, when $P > P_L$, at a predatory equilibrium firm i sets a higher price and obtains higher profits than at any non-predatory equilibrium; profits are only equal where $P = P_L$.

(b) We need to show that whenever a duopolist preys, a monopolist would to, and there are situations where a monopolist preys and a duopolist does not. At P_L, the lowest value duopoly predation can occur at, we know $p_i = P/2$ for all i. Here each firm is indifferent between preying and accommodating; as each gains (t/4 - t/9) from preying, they must be prepared to give up (t/4 - t/9) = 5t/36. Thus profits are - t/36 in the first period. A monopolist gains (t - t/4), thus will tolerate profits as low as - t/2 in the first period. From lemma 2, we know that for any π_E, the required $P = 4\sqrt{\pi_E}\sqrt{t} + 2c - 2t/3$. Thus p_i must equal: $2\sqrt{\pi_E}\sqrt{t} + c - t/3$. We can show that, for a monopoly, the required cut-off level, which we call p_M, is: $2\sqrt{\pi_E}\sqrt{t} + c - t/3$. Thus, at the lowest predatory equilibrium: $p_i = p_M + t/6 = p_j$. For the duopoly, we know first period profits are: $(p_i - c)\,[((p_N + p_i)/2 + t/3 - p_i)/t] = - t/36$. We substitute for p_N and obtain: $(p_i - c)\,[(- p_i/4 + c/4 + 5t/12)/t] = - t/36$. If our propostion is true, then at π_M, the monopolist's profits must be strictly greater than - t/2; the monopolist is prepared to prey more and earn lower first period profits to drive out firm N. Thus: $(p_i - t/6 - c)[(- p_i/2 + c/2 + 5t/6)/t] > - t/2$. We note that $(- p_i/2 + c/2 + 5t/6)/t = 2(- p_i/4 + c/4 + 5t/12)/t$. We label $(- p_i/4 + c/4 + 5t/12)/t$ as X; and note $(p_i - c)X = - t/36$. This implies $X = - t/[36(18(p_i - c)$. Substituting back, we obtain: $(p_i - t/6 - c)\{- t/[18(p_i - c)] > - t/2$. After manipulation and reversing the sign of the inequality, we have: $t^2/6 < - 8p_it + 8tc$, which implies: $t^2 < 48t(c - p_i)$. From lemma 2, we have: $p_i = 2\sqrt{\pi_E}\sqrt{t} + c - t/3$, which means that $c - p_i = t/3 - \sqrt{\pi_E}\sqrt{t}$. This gives: $t < 48[t/3 - \sqrt{\pi_E}\sqrt{t}]$ implying $\sqrt{\pi_E}\sqrt{t} < 5t/16$. Thus, as long as $\sqrt{\pi_E}\sqrt{t} < 5t/16$, a duopoly will prey less frequently than a monopoly. Q.E.D.

Proof of lemma 6: We are attempting to find the same results as in lemma 3, but with an extra firm, k, in the market. For π_{iNP}, dq_i/dp_j is: $d[(p_k(p_j) + t/3 - p_i(p_j) + p_N(p_j) + t/3 - p_i(p_j))/2t]/dp_j$. As $dp_k(p_j)/dp_j$ is equal to 1/4, $d\pi_{iNP}/dp_j = 0(p_i - c) + q_i/4$. This is smaller than the case of three firms. For π_{iP}, we have $dq_i/dp_j = d[(p_j + p_k(p_j) + p_N(p_j) + 2t/3 - 2P)/2t]dp_j$. This leaves us with $d\pi_{iP}/dp_j = (9/8t)(p_i - c) - q_i$. This is smaller than for three firms. Q.E.D.

