How to Ration the Public Provision of Private Goods

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July 1997

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This paper considers alternative allocation mechanisms in the public provision of private goods for the purpose of redistribution. The benchmark policy is to ration by low quality. Better off households choose higher quality private over subsidised public provision. It is suboptimal to ration by queues where individuals suffer deadweight losses while waiting. In contrast, waiting lists where households consume on the private market while waiting are optimal. Under certain circumstances, there are qualifying restrictions to remaining on the waiting list.
3. Introduction

A growing literature has explored the role of publicly provided private goods in redistribution when household types are unobservable. Blackerby and Donaldson (1980), following Nichols and Zeckhauser (1982), argue that in-kind transfers can dominate cash redistribution when the good is valued by only one type of individual. Munro (1992) provides a general model showing how in-kind self-selection dominates cash transfers. Beasley and Coate (1995) argue that, in universal provision systems - such as the National Health Service in the UK - the choice of quality provided can lead to redistribution by self-selection in the take-up decision.

Given a universal provision system, aiming to target its take-up by self-selection, have to be of low quality? There are other methods of rationing public provision. Nichols, Smolensky and Tideman (1971) argue that, since individuals have different time costs, waiting time spent in a queue can be a desirable method of self-selection rationing. Individuals with low time cost - typically lower-income individuals - will join the queue for public provision. Lindsay and Faigenbaum (1988) develop a model of waiting lists. Here individuals do not waste time actually standing in a queue, but instead their demand may decay while waiting. They may not actually want the good by the time they reach the top of the waiting list.

In a related argument, Donaldson and Eaton (1981) emphasise how waiting time can lead to self-selection across individuals with different time discount rates.

In section 2, we present a model where the effects of the public provision of private goods, for the purposes of redistribution across a continuum of households, can be examined easily. We apply this to the public authority's choice of quality and subsidy of provision, in the absence of queuing or waiting lists. In section 3, we consider allocation by queues which involve actual time wasted while queuing. We show that this sort of deadweight loss is dominated by rationing by low quality of provision. Rather than rationing a high quality public swimming pool - for example - by queuing, the public authority should build a less attractive swimming pool. Only if quality is at the minimum possible level should queues be used. In section 4, we introduce waiting lists as in Lindsay and Faigenbaum (1988). An important issue with respect to waiting lists is what is happening while the individual is waiting. While waiting for public housing, individuals typically rent in the private sector. In this situation, it turns out that a waiting list is an optimal form of rationing. In section 5, we observe that housing authorities often use points systems - taking account of the household's current housing and housing needs - rather than a straightforward waiting list. Like queues, these additional devices are only optimal if the authority is offering the lowest possible quality of housing. Section 6 considers the example of health care and presents our conclusions.

2. Rationing by Low Quality

We consider the public provision of private goods for the purpose of redistribution. Our analysis applies generally to a broad class of goods, and we will in turn consider the examples of public swimming pools, housing and health services. To allow for self-selection mechanisms, there must be no resale and no ability to augment the publicly provided quality and quantity.

Although we have a continuum of types of households, we examine the provision of a single good. Write qR as the number of units of the good
actually consumed by household $i$. We will refer to $q$ as the "quality" of provision, although it encompasses both quality and quantity features. For housing, for example, $q$ includes measures of space (such as the number of bedrooms), facilities, location and standard of build and maintenance.

There is a private market for the good and, if household $i$ was to acquire its consumption of the good in the private sector at market prices, it would choose an optimal quality level $q_i^*$. Households have different optimal private sector consumption levels. They are distributed in the economy over some support $q_i \in [q_l, q_u]$ with the distribution function $F(q_i)$. We will refer to $q_i$ as the type of household. For the good to be a suitable candidate for public provision for the purposes of redistribution, $q_i$ needs to be correlated with household income or some other redistributive aim.

