

EC 5555 Problem Set 2: Differentiation, Logs and Exponentials

Differentiation and Partial Differentiation

1. Differentiate the following from 1st principles

i) $2x$ ii) $x^2 + 3$

2. Differentiate the following functions with respect to x.

(i) $f(x) = 4x$ (ii) $f(x) = 5x + 5$ (iii) $f(x) = 1 + 2x + 4x^2$

(iv) $f(x) = 10x + 2/x$ (v) $f(x) = \sqrt{x}$ (vi) $f(x) = 3/x^3 + 30$

vii) Total Cost = $Q^3 - 3Q^2 + 9Q + 50$

3. Differentiate the following functions with respect to x

i) $y = 3x^4(2x - 5)$ ii) $y = (4x^2 - 3)(2x^5)$

iii) Average Revenue = $20 - 2Q$, find marginal revenue

iv) $y = \frac{10x^8 - 6x^7}{2x}$ v) $y = \frac{5x^2 - 9x + 8}{x^2 + 1}$

vi) Total Cost = $2Q^3 - 12Q^2 + 30Q$

Find Average Cost and the rate of change of the Average Cost Curve with respect to output, Q

4. Differentiate the following functions with respect to x

i) $y = (3x^4 + 5)^2$ ii) $y = -3(x^2 - 8x + 7)^3$

iii) Find the marginal product of labour and the marginal revenue product of labour if Output $Q = 2L^2$ and Price = $15 - Q$

5. Differentiate the function $z = f(xy) = x^2y$ partially with respect to x from first principles

6. Find the partial derivative with respect to x

(i) $z = 2x + y$ (ii) $z = xy + 4x^3 - 2y/x$ (iii) $z = x(x + y)^2$

(iv) $z = x^{0.5}y^{0.5}$ (v) $z = (\frac{1}{2})\ln(x) + (\frac{1}{2})\ln(y)$

vi) Given a utility function $U = U(x, y) = (x + 2)^2(y + 3)^2$

Find the marginal utility of x and of y

Logs and Exponentials

7. Evaluate and then sketch the following curves

- i) $y = \text{Ln } x$ ii) $y = e^x$
 iii) $y = e^{-x}$ iv) $y = -e^{-x}$

for $-2 \leq x \leq 4$

X	Ln x	e^x	e^{-x}	$-e^{-x}$
-2	n/a	.135	7.389	-7.389
-1	n/a	.368	2.718	-2.718
0	n/a	1	1	-1
1	0	2.718	.368	-.368
2	.693	7.389	.135	-.135
3	1.099	20.085	.049	-.049
4	1.386	54.598	.018	-.018

8. Rewrite these expressions in terms of $\log(x)$ and $\log(y)$

- (i) $\log(1/x)$ (ii) $\log(xy)$ (iii) $\log(x^3/y)$

9. Rewrite these expression in terms of e^x and e^y :

- (i) e^{-x} (ii) e^{x+y} (iii) e^{3y-x} (iv) e^{5x+2y}

10. Find the derivatives with respect to x:

- (i) e^{-x} (ii) $4e^{x-3}$ (iii) $3e^{-x^2} - x$ (iv) $x \log(4x)$ (v) $e^{\log x}$

11. What is the present value of £100 in 10 years time if interest rate is a) 10% b) 2%

12. What is the equivalent of £100 now in 10 years time if the interest rate is a) 10% b) 2% (and compounded continuously)

13. If the level of GDP grows according to $= 2^t$ find the rate of growth

14. If $Y=F(K,L)$ and $Y=10L^{.7}K^{.3}$

what is the elasticity of output with respect to labour?

15. Utility $U = U = 0.4x^2 + 0.6y^3$

Find the point where the marginal rate of substitution equals the ratio of marginal utilities

16. Find the point where the marginal rate of technical substitution equals the ratio of marginal products using the production function in question 14

17. What is:

- (i) $1!$ (ii) $5!$

18. Set out the second order Taylor's series for $\log(1-x)$ around $x_0 = 0$

19. Calculate a second order Taylor's approximation for $\log(1.5)$. How close is your answer to the true value?