ENDNOTES

1. See Tirole (1988), Ch 9, or Ordover and Saloner (1990), for a summary of economic approaches to the problem of predation.
2. The two other main types are: (i) The reputation approach, where firms prey on new entrants in order to gain a reputation for harshness. This is usually only sustainable either in an infinitely repeated context or in a situation where entrants might believe an incumbent actually like to "fight". See Kreps and Wilson (1982) and Milgrom and Roberts (1982b). (ii) The signalling approach, wher the incumbent preys in order to attempt to convince the newcomer that it would be unprofitable to remain in the industry by influencing its opponents beliefs about the predator's costs or the nature of industry demand. See Scharfstein (1984) for a basic analysis and Milgrom and Roberts (1982a) for a discussion of the related limit-pricing phenomenon.
3. This relates to the literature on whether financial obligations lead to higher or lower prices in the product market. See Brander and Lewis (1984) for a model where debt also leads to lower prices and higher quantities.
4. The Chunnel's disastrous financial state has led to an ongoing debate about whether the British government should underwrite all its losses in order to keep it in operation.
5. For a comprehensive discussion of the nature of entry-deterrence, see Tirole (1988), Ch 8.
6. We assume that the surviving firms are able to costlessly relocate to points on the circle which allow them to earn the maximum second-period profit. In practice, this may be unlikely, but even if they remain in the same positions they will still earn a fixed second period profit. This is likely to be less than what they would have earned had they re-located, and may result in the gains from predation, and the range of parameters where it occurs, falling.
7. The legal literature on predation is voluminous. Note that this "natural predation" does not satisfy the test of predatory behavior proposed by Ordover and Willig (1981), and would surely not be considered predatory by the courts.
8. For two of the most well-known attempts to measure and detect predatory pricing, see Areeda and Turner (1975) and Ordover and Willig (1981).
9. Kovenock and Roy have a similar condition on product differentiation in their model - firm must be sufficiently differentiated for type 2 underinvestment to occur.
10. The analogy to judo economics is not perfect, as the weak firm does not consciously attempt to weaken itself or to restrict its output.
11. Note that, as in Waldman, any interior equilibrium must be symmetric. If E(P) = 0, then there may exist non-symmetric corner equilibria.
12. For a discussion of this case and the legal attitudes toward multi-firm predation, see Elzinga (1994).
13. Persson (1998) studies a model which considers mergers and predation from a multi-firm perspective.

REFERENCES

Applebaum, E., and S. Weber, (1992), "A note on the free rider problem in oligopoly", *Economics Letters*, 40, pp 473-480.

Areeda, P., and D. Turner, (1975), "Predatory pricing and related practices under section 2 of the Sherman act", *Harvard Law Review*, 88, pp 697-733.

Bolton, P., and D. Scharfstein, (1990), "A theory of predation based on agency problems in financial contracting", *American Economic Review*, 80, pp 93-107.
Brander, J., and T. Lewis, (1986), "Oligopoly and financial structure", *American Economic Review*, 76, pp 956-970.
Cabral, L., and M. Riordan, (1997), "The learning curve, predation, antitrust, and welfare", *Journal of Industrial Economics*, 45, pp 155-169.
Elzinga, K., (1994), "Collusive predation: *Matsushita v Zenith* 1986", in J. Kwoka and L. White (eds) *The Antitrust Revolution*, 2nd edition, Harper Collins.
Fudenberg, D., and J. Tirole, (1985), "Predation without reputation", Working Paper 377, Massachusetts Institute of Technology..
Gelman, J., and S. Salop, (1983), "Judo economics: capacity limitation and coupon competition", *Bell Journal of Economics*, 14, pp 315-325.
Gilbert, R., and X. Vives, (1986), "Entry deterrence and the free-rider problem", *Review of Economic Studies*, 53, pp 71-83.
Kovenock, D. and S. Roy, (1997), "Free riding in non-cooperative entry deterrence with differentiated products", mimeo.
Kreps, D., and R.Wilson, (1982), "Reputation and imperfect information", *Journal of Economic Theory*, 27, pp 253-279.
McGee, J., (1980), "Predatory pricing revisited", *Journal of Law and Economics*, 23, pp 289-330.
Milgrom, P., and D. Roberts, (1982a), "Limit pricing and entry under incomplete information: an equilibrium analysis", *Econometrica*, 50, pp 443-460.
Milgrom, P., and D. Roberts, (1982b), "Predation, reputation and entry deterrence", *Journal of Economic Theory*, 27, pp 280-312.
Ordover, J., and G. Saloner, (1989), "Predation, monopolization, and antitrust", Ch. 9 in the *Handbook of Industrial Organization*, Vol. 1, North Holland.
Ordover, J., and R. Willig, (1981), "An economic definition of predation: pricing and product innovation", *Yale Law Journal*, 91, pp 8-53.
Persson, L., (1998), "Predation and mergers: is merger law counterproductive ?", mimeo.
Salop, S., (1979), "Monopolistic competition with outside goods", *Bell Journal of Economics*, 10, pp 141-156.
Scharfstein, D., (1984), "A policy to test market predation", *Rand Journal of Economics*, 15, pp 229-243.
Tirole, J., (1988), *The theory of industrial organization*, MIT Press.
Waldman, M., (1987), "Non-cooperative entry deterrence, uncertainty and the free rider problem", *Review of Economic Studies*, 54, pp 301-310.