The basic policy measure open to the public authority in our model is to make available public provision of the good at a quality level $q$ and a subsidised charge. It is easier to think of the charge as being at the market level less a subsidy $s$. Were the household to consume a suboptimal level at market prices, its loss - relative to its optimal consumption $q_i^*$ - can be approximated by a quadratic function. A household consuming the public quality level $q$ rather than $q_i^*$, paying market prices but receiving a subsidy $s$ from the public authority, has a net gain of:

$$ G(q, s, q_i) = s - q(q - q_i^*)^2 $$

The household chooses to participate in the public provision scheme if $G(q, s, q) < 0$. From the quadratic loss function, participation is symmetric around $q$ and households in the range $[q - (s/a)^{1/2}, q + (s/a)^{1/2}]$ choose the subsidised public provision over their unsubsidised market consumption level.

We now consider the government's redistribution aims. We assume that the government wishes to increase the welfare of the different household types $q_i$ by at least the associated amounts in a redistribution function $T(q_i) > 0$, but to do this at the least total expenditure. On the assumption that the government wishes to provide greatest assistance to the least well off, the function $T(q_i)$ is decreasing. We also assume that $T(q_i)$ is a continuous, (weakly) convex function and there is a household $q_i^*$ that is sufficiently well off that the government does not wish to provide any assistance: $T(q_i^*) = 0$ for $q_i > q_i^*$.

If the government was able to observe each household's type, it could meet its redistribution objectives by giving each household the appropriate cash transfer $T(q_i)$. This full information solution achieves the redistribution aim at the least total cost. With unobservable types, the authority could give the same cash transfer $T(q_i)$ - the desired redistribution sum for the least well off household $q_i$ - to all households.

This gives too large a transfer to all but the least well off households. An alternative is to adopt an in-kind transfer programme of low quality provision, as in Banerjee and Coate (1992), with the aim of targeting participation.

To see how this can have no expenditure, suppose that the redistribution function takes on the form $T(q_i) = \ln(q_i)$ shown in Figure 1. Redistribution can be targeted by setting quality at the optimal level of the least well off households, $q_i^*$, and the subsidy level $s$ at the required redistribution level for those households, $T(q_i^*)$. The curve $q_i(q_i) = G(q_i, T(q_i), q_i)$ shows the gain from participation by households of different types. As seen from the figure, the policy meets the government's redistribution aims since $q_i(q_i) > T(q_i)$ for all households $q_i \geq q_i^*$.
Expenditure is saved through targeting since better-off households (with \( q_i > q_0 \) in the figure) decline to participate.

The redistribution function \( T(q) \) is steep since the government wishes to focus redistribution on the very worst-off households. As a result, the optimal policy is to target very low quality housing. If the redistribution function takes on a flatter form such as \( T(q) \) in Figure 1, public provision of quality \( q_0 \) at subsidy \( s = T(q) \) fails to meet all the redistribution aims. The government could maintain the low quality provision \( q_0 \) and raise the subsidy above \( T(q) \), or it could simultaneously raise the subsidy and the quality as shown in the curve \( q, q_0 \).

To determine the characteristics of the optimal policy, define the excess transfer function \( 
\begin{align*}
E(q, a, q_0) &= \Theta(q, a, q_0) - T(q). 
\end{align*}
\)
A policy is defined as feasible if it meets the transfer requirement \( E(.) \geq 0 \) for households \( i \) such that \( q_i \leq q_0 \). In fact, since \( E(.) \) is a strictly concave function (as the difference between the strictly concave \( \Theta(.) \) and the convex \( T(.) \), feasibility will hold provided \( E(.) \) is nonnegative at the endpoints \( q_0 \) and \( q_0 \). We show by contradiction that, if \( q_0 T(q) \) is not a feasible policy, then in any optimal policy, both \( E(q, a, q_0) = 0 \) and \( E(q, a, q_0) > 0 \). Suppose instead that \( E(q, a, q_0) > 0 \). Then a small lowering of \( a \) saves on expenditure, while maintaining the feasibility of the policy. If \( E(q, a, q_0) > 0 \) and \( E(q, a, q_0) > 0 \), then \( q \) can be raised by some small amount \( c \) allowing \( a \) to be lowered by an amount \( \delta \) such that \( E(q, a, q_0, c, \delta) = 0 \). If \( E(q, a, q_0) = 0 \) and \( E(q, a, q_0) > 0 \), then \( q \) can be lowered by some small amount \( e \) allowing \( a \) to be lowered by \( \lambda \) such that \( E(q, a, q_0, c, \delta) = 0 \).

