

Problem Set 3; 2nd Order conditions for Optimisation: Concavity and Optimisation

1. Work out whether the following functions are increasing, decreasing or stationary at $x = 4$

a) $f(x) = 3x^2 - 14x + 8$ b) $f(x) = x^3 - 7x^2 + 6x - 1$ c) $f(x) = x^4 - 6x^3 + 4x^2 - 12$

2. Are the following functions concave or convex at $x = 2$

a) $f(x) = -2x^3 + 4x^2 + 9x - 14$ b) $f(x) = (5x^2 - 8)^2$

3. In the lecture we saw the conditions on the Hessian for a function to be concave. Find and write down the equivalent conditions for a function to be convex when f has (i) two variables; (ii) three variables

4. Draw a function that is quasi-concave but not concave and explain why.

5. Is f concave, convex or neither?

i. $f(x_1, x_2) = x_1 - 2x_1^2 + 2x_2 - 3x_2^2$ ii. $f(x_1, x_2) = x_1 - 2x_1^2 + 2x_2 - 3x_2^2$

6. f is convex. Show the following.

i. $-f$ is concave ii. $2f$ is also convex

7. Which (if any) of these matrices are negative semi-definite?

i. $\begin{pmatrix} -4 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 2 & -1 \end{pmatrix}$ ii. $\begin{pmatrix} -3 & 1 & -1 \\ -2 & 0 & 2 \\ 4 & 0 & 0 \end{pmatrix}$

8. Find the maximum of the following functions. Remember to check the second order conditions.

(i) $z = 100 - x^2 - 5y^2$ (ii) $z = 20x - 2xy - y^2 - 3x^2$

Make sure you check the second order conditions.

Continued over

9. Total Cost = $2q_1^2 + q_1q_2 + 2q_2^2$
Find the values of q_1 and q_2 that minimise costs

1st order conditions

10. Total Cost = $3q_1^2 - q_1q_2 + 2q_2^2 - 4q_1 - 7q_2 + 12$

Find the values of q_1 and q_2 that minimise costs

11. Find the maximum of f where

i. $f(x_1, x_2) = x_1 - 2x_1^2 + 2x_2 - 3x_2^2$

ii. $f(x_1, x_2, x_3) = -5x_1^2 + 10x_1 + x_1x_3 - 2x_2^2 + 4x_2 + 2x_1x_3 - 4x_3^2$

iii. $f(x_1, x_2, x_3) = x_1 - 0.5x_1^2 + 6x_3x_2 - 3x_2^2 - x_3^2$

iv. $f(x_1, x_2, \lambda) = \ln(x_1) + \ln(x_2) + \lambda(100 - 2x_1 - 5x_2)$

(In each case check the second order conditions.)

Eigenvalues & Eigenvectors

Find the eigenvalues (characteristic roots) and hence the sign definiteness of

$$\text{i) } A = \begin{bmatrix} -4 & -2 \\ -2 & -6 \end{bmatrix}$$

$$\text{ii) } A = \begin{bmatrix} 6 & 2 \\ 2 & 2 \end{bmatrix}$$

Comparative statics and the implicit function theorem

1. What is the difference between a Hessian and a Jacobian matrix?

2. Given the implicit functions

$$\text{i) } 7x^2 - y = 0$$

$$\text{ii) } 3x^4 - 7y^5 - 86 = 0$$

Find dy/dx using the implicit function rule

3. Given an implicit utility function $(U, x_1, x_2) = 0 = U - x^{3/4} y^{1/4}$

Use the implicit function rule to find

i) marginal utility of x_1

ii) marginal utility of x_2

iii) the marginal rate of substitution (slope of the indifference curve)

4. Using the 3 equation national income model given in the lectures

Explicit

$$Y = C + I + G$$

$$C = a + b(Y-T)$$

$$T = g + dY$$

Implicit

$$Y - C - I - G = 0 = F^1$$

$$C - a - b(Y-T) = 0 = F^2$$

$$T - g - dY = 0 = F^3$$

Find $\delta Y / \delta I$