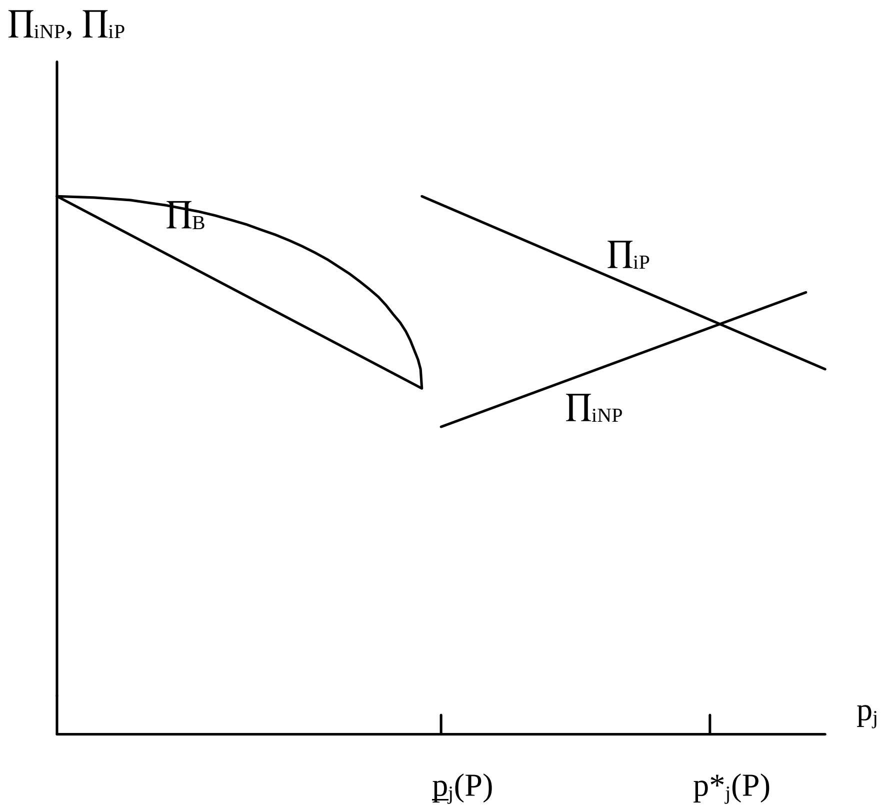

Figure 1: Firm i's maximum level of profit from preying and from not preying as a function of firm j's prices.