The optimal policy has the characteristics of either \( q, q_0 \) or \( q_0, q_0 \) in Figure 1. If the desired transfer to the least well-off household is very large relative to other households, the authority adopts the corner solution policy at the lowest quality level \( q < q_0 \) and the subsidy \( s = T(q_0) \). If the redistribution function is flatter, the authority sets a higher quality level and a higher subsidy such that both \( E(q, a, q_0) = 0 \) and \( E(q, a, q_0) = 0 \). In either case, targeting is not perfect; all but the worst off and the best off participating households gain a higher transfer than desired by the authority. Further, households consume a different quality than they would in the market (some higher and some lower). The introduction of other allocation mechanisms - as examined in the remainder of the paper - may cut these deadweight losses.

3. Rationing by Queues

Nichols, Sorensen and Yiderman (1972) argue that queues can be an effective rationing device for publicly provided goods. Individuals could join a private pool club, or they could use the public pool, recognizing that they will often have to queue for entry on summer days. In our framework it is natural to ask whether rationing by queues is optimal, or whether the authority should lower the quality of provision by building a less attractive pool.

For a queue to target, it must be less costly for the poor than for the better off. Write \( c(q) \) as the per unit time cost of waiting in line for the households of different types. Assume that \( c(q) \) is convex increasing function. We argue that rationing by a queue is suboptimal unless its quality is already at the minimum level \( q_0 \).

Suppose that the public authority has a policy of providing the quality \( q > q_0 \) at a subsidy \( s \) with a queue of line length \( \lambda > 0 \). Write \( E(q, a, q_0, \lambda) = E(q, a, q) - \lambda c(q) \) as the excess transfer to households net of queueing costs, where \( E(.) \) is defined in the previous section. Using
the same argument as in the previous section (but now for a fixed value of \( \lambda \)), the policy can only be optimal if \( E[q_{t+1},s,q] = E[q_{t+1},u,q] = 0 \). Then, if \( \lambda \) is lowered by a small amount, \( q \) can fall by some amount \( \delta \) and still maintain \( E[q_{t+1},s,q-\delta,\lambda] = 0 \). But \( \delta E[q_{t+1},\lambda] / \delta q < 0 \) implies that \( E[q_{t+1},s,q-\delta,\lambda] > 0 \) and, from the strict convexity of \( E[q_{t+1},s,q-\delta,\lambda] \), \( E[q_{t+1},s,q-\delta,\lambda] > 0 \) for all households \( q_{t+1} < q \). But then we know that there is a less costly policy with a lower \( s \) meeting \( E[q_{t+1},s,q-\delta,\lambda] = E[q_{t+1},s,q] = 0 \) that also achieves the government's redistribution aim.

A queue introduces a deadweight loss that affects all households (although to differing degrees) and requires a compensating rise in the subsidy to achieve the same transfer as in the absence of a queue. It is better to target the less well off by lowering quality which - for the worst off households where the government is seeking to target the largest transfer - has no deadweight loss. It is only if quality is at the lowest level \( q_{t+1} \) that a queue can be optimal. The introduction of a queue of length \( \lambda \) requires that the subsidy rise by \( \lambda E[q_{t+1}] \) so that the worst off households are unaffected. But this increase in subsidy does not fully compensate better off participants and some of these drop out of public provision. This say outweighs the additional costs from the higher subsidy level paid to the remaining participants.

4. Waiting Lists

A waiting list as analyzed by Lindsey and Feigenbaum (1984) is different to a queue of the sort described in the previous section. A household on the waiting list is not spending actual time in a queue and therefore has no direct queuing costs. Instead, the household suffers a wait in receiving the public provision of the good and will therefore discount its value. Examples of waiting lists include those for health care in the UK National Health Service, and public housing in both the UK and the US. In this section we consider a repeated purchase good such as housing. In Section 6 we consider to what extent the analysis holds for a single purchase such as a medical operation.

