

EC5555. Problem Set 1: Vectors & Matrices

Reading. Chiang and Wainwright, chapter 4

1. Write the following in vector form

- i. p_1 is £8, p_2 is £1.5.
- ii. p_1 is £4, p_2 is £0.5 and p_3 is £12
- iii. Anne buys four units of good 1, 6 units of good 2 and nine units of good 3.
- iv. The height of the building is z , the width is 12 and the depth is x

2. What are the dimensions of the following matrices and vectors?

i. $\begin{pmatrix} 1 & 0 \\ 4 & 0 \\ 2 & 1 \end{pmatrix}$ ii. $\begin{pmatrix} 3 & 6 & 1 \\ 2 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ iii. $\begin{pmatrix} 2 & 7 & 7 \\ 0 & x & 6 \end{pmatrix}$ iv. $(1 \ 0 \ 0 \ 2 \ 1)$

v. (1)

3. Plot the following vectors:

i. $x = (4 \ 0)$ ii. $y = (3 \ 1)$ iii. $z = (2 \ 2)$

4. Using the vectors from question 3, find and plot

$x + y$
 $x - y$
and
 $2z + y$

5. In each case find a where:

i. $x = (4 \ 0)$ and $a = 2$ ii. $x = (3 \ 1)$ and $a = 4$ iii. $x = (2 \ 2)$ and $a = b$
iv. $x = (4 \ 1)$ and $a = 0$.

6. Show that a , b , and c are not linearly independent

i. $a = (4 \ 0)$ $b = (2 \ 0)$ $c = (1 \ 1)$
ii. $a = (6 \ 0)$ $b = (3 \ 4)$ $c = (3 \ 2)$
iii. $a = (4 \ 1 \ 1)$ $b = (2 \ 3 \ 0)$ $c = (0 \ -5 \ 1)$

7. Show that a and b are linearly independent.

i. $a = (4 \ 0)$ $b = (1 \ 1)$
ii. $a = (4 \ 1 \ 1)$ $b = (2 \ 3 \ 0)$

8. Find a.b (the dot product) in each case

- i. $a = (4 \ 0)$ $b = (1 \ 1)$
- ii. $a = (4 \ 1 \ 1)$ $b = (2 \ 3 \ 0)$

9. You have three $1 \times n$ vectors, a b and c
Will the following two operations produce the same result?

- i. Find a.b then multiply c by the resulting scalar.
- ii. Find b.c then multiply a by the resulting scalar.

10. Consider the following matrices

$$\text{i. } A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 2 & 1 \end{pmatrix} \quad \text{ii. } B = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix} \quad \text{iii. } C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \end{pmatrix} \quad \text{iv. } D = (1 \ 0 \ 2 \ 2 \ 1)$$

Which pairs can you multiply?
(note: AB is not the same as BA etc. so you need to consider AB and BA separately).

11. Given

$$A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \end{pmatrix} \quad D = (1 \ 0 \ 2 \ 2 \ 1)$$

Find i. BA ii. BB' iii. CB and iv. AD

$$12. \text{ Let } A = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 \\ -2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 2 \\ 3 & 1 \end{pmatrix} \quad D = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

Find

- i. det. A
- ii. det. B.
- iii. det. A'
- iv. det. C.
- v. det. D

13. Find the trace of B, the trace of B' and the trace of A +B
What do you find?

14. Write these systems of equations in matrix form

- i. $x_1 + x_2 = 2$; $x_1 - 3x_2 = 4$
- ii. $x_1 + x_2 + x_3 = 1$; $x_1 - x_2 = 4$; $x_2 + x_3 = 7$.

Example:

$$3x_1 = 7; 2x_1 + 6x_2 = 0$$

becomes:

$$\begin{pmatrix} 3 & 0 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

15. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 4 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 4 & 0 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 10 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Find

i. det. A ii. det. B. iii. det. A' and iv. det. C

v. What do you conclude about the relationship between the determinant of a matrix and the determinant of the transpose of the same matrix?

vi. What is the the effect of swapping rows on the determinant of a matrix?

vii. the determinant of D?

16. Do the following have a unique solution?

$$\text{i. } \begin{pmatrix} 3 & 0 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad \text{ii. } \begin{pmatrix} 3 & 9 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \text{iii. } \begin{pmatrix} 1 & 1 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

17. Let

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 \\ -2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 2 \\ 3 & 1 \end{pmatrix}$$

Find the inverses of any non-singular matrices in (you may need to look back in your notes for the definition of non-singular)

18. Let

$$A = \begin{pmatrix} -4 & 10 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 6 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

What is the rank of A?

What is the rank of B?

19. Find the inverses of the following matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 4 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 4 & 0 \end{pmatrix}$$

20. Find the solution for x_1 , x_2 and x_3

$$\text{i. } \begin{pmatrix} 4 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \quad \text{ii. } \begin{pmatrix} 3 & 1 & -1 \\ 2 & 0 & 2 \\ 4 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad \text{iii. } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

21. Use Cramer's rule to find x_2 in the following system of equations

$$\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

22. Use Cramer's rule to find consumption (C) in i. and x_2 in ii.
What happens to consumption if exports (X) rise marginally?

$$\text{i. } \begin{pmatrix} 1 & -b & 0 & 0 \\ -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -m \\ 0 & 0 & 0 & -d \end{pmatrix} \begin{pmatrix} C \\ Y \\ M \\ I_1 \end{pmatrix} = \begin{pmatrix} a \\ X + G + I_0 \\ 0 \\ r \end{pmatrix} \quad \text{ii. } \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

23. Given a markov model of labour force dynamics

$$x_t = Mx_{t-1}$$

$$\text{where } x_0 = \begin{bmatrix} \text{employed}_0 \\ \text{unemployed}_0 \end{bmatrix} = \begin{bmatrix} 900 \\ 100 \end{bmatrix} \quad M_t = \begin{bmatrix} ee & ue \\ eu & uu \end{bmatrix} = \begin{bmatrix} .95 & .30 \\ .05 & .70 \end{bmatrix}$$

Find the 3rd period levels of employment and unemployment as the model converges towards its steady state values

In a steady state these flows should be equal. What are the implied flows for this 3rd order round?