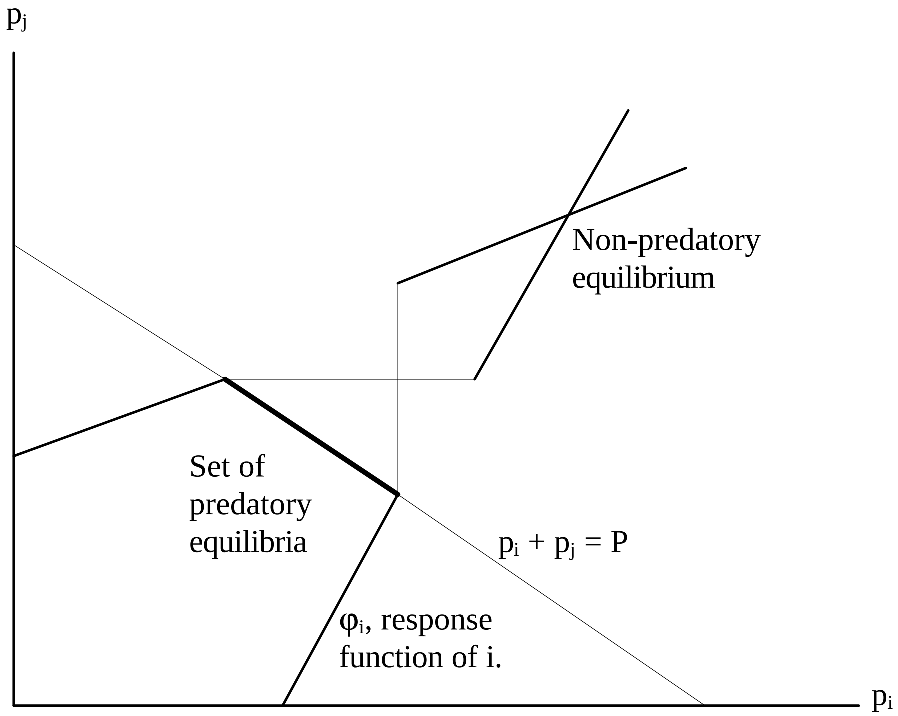

Figure 2: Firm i's best response function, φ_i, and the set of equilibria

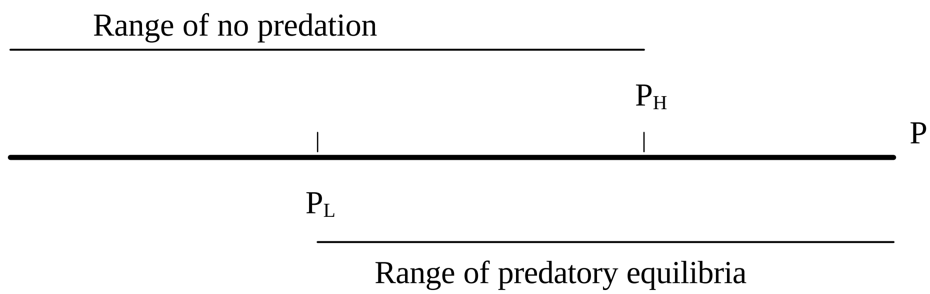

Figure 3: The set of predatory and non-predatory equilibria. Note we have both types coexisting when $P_L \leq P \leq P_H$.

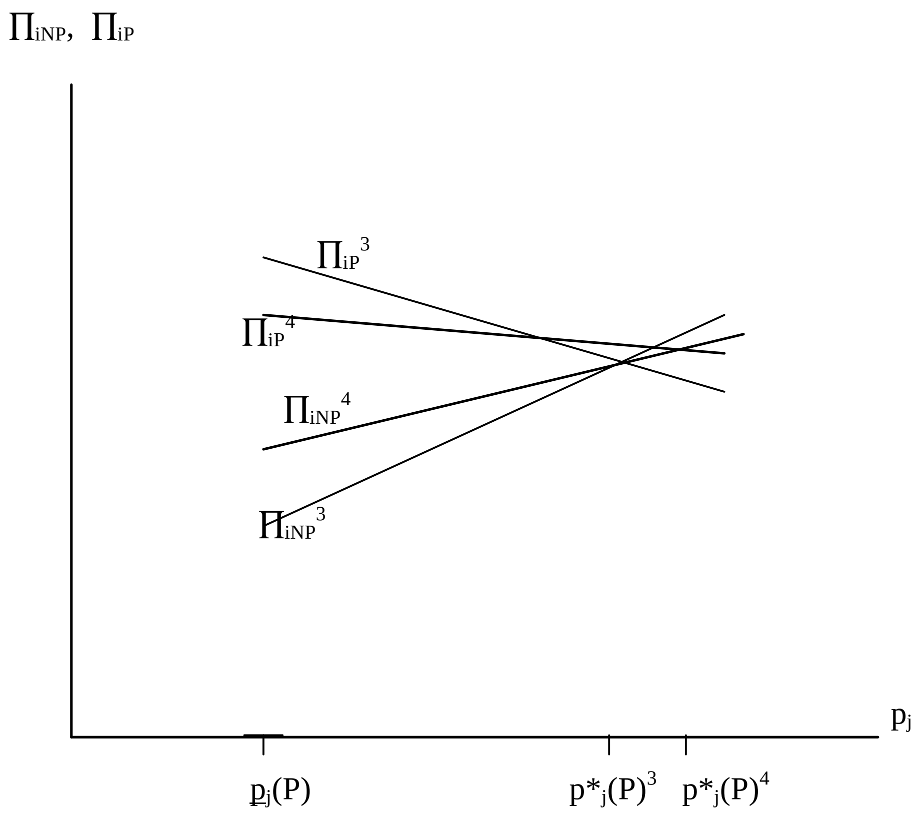

Figure 4: Maximum profits (with and without predation) with three firms and with four. Profits with three firms are represented by the dashed line. In this example, note that $p^*_j(P)$ is greater for four than for three.