We reformulate the model of Section 2 into continuous time. Households have an infinite horizon but discount at the rate \( \rho \). Government provision is offered on a waiting list system. A household puts down its name and waits until time \( W \) before receiving in perpetuity public provision at quality \( q \) and subsidy \( s \). During the waiting period, the household obtains its provision on the private market. In the absence of any restrictions, the household will acquire its optimal private consumption \( q_{t+1} \) until time \( W \). Lifetime discounted gain for a participating household is:

\[
(1 - \rho) E[q_{t+1},s,q,W] = e^{-\rho W} G[q_{t+1},s,q] / \rho
\]

It is straightforward to reinterpret the government's desired transfer function \( G[q_{t+1},s,q] \) as now representing a lifetime discounted transfer \( T[q_{t+1}]/\rho \). The government meets its redistribution objectives with a policy such that \( G[q_{t+1},s,q,W] = T[q_{t+1}]/\rho \) for \( q_{t+1} = q_{t+1} \). The optimal policy meets this at the minimum discounted cost to the government of the subsidies paid to participants. We assume that the government discounts at the same rate \( \rho \) as households.

How does a government policy with the same \( q \) and lifetime discounted subsidy, but with a wait time \( W > 0 \), differ from one without a waiting list? For the lifetime discounted subsidy to be the same, the

2 Lindsey and Feigenbaum observe that the household's tastes may change while waiting and claim that this decay factor represents an important difference in household discount rates. Since it is not clear whether better or worse-off households have a higher decay or other underlying reason for discounting, we assume that all households discount at the same rate.
instantaneous subsidy levels $a_b$ (associated with the waiting list) and $a$ (when $W = 0$) must meet $e^{-\alpha X_b} = a$. Then, from (1) and the definition of $Q(.)$ in (1), we have:

$$Q(q; s, q, w) = a/p - e^{-\alpha X_b}q - q^2/2$$

For a given lifetime discounted subsidy $a/p$, the waiting list scheme raises the lifetime discounted gain for all types of households other than $q^* > q$.

The effects are shown in Figure 2. The function $\Gamma(.)$ becomes flatter. Indeed, in the limit as $W = \infty$, all households gain $a/p$. The loss due to being in the wrong quality of housing (relative to the preferred market level) is indefinitely deferred, while the lifetime discounted subsidy $a/p$ remains the same.

Provided that the optimal policy in the absence of a waiting list has $q > q^*$. The introduction of a waiting list allows the government to lower the lifetime discounted subsidy it pays, while still meeting its redistribution objectives. The optimal no waiting list policy has - for the reasons discussed in previous sections - $Q(q^*; s, q, w) - \tau(q^*_b)/p - 0$.

Introducing a waiting list $W > 0$, while maintaining the same lifetime subsidy and $q$, we know from (1) that both $Q(q^*; s, q, w) - \tau(q^*_b)/p$ and $Q(q^*; s, q, w) - \tau(q^*_b)/p$ are strictly positive. But then the authority can lower the lifetime subsidy and save on expenditure.

5. Allocation by Points Systems

The Institute of Housing (1990) carried out a detailed survey about the methods of allocation of public housing by local authorities in England and Wales. They found that 21% of authorities used date order waiting lists of the sort described in the previous section. However, 72% used points schemes where the applicant’s priority depends upon the number of points assigned for housing need and circumstance attributes. Of the points schemes, 80% took time into account and 30% of authorities had minimum wait periods, the most common being 6 and 12 months. A specific example of a points scheme was that operated by the London Borough of Lambeth in 1995. Lambeth Households receive points depending upon a number of factors including the number of rooms in the current home, whether there are adequate kitchen and bathroom facilities, whether a family with young children or elderly is housed above the ground floor without a lift, whether the family is separated due to lack of accommodation, and the effects of medical condition on housing needs.

A points system is a way of operating a waiting list that advantages households that are currently in low quality housing. The simplest such scheme is one where joining the waiting list depends upon meeting qualifying restrictions. In the framework of the last section, such a scheme might require a wait $W$ where the waiting period must be spent in housing of quality no greater than $q_1$. The effects of this policy are shown in Figure 2. The qualifying restriction has no impact on households with $q^*_2 < q_1$, so the gains to these households remain on the curve $Q(q^*_b; s, q, w)$. However, the qualifying restriction lowers the gain for households with $q_1 < q^*_2$ as shown in the part of the curve $Q(.)$ since these households are not in their optimal market housing level during the waiting period.

Are these sorts of qualifying restrictions part of an optimal policy? If the optimal policy has $q > q^*_2$, then introducing qualifying restrictions

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3 This is in fact precisely the same $q$ and $a$ as for the single period problem in Section 2.

to a waiting list is suboptimal. The reason is the same as that for why
queues were suboptimal in Section 3. Rather than impose a deadweight loss
on the better off households participating in public provision, the
authority can lower the quality of provision. The lowering of quality of
provision - while having the same deterrent effect on better off households
- also has a direct positive effect on the least well off households.
While receiving the given public subsidy, the least well off households
obtain (and pay for) a quality of housing closer to their market level.

Suppose however that \( q_0 > q_k \) was the optimal policy in Section 2 and
- as in Figure 1 - this policy induced the participation of better off
households \( q_0 > q_k \). In this situation, the introduction of a waiting list
with qualifying restrictions is optimal. A waiting list of wait \( W \) and
qualifying restriction quality \( q_0 \) has no effect on the least well off
household. However, it lowers the gain to better off participating
households and induces some to drop out of public provision. The
government saves the subsidy it was paying to these undesired recipients.

More complicated points systems, such as the one operated in Lambeth,
have similar effects to a waiting list with qualifying restrictions. If
households seeking public provision and vacant houses arrive randomly, the
Lambeth system is equivalent to one where waiting times are stochastic and
a decreasing function of the household's points. Households with a large
number of points have a short waiting time, while those with low points
are unlikely to ever be housed. As a result, the gain to participation is
unaffected for the least well off household, but is diminished for better
off households.

f. Conclusions

The purpose of this paper has been to consider alternative ways of
allocating the public provision of private goods for the purposes of
redistribution. Self-selection by quality is better than rationing by
deadweight queues and by implication other forms of "ordnance" such as those
described by Nichols and Schleizer (1982). In contrast, rationing by a
waiting list where the household obtains the good on the private market
while waiting, can be desirable. Qualifying restrictions on the waiting
list are only optimal if the public quality of provision is at the minimum
possible level.

In practice, we often observe queues at publicly provided facilities
and, as observed in Section 5, authorities often adopt points systems or
qualifying restrictions for public provision. From our model, these
rationing devices are optimal if quality is at the minimum level \( q_k \).
Alternatively, the authorities may view many of the publicly provided goods
as merit goods and not wish to lower their quality below a certain level.
When that level is reached, queuing and qualifying restrictions become
optimal as further self-selection devices.

This line of argument may apply to the use of waiting lists for
public medical care. Nonurgent medical care (for example, hip
replacements) from the UK National Health Service is rationed by waiting
lists, as observed by Lindsay and Feigenbaum (1984). However, as argued by
Poppin (1995), patients awaiting treatment suffer pain and Incapacity.
Waiting lists for operations differ from those in Section 4 where
households consume in the private market while waiting and are more like
the queues in Section 3. Deadweight losses are suffered by both the least
well off and the better off households. For the reasons we have discussed,
policy should avoid such deadweight losses and would do better to return by
quality to the extent possible. This can be done in ways that do not
directly affect the adequacy of health care. An example of an appropriate
form of "low quality" in the National Health Service is the use of large wards rather than private rooms. It is only if there needs to be additional rationing that NHS waiting lists should be used.

REFERENCES


Institute of Housing, Housing Allocations, September 1990


Figure 1. Feasible optimal policies associated with different redistribution functions.

Figure 2. The introduction of a waiting list raises the gain for all types for a given lifetime discounted subsidy.